Records Arithmetick:

OR,

THE GROUND OF ARTS;

TEACHING

The perfect work and practice of Arithmetick, both in whole Numbers and Fractions, after a more easie and exact forme then in former time hath been set forth: Made by M. *Robert Record*, D. in Physick.

Afterward, augmented by M. John Dee.

And since enlarged with a third part of Rules of Practice abridged into a briefer method then hitherto hath been published, with divers necessary Rules incident to the Trade of Merchandise: with Tables of the valuation of all Coyns, as they are currant at this present time,

By JOHN MELLIS.

And now diligently perused, corrected, illustrated, and inlarged; with an *Appendix* of figurative Numbers, and the extraction of their Roots, according to the method of *Christian Urstitius*: with Tables of Board and Timber measure; and new tables of Interest, after 10. 8. and 6. per 100; with the true value of Annuities to be bought or sould present, Respited or in Reversion: the first calculated by R. C. but corrected, and the latter diligently calculated by *Ro. Hartwell Philomathemat.*

Scientia non habet inimicum nisi ignorantem.

Fide ——— sed ——— Vide.

LONDON,

Printed by *James Flesher*, and are to be sold by *Joseph Cranford*, at the signe of the Gunn in St. *Pauls* Church-yard. 1662.

That which my friend hath well begun
For very love to common-weale,
Need not all whole to be new done,
But now increaſe I doe reveale.

Something herein I once redreſt,
And now againe for thy behoof,
Of zeale I doe, and at requeſt,
Both mend and adde, fit for all proof.

Of numbers uſe, the endleſs might,
No wit nor language can expreſs:
Apply and try both day and night,
And then this truth thou wilt confeſs.

J. Dee.

The Books verdict.

To pleaſe or diſpleaſe ſure I am,
But not of one ſort to every man:
To pleaſe the beſt ſort would I fain,
The froward diſpleaſe ſhall I certain;
Yet wiſh I well though not with hope,
All eares or mouthes to pleaſe or ſtop.

To the most mightie Prince,

Edward the 6th by the grace of God,

King of *England*, *France*, and *Ireland*, &c.

THe Excellency of mans nature being such, as it is by Gods divine favour (most mighty Prince) not onely created in highness of degree far above all other corporall things, but by perfection, reason, and search of wit, much approaching toward the image of God, as not onely the holy Scriptures doe testifie, but also those natural Philosophers, which exactly did consider the nature of man, and namely, the far reach and infinite compass of the words of the mind, were inforced to confess, that man scarcely was able to know himself. And if he would duely ponder the nature of himself, he would find it so strange, that it might seem to him a very miracle: And thereof sprang that saying; *Magnum miraculum est homo, maximum miraculum sapiens homo.* For undoubtedly, as man is one of the greatest meracles that ever God wrought, so a wise man is plainly the greatest.

And therefore was it that some did account the head of a man the greatest miracle in the world, because not onely of the strange workmanship that is in it, but much more of the efficacy of reason, wit, memory, imagination, & such other powers, and works of the mind, which can more easily conceive any thing in a manner then understand it self. Amongst all the creatures of God, it findeth none more difficult to be perceived then the same powers of it self; whereby it doth conceive and judge, as it may be well conjectured by the diversity of opinions, that the wisest Philosophers did utter touching the spirit of man, and the substance of it: whereof I now intend to make no rehearsal; but who so listeth to read.

thereof, may find it largly ſet forth, not onely in *Ariſtotle* his bookes *de Anima,* but alſo in *Galen* his booke called *Hiſtoria Philoſophica* : and againe in *Plutarch's* work, *De Philoſophorum placitis* , whoſe words are alſo repeated of *Euſebius* in the xv. booke, Τῆς εὐαγγελικῆς προπαρασκευῆς , unto whom I remit them that have deſired to underſtand intricate difficulty of knowing our own ſelves, as touching our beſt part, and that part whereby we deſerve to bear the name of men.

This matter ſeemed ſo obſcure and difficult in knowledge; that *Galen* who for his excellent wiſdome and judgment in naturall works, is called of many men, a Miracle in Nature, yet in ſearching the nature and ſubſtance of the ſpirit of man, he not onely confeſſeth himſelfe ignorant, but counteth it plain temerity to attempt to find it. So far above the hope of mans knowledge is that part whereby man doth know and judge of things. And although the ignorant ſort (which hate all things that they know not) doe little eſteem the profoundneſs of a mans ſpirit and reaſon , the chief power and faculty of it : yet as there is a kind of feare and obedience of unreaſonable beaſts unto man, by the working power of God, ſo is there in thoſe ſmall reaſoned perſons a certain kind of reverence toward wiſdome and reaſon, which they doe ſhew oftentimes, and by power of perſwaſion, are enforced to obey reaſon, will they nill they. And hereby it came to paſs, that the rudeneſs of the firſt age of man was brought unto ſome more civill trade as it is well declared by *Cicero,* in the beginning of his firſt book, *De Inventione Rhetorica,* where he ſaith thus, *Nam fuit quoddam tempus quum in agris homines paſſim beſtiarum more vagabantur, & ſibi victu ferino vitam propagabant, nec ratione animi quicquam, ſed pleraq; viribus corporis adminiſtrabant. Nondum divinæ religionis, non humanæ ratio colebatur. Nemo legitimas viderat nuptias, non certos quiſquam inſpexerat liberos ; non jus æquabile quid utilitatis haberet, acceperat : ita propter errorem atq; inſcitiam cæca ac temeraria dominatrix animi cupiditas, ad ſe explendum viribus corporis abutebatur pernicioſiſſimis ſatellitibus. Quo tempore quidam magnus videlicet vir & ſapiens, cognovit quæ materia eſſet, & quanta ad maximas res opportunitas in animis ineſſet hominum, ſi quis eam poſſit elicere & præcipiendo meliorem reddere. Qui diſperſos homines in agris & in tectis ſylveſtribus abditos, ratione quadam compulit in unum locum, & congregavit: & eos in unamquamque rem inducens utilem atque honeſtam, primo propter inſolentiam reclamantes, deinde propter rati-*

onem

onem atque orationem studiosius audientes, ex feris & immanibus, mites reddidit & mansuetos.

This long repetition of *Tullies* words will seem tedious to them that love but little, and care much less for the knowledge of reason, but unto your Majesty (I dare say) it is a delectable remembrance, and unto me it seemed so pleasant, that I could scarce stay my pen from writing all that mine eyes did so greedily read.

This sentence of *Cicero* am I loath to translate into English, partly for that unto your Majesty it needeth no translation, but especially knowing how far the grace of *Tullies* eloquence doth excell any English mans tongue, and much more exceedeth the baseness of my barbarous stile: yet for the fruit of my sentence, I had rather unto my meer English Country-men utter the rudeness of my translation, then to defraud them the benefit of so good a lesson, trusting they will so learne to love reason, that they will also gladly and greedily embrace all good Sciences, that may help to the just furniture of the same, when they consider that informed reason was the onely instrument, or at least the chiefest meanes to bring men into civill regiment, from barbarous manners, and beastly conditions. "For the time was (saith *Tully*) that men "wandred abroad in the fields up and down like beasts, and used "no better order in feeding then they: so that by reasons rule "they wrought nothing, but most of their doings did they atchive "by force of strength. At this time there was no just regard of "Riligion towards God nor of duty towards man. No man had "seen right use of marriage, neither did any man know their own "children from other; nor no man had felt the commodity of "just Lawes: so that through error and ignorance, willfull lust, "like a blind and heady ruler, abused bodily strength as a most "mortall minister for the satisfying of his desire. At that time "was there one which not onely in power, but also in wis-"dome was great, and he considered how that in the mind of "men was both apt instrumenas, and great occasion to the due "accomplishment of most weighty affaires, if a man could apply "them to use, and by teaching of rules frame them to better trade. "This man with perswasion of reason gathered into one place "the people that were wandring about the fields, and lay lurk-"ing in wild cottages, and woods, and bringing them in one com-"mon society, did trade them to all such things, as either were "profitable or honest, although not without repining at the first,

" by reason then they had not been so accustomed before: Yet at
" length through reason and perswasion of words they obeyed
" him more diligently, and so of a wild and cruell people he made
" them courteous and gentle.

Thus hath *Tully* set forth the efficacy of reason and perswasion, how it was able to convert wild people to a mildness, and to change their furious cruelness into gentle courtesie; were it not now a great reproach in this our time (when knowledge reigneth so large) that men should shew themselves less obsequious to reason? Unless it may be thought that now every man having sufficient knowledge of himself, needeth not to hearken to the perswasion of others.

Indeed he that thinketh himself wise, will not esteeme the reason of any other, be he never so wise; so that of such a one it may well be said: He that thinketh himself wiser then he is, may justly be counted a double fool. Wherefore such men are not be permitted in open audience to talke, but must be put to silence, and be made to give ear to reason; which reason consisteth not in the multitude of words heaped rashly together, and applied for one purpose, but reason is the expressing of a just matter with witty perswasions, furnished with learned knowledg; such knowledg had *Moses*, being expert in all learning of the *Ægyptians*, as the Scriptures declare, and therefore was able to perswade the stubborn people of the Jewes, although not without paine. Such knowledg, and such reasons did *Druys* shew which was the first Lawmaker of all the west part of *Europe*. Like reason and wisdome did *Xamolxis* amongst the Goths. *Licurgus* unto the Lacedemonians. *Zeleucus* to the Locrians. *Solon* to the Athenienses, and *Dunwallo Molmutius* two thousand yeares past amongst the old Brittains of this Realm. And hereby came it to pass, that their Laws continued long till more perfect reason altered many of them, and wilfull power oppressed most of them.

Druys was son to King *Sarron*, and suceeded him in his kingdome.

At the beginning when these wise men perceived how hard it was to bring the rude people to understand reason, they judged the best meanes to attain this honest purpose, to depend of learning in every kind: for by learning (as *Ovid* saith) *Pectora mitescunt, asperitásque fugit*; Stout stomachs doe wax mild, and sharp fierceness is exiled. Therefore as *Berosus* doth testifie, *Sarron* that was the third King over all this west part of *Europe*, for to bring the people from beastly rage to manly reason, did erect schooles of liberall arts, which tooke such good success, that his name

continued

continued in that ſort famous above two thouſand years after: for *Diodorus Siculus* which was in the time of *Julius Cæſar*, maketh mention of the learned men of Goths or Celtes, and nameth them *Sarronides*, that is to ſay, *Sarron his Scholars and followers.*

Amongſt theſe Arts that then were taught, ſome did informe the tongue, and make them able both to utter aptly their mind, and alſo to perſwade; as Grammar, Logick, and Rhetorick, although not ſo curiouſly as in this time; ſome other did appertain to the juſt order of partition of Lands, the true uſing of Weights, Meaſures, and reckonings of all ſorts of bargaines, and for order of building and ſundry other uſes; thoſe were Arithmetick and Geometry. Again, to encourage men to the honour of God, they taught Aſtronomy, whereby the wonderfull works of God were ſo manifeſtly ſet forth, that no mans tongue, nor pen can in like ſort expreſs his infinite Power, his unſpeakable wiſdome, and his exceeding goodneſs toward man, whereby he doth bountifully provide for man all neceſſaries, not onely to live, but alſo to live pleaſantly. And ſo was their confidence in Gods providence ſtrongly ſtayed, knowing his goodneſs to be ſuch, that he would help man as he could, & his power to be ſo great, that he would do nothing but that that was beſt. Beſides theſe Sciences they taught alſo Muſick, which moſt commonly they did apply partly to religious ſervices, to draw men to delight therein, and partly to ſongs made of the manners of men, in praiſe of vertue and diſcommendation of vice, whereby it came to paſs, that no man would diſpleaſe them, nor doe any thing evill that may come to their hearing: for their ſongs made evill men more abhorred in that time then any excōmunication doth in this time. The poſterity of theſe Muſitians continue yet both in Wales and Ireland, called *Bardes* unto this day, by the ancient name of *Bardus* the firſt Founder.

This *Bardus Druydius* the 5. King of the Celtes, reigned 60. yeares, and died 1832 yeares before Chriſt.

And as theſe Sciences did encreaſe, ſo did vertue encreaſe thereby. Againe as thoſe Sciences did decay, ſo vertue loſt her eſtimation, and conſequently was little in uſe: whereof to make a full declaration were a thing meet for a Prince for to heare, but it would require a peculiar Treatiſe. Wherefore at this preſent I count it ſufficient, lightly to have touched this matter in generall words, and to ſay no more of the particularity thereof, but onely touching one of thoſe Sciences, that is Arithmetick, by which not onely juſt partition of Lands was made, but alſo touching buying and ſelling, all Aſſiſes, Weights, and Meaſures were deviſed

ed, and all recknonings and accounts driven; yea by proportion of it were the true orders of justice limited, as *Aristotle* in his Ethicks doth declare, and the degrees of estates in the common-wealth establiſhed; although that proportion be called Geometricall and not Arithmeticall, yet doth that proportion appertain to the art of Arithmetick, and in Arithmetick is taught the progreſſion of ſuch proportions, and all things thereto belonging. Wherefore I may well ſay, that ſeeing Arithmetick is ſo many waies needfull unto the firſt planting of a common-wealth, it muſt needs be as much required to the preſervation of it alſo: for by the ſame meanes is any common-wealth continued, by which it was erected and eſtabliſhed. And if I ſhall in ſmall matters in appearance, but indeed very weighty, put one example or two. What ſhall we ſay for the Statutes of this Realm, which be the onely ſtay of good order, in manner, now? As touching the meaſuring of ground by lengh and breadth, there is a good and an antient Statute made by art of Arithmetick; and now it ſhall be to little uſe, if by the ſame art it be not practiſed and tried, For the aſſiſe of bread and drink, the two moſt common and moſt neceſſary things for ſuſtentation of man, there was a goodly ordinance in the Law made, which by ignorance hath ſo grown out of knowledge, and uſe, that few men do underſtand it, and therefore the Statute books wonderfully corrupted, and the Commons cruelly oppreſſed: notwithſtanding ſome men have written that it is too doubtfull a matter to execute thoſe aſſiſes by thoſe Statutes, by reaſon they depend of the ſtandard of the coyne, which is much chang'd from the ſtate of that time, when thoſe Statutes were made. Thus ſhall every man read (that liſteth) in the Abidgement of the Statutes, in the title of Weights and Meaſures, in the ſeventh number of the Engliſh Booke, where he ſhould have tranſlated a good ordinance which is ſet forth in the French Booke: but no marvell if the Abridgment doth omit it ſeeing the great Booke of Statutes doth omit the ſame Statute, as it hath done many other very good Lawes. And this is the fruit of ignorance to reject and condemn all that it underſtandeth not, although they uſe ſome cloaks for it, but ſuch cloaks as being allowed, might ſerve to repell all good Laws; which God forbid.

Again, there is an ancient order for aſſiſe for fire-wood and coals, which was renued not many yeares paſt; and now how avarice and ignorance doth canvace that Statute, it is too pitifull to talk of, and more miſerable to feel.

Further

Furthermore, for the Statute of Coynage, and the ſtandard thereof, if the people underſtood rightly the Statute, they ſhould not nor would (as they often doe) gather an excuſe for their folly thereby : but as I ſaid, theſe Statutes by wiſdome and good knowledg of Arithmetick were made, & by the ſame muſt they be continued. And let not ignorance no more meddle with the uſe of them, then it did with the making of them. Oh in how miſerable caſe is that Realm, where the miniſters and interpreters of the Law are deſtitute of all good Sciences, which be the keys of the Lawes! How can they either make good lawes, or maintain them that lack the true knowledge whereby to judge them ? And happy may that Realm be accounted, where the Prince himſelfe is ſtudious of learning, and deſireth to underſtand equity in all Lawes. Therefore moſt happy are we the loving Subjects of your Majeſty, which may ſee in your Highneſs not onely ſuch towardneſs, but alſo ſuch knowledge of divers arts, as ſeldome hath been ſeen in any Prince of ſuch yeares, whereby we are enforced to conceive this hope certainly, that he which in thoſe yeares ſeeketh knowledg when knowledge is leaſt eſteemed, and of ſuch an age can diſcern them to be enemies both to his royall Perſon and to his Realm, which labour to withdraw him from knowledge to exceſſive paſtime, and from reaſonable ſtudy to idle or noyſome pleaſures, he muſt needs when he cometh to more mature yeares, be a moſt prudent Prince, a moſt juſt Governor, and a right judge, not onely of his ſubjects commonly, but alſo of the miniſters of his Lawes, yea, and of the Lawes themſelves : and to be able to conceive the true equity and exact underſtanding of all his Laws and Statutes, to the comfort of his good ſubjects, and the confuſion and reproach of them which labour to obſcure or prevent the equity of the ſame Lawes and Statutes. How ſome of theſe Statutes may be applyed to uſe, as well in this our time, as in any other time, I have peculiarly declared in this Book, and ſome other I have omitted for juſt conſiderations, till I may offer them firſt unto your Majeſty to weigh them as to your Highneſs ſhall ſeem good : for many things in them are not to be publiſhed without your Highneſs knowledge and approbation ; namely, becauſe in them is declared all the rates of allayes for all ſtandards from one ounce upward, with other myſteries of Mint matters, & alſo moſt part of the varieties of coyns that have been currant in this your Majeſties Realm by the ſpace almoſt of ſix hundred yeares laſt paſt, and many of them that were currant in the time that the Romans ruled here.

All which with the ancient deſcription of England and Ireland, and my ſimple cenſure of the ſame, I have almoſt compleated to be exhibited to your Highneſs: In the meane ſeaſon moſt humbly beſeeching your Majeſty to accept this ſimple Treatiſe, not worthy to be preſented to ſo high a Prince, but that my lowly requeſt to your Majeſty is, that this amongſt other of my Books may paſs under the protection of your Highneſs, whom I beſeech God moſt earneſtly and dayly, according to my duty, to advance in all honour, and Princely Regality, and to encreaſe in all knowledg, juſtice and godly policy, Amen.

Your Majeſties moſt obedient
ſubject and ſervant,

Robert Record.

To

To the loving READERS

The Preface of Mr Robert Record.

SOre oft times have I lamented with my ſelf the unfortunate condition of England, ſeeing ſo many great Clerks to ariſe in ſundry other parts of the world, and ſo few to appeare in this our Nation: whereas for pregnancy of naturall wit (I think) few Nations doe excell Engliſhmen: But I cannot impute the cauſe to any other thing, then to the contempt, or miſregard of learning. For as Engliſhmen are inferiour to no men in mother wit, ſo they paſs all men in vain pleaſures, to which they may attain with great pain and labour: and are ſlack to any never ſo great commodity, if there hang of it any painfull ſtudy or travelſome labour.

Howbeit, yet all men are not of that ſort, though the moſt part be, the more pity it is, but of them that are ſo glad, not onely with painfull ſtudy, and ſtudious paine to attain learning, but alſo with as great ſtudy and pain to communicate their learning to others, & make all England (if it might be) partakers of the ſame; the moſt part are ſuch that unneath they can ſupport their own neceſſary charges, ſo that they are not able to beare any charges, in doing of that good that elſe they deſire to do.

But a greater cauſe of lamentation is this, that when learned men have taken paines to do things for the aid

of

of the unlearned, ſcarce they ſhall be allowed for their well doing, but derided and ſcorned, and ſo utterly diſcouraged to take in hand any like enterpriſe againe. So that if any be found (as here are ſome) that doe favour learning and learned wits, and can be contented to further knowledge, yea onely with their word; ſuch perſons though they be rare, yet ſhall they encourage learned men to enterpriſe ſomething at the leaſt that England may rejoyce of. And I have good hope that England will (after ſhe hath taken ſome ſure taſte of learning) not onely bring forth more favourers of it, but alſo ſuch learned men, that ſhe ſhall be able to compare with any Realm in the world. But in the mean ſeaſon where ſo few regarders of learning are, how greatly they are to be eſteemed that doe favour and further it, my pen will not ſuffice at full to declare.

Therefore, gentle Reader, wheras I doe upon moſt juſt occaſion judge, yea and know aſſuredly, that there be ſome men in this Realm, which both love and alſo much deſire to further good learning, and am not well able to write their condign praiſe for the ſame, I think it better with ſilence to overpaſſe it, then either ſay too little of it, or provoke againſt them the malice of ſuch other, which doe nothing themſelves that is praiſe-worthy, and therefore cannot abide to heare the praiſe of any other mans good indeed.

And conſidering their great favour unto learning, though I my ſelfe be not worthy to be reckoned in the number of great learned men, yet am I bold to put my ſelfe in Preſſe, with ſuch ability as God hath lent me, though not with ſo great cunning as many men, yet with as great affection as any man to help my country men, and will not ceaſe dayly, (as much as my ſmall ability

will

will ſuffer me) to endite ſome ſuch thing, that ſhall be to the inſtruction, though not of learned men, yet at the leaſt of the vulgar ſort, whoſe argument alwaies ſhall be ſuch as it ſhall delight all learned wits though they doe not learne any great thing out of it.

But to ſpeak of this preſent booke of Arithmetick, I dare not, nor will not ſet it forth with any words, but remit it to the judgment of all gentle Readers, and namely ſuch as love good learning, beſeeching them ſo to eſteem it, as it doth ſeem worthy. And ſo either to accept the thing for it ſelfe, either at the leaſt to allow my good endeavour. But I perceive I need not uſe any perſwaſions unto them, whoſe gentle nature and favourable mind is ready to receive thankfully and interpret to the beſt of all ſuch enterpriſes attempted for ſo good an end, though the thing doe not alwaies ſatisfie mens expectation. This conſidered, did bolden me to publiſh abroad this little booke of the Art of numbring, which if you ſhall receive favourably, you ſhall encourage me to gratifie you hereafter with ſome greater thing.

And as I judg ſome men of ſo loving a mind to their native Countrey, that they would much rejoyce to ſee it proſper in good learning, and witty Arts: ſo I hope well of all the reſt of Engliſhmen, that they will not be unmindfull of his due praiſe, by whoſe meanes they are helped and further'd in any thing. Neither ought they to eſteem this thing of ſo little value, as many men of little diſcretion often times doe. For who ſo ſetteth ſmall price by the witty device and knowledge of numbering, he little conſidereth it to be the chief point, (in manner) whereby men differ from all brute beaſts: for as in all other things (almoſt) beaſts are partakers with us, ſo

so in numbring we differ clean from them, and in manner peculiarly, sith that in many things they excell us again.

The Fox in crafty wit exceedeth most men,
A Dog in smelling hath no man his peer.
To foresight of weather if you look then,
Many beasts excell men; this is cleer.
The wittiness of Elephants doth letters attaine,
But what cunning doth there in the Bee remain?
The Emmat foreseeing the hardness of winter,
Provideth victuals in the time of summer.
The Nightingale, the Linet, the Thrush, the Larke,
In Musicall harmony pass many a Clark.
The Hedghog of Astronomy seemeth to know,
And stoppeth his cave where the wind will blow.
The Spider in weaving such art doth show
No man can him mend, nor follow I trow,
When a house will fall, the Mice right quick
Flee thence before; can man doe the like?

Many things else of the wittiness of Beasts and Birds might I here say, save that another time of them I intend to write, wherein they excell in manner all men as it is dayly seen: but in number was there never beast found so cunning, that could know or discerne one thing from many, by dayly experience you many well consider, when a Bitch hath many whelps, or a Hen many chickens: and likewise of other what soever they be, take from them all their young saving onely one, and you shall perceive plainly that they miss none, though they will resist you in taking them away, and will seek them again if they may know where they be; but else they will never

never misse them truly; but take away that one that is left, and then will they cry and complain; and restore to them that one, then are they pleased again. So that of number, this may I justly say, it is the onely thing almost that separateth man from beasts. He therefore that shall contemn number, declareth himselfe as brutish as a beast, and unworthy to be counted in the Fellowship of men. But I trust there is no man so foul overseen, though many right smally doe it regard.

Therefore will I now stay to write against such, and returne again to this my Book, which I have written in the forme of a Dialogue, because I judge that to be the easiest way of instruction, when the Scholar may aske every doubt orderly, and the master may answer to his question plainly. Why the Author wrote in Dialogue wise.

Howbeit I think not the contrary, but as it is easier to make another mans work then to make the like; so there will be some that will find fault because I writ in a Dialogue: but as I conjecture those shall be such as do not, cannot, or will not perceive the reason of right teaching, and therefore are unmeet to be answered unto, for such men with no reason well be satisfied.

And if any man object, that other Books have been written of Arithmetick *already so sufficiently that I needed not now to put pen to the book, except I will condemn other mens writings: To them I answer, that as I condemn no mans diligence, so I know that no one man can satisfie every man: and therefore like as many doe esteeme greatly other books, so I doubt not but some will like this my book above any other* English Arithmetick *hitherto written; and namely such as shall lack instructors for whose sake I have so plainly set forth the Examples, as no book that I have seen hath done hitherto*

thereto: which thing shal be great ease to the Rude Readers.

Therefore (gentle Reader) though this Booke can be but small aid to the learned sort, yet unto the simple ignorant (which needeth most help) it may be a good furtherance and mean unto knowledge.

And though unto the King His Majesty privately I doe it dedicate, yet I doubt not (such is his clemency) but that he can be content, yea, and much desirous, that all his loving subjects shall take the use of it, and imploy the same to their most profit. Which thing if I perceive that they thankfully doe, and receive with as good will as it was written, then will I shortly with no lesse kindness set forth such introductions into Geometry *and* Cosmography, *as I have at times promised, and as hitherto in* English *hath not been enterprised, wherewith I dare say all honest hearts will be pleased, and all studious wits greatly delighted.*

I will say no more, but let every man judge as he shall see cause. And thus for this time I will stay my Pen, committing you all to that true fountain of perfect number, which wrought the whole world by number and measure: He is Trinity in Unity, and Glory, Amen.

Here

Here followeth a Table of the whole *Contents of this Book.*

The Contents of the first Dialogue.

The second Dialogue containeth

The Contents of the second part touching Fractions.

The Contents of the third part.

The

A Col-

A Collection of ſuch Tables as are *Contained in this Treatiſe.*

The Contents.

Before the Introduction of Arithmetick it were very good to have ſome underſtanding and knowledge of theſe Figures and Notes.

j	1	*one*	xx	20	*twenty*
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&c.	&c.	*&c.*	&c.	&c.	*&c.*

A

A Dialogue between the Master and the Scholar: teaching the *Art* and use of *Arithmetick* with *Pen.*

The Scholar speaketh.

SIR, such is your authority in mine estimation, that I am content to consent to your saying, and to receive it as truth, though I see none other reason that doth lead me thereunto: whereas else in mine owne conceit it appeareth but vaine, to bestow any time privately in learning of that thing that every Childe may and doth learne at all times and houres, when he doth any thing himself alone, and much more when he talketh or reasoneth with others.

M. Lo, this is the fashion and chance of all them that seek to defend their blind ignorance, that when they think they have made strong reason for themselves, then have they proved quite contrary. For if numbering be so common (as you grant it to be) that no man can doe any thing alone, and much less talk or bargain with other, but he shall still have to doe with number: this proveth not number to be contemptible and vile, but rather right excellent

lent and of high reputation, sith it is the ground of all mens affaires, in that without it no tale can be told, no communication without it can be continued, no bargaining without it can duly be ended, or no business that man hath justly compleated. These commodities if there were none other, are sufficient to approve the worthiness of number. But there are other innumerable, far passing all these, which declare number to exceed all praise. Wherefore in all great works are Clerks so much desired? Wherefore are Auditors so richly fed? What causeth Geometricians so highly to be inhaunced? Why are Astronomers so greatly advanced? Because that by number such things they finde, which else would far excell mans minde.

Scholar. Verily, Sir, if it be so, that these men by numbring, their cunning do attain, at whose great works most men doe wonder, then I see well I was much deceived, and numbring is a more cunning thing then I took it to be.

Master. If number were so vile a thing as you did esteem it, then need it not to be used so much in mens communication. Exclude number, and answer to this question: How many yeares old are you?

Scholar. Mum.

Master. How many dayes in a weeke? How many weekes in a year? What lands hath your Father? How many men doth hee keep? How long is it since you came from him to me?

Scholar. Mum.

Master. So that if number want, you answer all by mummes: How many miles to London?

Scholar.

Scholar. A poak full of plums.

Master. Why, thus you may see, what rule number beareth, and that if number be lacking it maketh men dumb, so that to most questions they must answer mum.

Scholar. This is the cause sir, that I judged it so vile, because it is so common in talking every while: Nor plenty is not dainty, as the common saying is.

Master. No, nor store is no sore, perceive you this? The more common that the thing is being needfully required, the better is the thing, and the more to be desired. But in numbring as some of it is light and plain, so the most part is difficult and not easie to attain. The easier part serveth all men in common, and the other requireth some learning. Wherefore as without numbring a man can doe almost nothing, so with the help of it you may attaine to all things.

Scholar. Yea sir, why then it were best to learn the Art of numbring, first of all other learning, & then a man need learn no more if all other come with it.

M. Nay, not so: but if it be first learned, then shall a man be able (I mean) to learn, perceive & attain to other Sciences; which without it he could never get.

Scholar. I perceive by your former words, that Astronomy and Geometry depend much on the help of numbring: but that other Sciences, as Musick, Physick, Law, Grammar, and such like, have any help of Arithmetick, I perceive not.

Master. I may perceive your great Clerkliness by the ordering of your Sciences: but I will let that pass now, because it toucheth not the matter that

I

I intend, and I will shew you how Arithmetick doth profit in all these, somewhat grosly, according to your small understanding; omitting other reasons more substantiall.

Musick. First (as you reckon them) musick hath not only great help of Arithmetick, but is made, and hath its perfectness of it: for all musick standeth by number and proportion: And in Physick, beside the calculation of criticall dayes, with other things which I omit, how can any man judge the pulse rightly, that is ignorant of the proportion of numbers.

Physick.

Law. And as for the Law, it is plain, that the man that is ignorant of Arithmetick, is neither meet to be a Judge, neither an Advocate, nor yet a Proctor. For how can he well understand another mans cause, appertaining to distribution of goods, or other debts, or of summes of money, if he be ignorant of Arithmetick? This oftentimes causeth right to be hindred, when the Judge either delighteth not to heare of a matter that he perceiveth not, or cannot judge for lack of understanding: this commeth by ignorance of Arithmetick.

Grammar. Now, as for Grammar, me thinketh you should not doubt in what it needeth number, sith you have learned that Nouns of all sorts, pronouns, Verbs and Participles are distinct diversly by numbers: besides the variety of Nouns of Number, and Adverbs. And if you take away number from Grammar, then is all the quantity of Syllables lost. And many other wayes doth number help Grammar. Whereby were all kinds of Meeters found and made? was it not by number?

But

But how needfull Arithmeticke is to all parts of Philosophy, they may soon see, that doe reade either Aristotle, Plato, or any other Philosophers writings. For all their examples almost, and their probations depend of Arithmetick. It is the saying of Aristotle, that he that is ignorant of Arithmetick, is meet for no Science. And Plato his Master wrote a little sentence over his School-house door, Let none enter in hither (quoth he) that is ignorant of Geometry. Seeing he would have all his Scholars expert in Geometry, much rather he would have the same in Arithmetick, without which Geometry cannot stand. Philosophy.

And how needfull Arithmetick is to Divinity, it appeareth, seeing so many Doctors gather so great mysteries out of number, and so much do write of it. And if I should go about to write all the commodities of Arithmetick in civill acts, as in governance of Common-weales in time of peace, and in due provision and order of Armies in time of war for numbring of the Host, summing of their wages, provision of Victuals, viewing of Artillerie, with other Armour; beside the cunningest point of all for casting of ground, for encamping of men, with such other like: And how may wayes also Arithmetick is conducible for all private Weales, of Lords and all Possessioners, of Merchants, and all other occupiers, and generally for all estates of men, besides Auditors, Treasurers, Receivers, Stewards, Bailiffes, and such like, whose Offices without Arithmetick are nothing: If I should (I say) particularly repeat all such commodities of the noble Science of Arithmetick, it were enough Divinity. Armie.

enough to make a verie great book.

Scholar. No, no, sir, you shall not need: For I doubt not but this, that you have said, were enough to perswade any man to think this Art to be right excellent and good: and so necessary for man, that (as I think now) so much as a man lacketh of it, so much he lacketh of his sense and wit.

Master. What, are you so farre changed since, by hearing these few commodities in generall? by likelihood you would be farre changed if you knew all the particular Commodities.

Scholar. I beseech you Sir, reserve those Commodities that rest yet behinde unto their place more convenient: and if ye will be so good as to utter at this time this excellent treasure, so that I may be somewhat inriched thereby, if ever I shall be able, I will requite your pain.

Master. I am very glad of your request, and will doe it speedily, sith that to learn it you be so ready.

The duty of a Scholar.

Scholar. And I to your authority my wit doe subdue, whatsoever you say, I take it for true.

Master. That is too much, and meet for no man to be believed in all things, without shewing of reason. Though I might of my Scholar some credence require, yet except I shew reason, I doe it not desire. But now sith you are so earnestly set this Art to attaine, best it is to omit no time, lest some other passion coole this great heat, and then you leave off before you see the end.

Perseverance in study.

Scholar. Though many there be so unconstant of mind, that flitter and turn with every winde, which often begin, and never come to the end, I am

none

none of this sort as I trust you partly know. For by my good will what I once begin, till I have it fully ended, I would never blin.

Master. So have I found you hitherto indeed, and I trust you will increase, rather then goe backe. For better it were never to assay, then to shrinke and fly in the mid way: But I trust you will not doe so; therefore tell me briefly: What call you the Science that you desire so greatly.

Scholar. Why sir, you know.

Master. That maketh no matter, I would heare whether you know, and therefore I aske you. For great rebuke it were to have studied a Science, and yet cannot tell how it is named.

Scholar. Some call it Arsemetrick, and some Augrime.

Master. And what doe these names betoken?

Sch. That if it please you, of you would I learn.

Master. Both names are corruptly written: Arsemetrick for Arithmetick, as the Greeks call it, and Augrime for Algorisme, as the Arabians found it: which doth betoken the Science of Numbring: for Arithmos in Greeke is called Number: and of it commeth Arithmetick, the Art of Numbering: So that Arithmeticke is a Science or Art teaching the manner and use of Numbering: This Art may be wrought diversly, with Pen, or with Counters. But I will first shew you the working with the pen, and then the other in order.

Αριθμητική.

Scholar. This will I remember. But how many things are to be learned to attain this Art fully?

Master. There are reckoned commonly seven parts or works of it.

Numeration,

Numeration, Addition, Substraction Multiplication, Division, Progression, and Extraction of roots: to these some men adde Duplation, Triplation, and Mediation. But as for these three last they are contained under the other seven: For Duplation, and Triplation are contained under Multiplication, as it shall appear in their place: And Mediation is contained under Division, as I will declare in his place also.

Scholar. Yet then there remain the first seven kinds of Numbering.

Master. So there doth: howbeit if I shall speake exactly of the parts of Numbering, I must make but five of them: for Progression is a compound Operation of Addition, Multiplication and Division, And so is the Extractions of roots, But it is no harme to name them as kinds severall, seeing they appeare to have some severall working. For it forceth not so much to contend for the number of them, as for the due knowledge and practising of them.

Scholar. Then you will that I shall name them, as seven kindes distinct, But now I desire you to Instruct me in the use of each of them.

Master. So I will, but it must be done in order: for you may not learn the last so soon as the first, but you must learne them in that order, as I did rehearse them, if you will learn them speedily and well.

Scholar. Even as you please. Then to begin: Numeration is the first in order, what shall I doe with it?

Master. First, you must know what the thing is, and then after learn the use of the same.

Numeration.

NUmeration *is that* Arithmeticall *skill, whererby we may duely value, expresse, and reade any* Number or Summe *propounded: or else in apt* Figures *and* Places *set down any* Number *known or named.*

Scholar. Why? then me thinketh you put a difference between the Value and the Figures.

Master. Yea so doe I. For the Value is one thing, and the Figures are another thing, and that cometh partly by the diversity of Figures, but chiefly in the places wherein they be set.

Scholar. Then must I know here 3. things, the Value, the Figure, and the Place.

Master. Even so. But yet adde Order to them as the fourth. And first mark, that there are but tenne Figures that are used in Arithmetick; and of those tenne, one doth signifie nothing, which is made like an o, and is privately called a Chypher, though all the other sometime be likewise named. The other nine are called signifying Figures, and be thus figured. A Cypher.

1. 2. 3. 4. 5. 6. 7. 8. 9.

And this is their value.

i. ii. iii. iiii. v. vi. vii. viii. ix.

But here you muſt mark, that every Figure hath two values: One alwayes certain, that it ſignifieth properly, which it hath of his form, and the other uncertain, which he taketh of his place.

A place. A place is called the ſeate or roome that a Figure ſtandeth in. And loke how many Figures are written in one ſumme, ſo many places hath that whole number. And that muſt bee called the firſt place, that is next to the right hand, and ſo reckoning by order towards the left hand, ſo that that place is laſt that is next to the left hand. As for example. If there ſtood before you ſix men in a row, ſide by ſide, and you ſhould tell them as they ſtand in order, beginning with the man that were next to your right hand; then he that were next him ſhould be called your ſecond, and ſo forth to the furtheſt from your right hand, which is the ſixth and the laſt.

Scholar. I perceive you well: ſo might I reckon Letters or any other thing. As if I ſhould write 8 Letters after this order, a, b, c, d, e, f, g, h. then muſt I ſay, h, is the firſt, g, the ſecond, f, the third, e, the fourth, d, the fifth, c, the ſixth, b, the ſeventh, a, the eighth.

Maſter. That is well done. And after the ſame ſort uſe hereafter, that what I declare by one example doe you expreſſe by another : and ſo ſhall I perceive whether you underſtand it or no. And ſo paſſe over nothing till you perceive it well, and be expert therein.

Schol. I pray you how many of theſe places be there in all ?

Maſter. There is no certain number of them, but they are ſometimes more, and ſometimes fewer, according to the ſum that is expreſſed. For ſo many as the figures are, ſo many are the places : and the laſt place is ſo called, not becauſe it is the laſt of all other, but it is the laſt of that preſent ſumme, and it may be the middle place in another ſumme.

Scholar. Me ſeemeth I perceive this very well, as touching the order of reckoning of the places : but as for the number of them, you ſay there is no certainty. Now there reſteth to declare the value of the figures by the diverſity of places, which you called the value uncertain. Value uncertain.

Maſter. But firſt let me hear whether you know perfectly the certain value. Value certain.

Scholar. Yes ſir, as you wrote them ſo I marked them.

Maſter. How write you then five?

Scholar. By this figure 5.

Maſter. And how ſix ?

Scholar. Thus 6.

Maſter. Write theſe three numbers, each by it ſelf, as I ſpeak them vii. iiii. iii.

Scholar. 7. 4. 3.

Master. How write you these foure other, ii.i.ix.viii.

Scholar. Thus (I trow) 2.1.6.8.

Master. Nay there you miss: look on mine example again.

Scholar. Sir, true it is, I was to blame, I take 6 for 9, but I will beware hereafter.

Master. Now then take héed, those certain values every figure representeth when it is alone written without other figures joyned to him. And also when it is in the first place, though many other doe follow: as for example, this figure 9 is ix, standing now alone.

Scholar. How is he alone, and standeth in the middle of so many letters?

Master. The letters are none of his fellows. For if you were in France in the middle of a thousand Frenchmen, if there were no English man with you, you would reckon your self to be alone.

Scholar. So it is. Then 9 without more figures of Arithmetick betokeneth ix. whatsoever other letters be about it.

Master. Even so, and so doth it, if it be in the first place joyned with other, how many soever do follow, as in this example, 3679. You sée 9 in the first place, and doth betoken nine as it were alone.

Scholar. I perceive that, and doth not 7 that standeth in the second place (betwéen 9 and 6 in the third place) betoken vii. and so 3 in the fourth place betoken three?

Master. Their figures be as you have said, but their values are not so. For as in the first place every figure betokeneth his own value certain only, so

in

in the second place every figure betokeneth his own value certain ten times: as in the example, 7 in the second place is seven times ten, and is lxx. And in the third place, every Figure betokeneth his own value an hundred times, so the 6 in that place betokeneth vi C. and in the fourth place every figure betokeneth his own value a M. times, as in the aforesaid number 3 in the fourth place standeth for 3 M. and in the fifth place every figure standeth for his own value x M. times, and in the sixth place a C M. times, and in the seventh place a M M. times, and in the eighth place xMM, so that every place exceedeth the former ten times.

Scholar. As thus: if I make this number at all adventures, 91359684. here are eight places. In the first place is 4 and betokeneth but foure: in the second place is 8 and betokeneth ten times 8 that is 80, in the third place is 6 and betokeneth six hundred: in the fourth place 9 is nine thousand, and 5 in the fifth place is xM times 5, that is fifty M. So 3 in the sixth place is a CM times 3, that is, CCCM. Then 1 in the seventh place, one MM, and 9 in the eighth place, ten thousand thousand times 9, that is, xcMM. But now I cannot easily nor quickly read it in order. A generall Rule.

Master. That shall you practise by this meanes. First, put a prick over the fourth figure and so over the seventh. And (if you have so many) over the tenth, thirteenth, sixteenth, and so forth, still leaving two figures between each two pricks. And those two roomes between the pricks are called Ternaries. Ternaries

Then begin at the last prick, and see how many

figures are betwéen him and the end, which cannot passe thrée, reckoning himself for one: then pronounce them as if they were written alone from the rest, and at the end of their value, so many times thousands as your numbers have pricks.

After that, come to the next thrée figures, and sound them as if they were apart from the rest, and adde to their value so many times thousands, as there are pricks betwéen them and the first place of your whole number. And so doe by every other thrée figures following, if you have more. As in example, 91359684 this was your number.

Put a prick over 9 in the fourth place, and over 1 in the seventh place, and then no more (for your places come not to ten) as thus: 91359684.

Now goe to the last prick over 1, and take it and the figure 9 that followeth it, & value them alone.

Scholar. 91, that is xci.

Master. So it is, then adde for the number of your pricks twice M.

Scholar. that is, xci. thousand thousand.

Master. So it is. Then take the thrée other figures from one to the next Prick, and value them.

Scholar. 359, that is CCC. lix.

Master. Now adde for the one prick, that is betwéen them and the first place, M.

Scholar. CCC. lix. thousand.

Master. Then come to the other 3 figures that emain.

Scholar. 684. that is, vi. C. lxxxiiii.

Master. Now have you valued all. And at the end of the last number you shall adde nothing, because there remaineth no prick nor number after it,

it, yet prove it in another number, as thus, 2308640891053 40.

Schol. 2308640891053 40. I have pricked them as you taught me, but I am in doubt whether I have done well or no, becauſe of the Cyphers, for I remember you told me that they do ſignifie nothing, and therefore I doubt whether I ſhould reckon them for a Figure in ſetting of the pricks; and againe, I know not wherefore they ſerve.

Maſter. That will I tell you now. Indeed they are of no value themſelves, but they ſerve to make up the number of places, and to make the Figure following them to be in a further place, and therefore to ſignifie the more value: as in this example, 90 the Cypher is of no value, but yet he occupieth the firſt place, and cauſeth 9 to be in the ſecond place, and ſo to ſignifie ten times 9, that is xc. ſo that two Cyphers thruſt the Figure following them into the third place, and ſo forth.

The uſe of Cyphers.

Scholar, then I perceive in the example above, I have pricked well enough, for though that Cypher that is pricked ſignifies nothing, yet muſt he have the prick becauſe he came in the thirteenth place. Then will I prove to number that ſumme, Firſt, there is 230, M, M, M, M, and then followeth 864, M, M, M. And what ſhall I now do? There is a Cypher in the third place, and no Figure after him, but they that I have reckoned.

Maſter. He did ſerve for them that you have already reckoned, to make them in a place further then they ſhould be, if he were away, and therefore now ye ſhall let him goe. And ſo doe alwayes when he occupieth that place next before any

prick, which is the last of that Ternary, and a Cypher in the last place doth nothing.

Scholar. Then shall I say but 89, M, M.

Master. So, but go forth.

Scholar. 105 thousand. Now are all my pricks spent, and yet remain 340, so that I must value them, CCC. xl. only.

Master. Now can you reckon after this sort: and remember that every such room, so parted, is called a Ternary, or Trinity, for you have numbred or valued the summe most truly, and by the aid of the pricks each denomination is distinct most plainly.

Trinity.

Denomination.

Scholar. What call you Denomination?

Master. It is the last value or name added to any summe. As when I say, an hundred two and twenty Pounds: Pounds is the Denomination. And likewise in saying, 25 men: Men is the Denomination, and so of other: But in this place (that I spake of before) the last number of every Ternary, is the Denomination of it. As for the first Ternary, the Denomination is Vnites, and of the second Ternary, the Denomination is thousands, and of the third Ternary, thousand thousands, or Millions; of the fourth, thousand thousand thousands or thousand millions: and so forth.

Scholar. And what shall I call the value of the three figures that may bee pronounced before the Denomination, as in saying, 203000000. that is, two hundred three millions: I perceive by your words, that millions is the Denominator: but what shall I call CCiii. joyned before the millions?

Numerator.

Summe or value.

Master. That is called the Numerator, or Valuer, and the whole summe that resulteth of them both, is called the Summe, Value, or Number.

Scholar,

Scholar. Now is there any thing elſe to be learned in Numeration? or elſe have I learned it fully?

Maſter. I might ſhew you here who were the firſt Inventers of this Art, and the reaſon of all theſe things that I have taught you, but that I will reſerve till ye have learned over all the practiſe of this Art, leſt I ſhould trouble you with over many things at the firſt.

Three kinds of numbers.

But yet this you muſt mark, that there are thrée kinds of Numbers, one called Digits, another Articles, and the third mixt numbers.

Digits.

A Digit is any number under ten, as theſe: 1.2.3.4.5.6.7.8.9.

Articles.

And 10 with all other that may be divided into ten parts juſt, & nothing remain, are called articles, ſuch as are 10.20.30. 40. 50. &c. 100. 200. &c. 1000. &c.

Mixt.

And that number is called mixt, that containeth articles, or at the leaſt one article, and a Digit, as 12.16.19.21.38.107.1005. and ſo forth, and for the more eaſe of underſtanding and remembrance, mark this. The Digit number is never written with more then one figure, but the article and the mixt number are ever written with more then one figure. And thus they differ, that the article hath evermore this Cypher 0 in the firſt place: and the mixt number hath ever there ſome Digit.

Scholar. By theſe laſt words I perceive it much better then I did before, and now (I think) I will never miſs to know thoſe thrée aſunder.

Maſter. If you remember now all that I have ſaid, you have learned ſufficiently this firſt kind of Arithmeticke called Numeration. Howbeit I will exhort you now to remember both this that I have

Vse maketh mastery.

said, and all that I shall say, and to exercise your self in the practise of it; for rules without practise, are but a light knowledge, and practise it is that maketh men perfect and prompt in all things.

And as you have learned to gather and expresse the value of a summe propounded and set down before you, so must you practise to marke, note, and write down with apt figures and in due places, any number only named, or recited to you, or if your self imagined; as for a proof. How note you, or write down this summe, five thousand two hundred fifty and seven?

S. This troubleth me now, whether I should begin at the first, or at the last. For reason (me thinketh) should cause me to begin at the first and yet if I write it as you speake it, I must begin at the last.

Master. When you know your places perfectly, you may begin where you list; but the more ease for your hand is to beginne with the last, that is to say, as I did speak them, yet for the more surety, a while you may begin at the first, repeating my words backwords thus seven, fifty, two hundred, five thousand: or else sounding them all by their digit or value, as thus; seven, five, two, five; for that way is easiest: But then must you look well whether there be any cypher in your summe, that he may be set in his place: as if the last valuer of your summe (as you speak it) be above 9, then is there a cypher in the first place. And if it be an hundred, or above then is there two cyphers, one in the first place and another in the second, and so forth.

But because this thing is such that cannot be set forth without many words; I think best here now

at

at the end of Numeration, to adde a Table easie and ready for the first exercise of it.

Lo this is the Table.

The right side or hand. The names of Digits, values certain, or valuers.

The denominators of the place or value uncertain.	Nine.	Eight.	Seven.	Six.	Five.	Foure.	Three.	Two.	One.	Cyph.	The order of places.
Unites.	9	8	7	6	5	4	3	2	1	0	First.
Tennes.	9	8	7	6	5	4	3	2	1	0	Second.
Hundreds.	9	8	7	6	5	4	3	2	1	0	Third.
Thousands.	9	8	7	6	5	4	3	2	1	0	Fourth.
X. of Thousands.	9	8	7	6	5	4	3	2	1	0	Fifth.
C. of Thousands.	9	8	7	6	5	4	3	2	1	0	Sixt.
Millions.	9	8	7	6	5	4	3	2	1	0	Seventh.
X. of Millions.	9	8	7	6	5	4	3	2	1	0	Eighth.
C. of Millions.	9	8	7	6	5	4	3	2	1	0	Ninth.
M. of Millions.	9	8	7	6	5	4	3	2	1	0	Tenth.
X. M. of Millions.	9	8	7	6	5	4	3	2	1	0	Eleventh.

The left hand or side.

This Table (as ye may see) hath eleven Places, and in each of them are set all the Digits, whose certain value is written on the right hand of the Table, and the value uncertain on the left hand; so that by this Table you may learne both how to

expresse

expreſs any Number that you liſt, if that it exceed not eleven places) that is to ſay, XC. thouſand Millions, and ſo may you by help of it, value all ſummes propoſed under the ſaid number.

For example: take the ſumme that I propoſed before, which was five thouſand two hundred fifty and ſeven. And if you will expreſs it, take the firſt number (as I ſpeak it) which is five M, whoſe valuer or certain value is v, and his uncertain value, or Denomination is M. Firſt, you ſhall ſeek at the right hand of the valuer 5. Then ſeek along under the title of Denomination toward the left hand till you finde thouſands, and under it, right at the foot of the Table, is the number of the place, that is in the fourth, wherein you muſt write your Digit, or valuer 5.

Afterward come to the ſecond part of the number two hundred, whoſe valuer is 2, and his Denomination C. Seek two at the right hand of the Table, and go along under the Denomination toward the left hand, till you come under C. then look to the foot of the Table, and there you ſhall ſee the number of the place, that is to ſay, the third, wherein you muſt ſet your Digit 2.

Then do ſo by your other two numbers that remain, and you ſhall finde 5 in the ſecond place for your fifty, and 7 in the firſt place for your ſeven. And thus you may do with other numbers.

Scholar. Maſter, I thank you heartily. I perceive you ſeek to inſtruct me moſt plainly and briefly, and not to hide your knowledge with ſubtile words, as many do. For this rule is ſo plain, that I can deſire it no plainer. And though it ſeem ſomewhat

what long, yet I perceive it to be a sure way.

Master. So it is, and though it be long, yet it is neither too long, neither too plain for young learners that lack practise: for this Table is in stead of a Teacher to them that lack one. But now I trust I have said enough of Numeration: which after you have well practised, then may you learn forth.

Scholar. Yet I pray you in one thing to tell me your judgment. Why do men reckon the order of the places backward, from the right hand to the left?

Why numbers are written backward.

Master. In that thing all men do agree, that the Chaldees, which first invented this Art, did set these Figures as they set all their Letters, for they write backward, as you terme it, and so do they read. And that may appear in all Hebrew, Chaldee and Arabick Books; for they be not only written from the right hand to the left, and so must be read, but also the right end of the book is the beginning of it, whereas the Greeks, Latins, and all Nations of Europe, doe write and read from the left hand toward the right: and all their Books beginne at the left side.

Scholar. That reason hath satisfied me.

Master. It neither satisfieth me, neither liketh me well, because I see that the Chaldees and Hebrews doe not so use their own Numbers, as at another time I will declare. But this plain reason may best satisfie you presently, that seeing in pronouncing of Numbers wee keep the order of our own reading, from the left hand to the right: and again, wee doe ever name the greater numbers

before

before the smaller: It was reason that the lesser places, containing the lesser numbers, should be set on the right hand, and the greater places containing the greater Numbers to proceed toward the left hand.

Scholar. This reason is to me so plain, that it seemeth now against reason to make a doubt of that order. So that now for Numeration I am satisfied: hoping that practise shall make me fully ready and expert in it. And in the mean season I desire to learn the other kinds of Arithmetick.

Master. That is well said: but what should you next learn? can you tell?

Scholar. I remember you said, that Addition was next.

Master. Even so, and what that is, must you first know.

Addition.

Addition is the gathering together and bringing of two numbers, or more, into one summe. As if I have 106 Books in the Latine tongue, and 136 in the Greeke tongue, and would know how many they bee in all, I must write these two numbers one over another, writing the greatest number highest, so that the first figure of the one being under the first figure of the other, and the second under the second, and so forth in order.

When you have so done, draw under them a right line, then will they stand thus.

Now begin at the first places toward the right hand alwayes, and put toge-

$$\begin{array}{r} 160 \\ \underline{136} \end{array}$$

ther

ther the two first figures of these two numbers, and looke what commeth of them write under them, right under the line. As in saying 6 and 0 is 6, write 6 under 6, as thus:

160
136
6

And then goe to the second figures, and doe likewise: as saying 3 and 6 is 9, write 9 under 6 and 3, as here you see.

160
136
96

And likewise doe you with the figures that be in the third place, saying 1 and 1 be 2, write 2 under them, and then will your whole summe appear thus:

160
136
296

So that now you see that 160, and 136, doe make in all 296.

Scholar. What? this is very easie to doe, me thinketh I can doe it even since.

There came through Cheapside two droves of Cattell: in the first was 848 sheep, and in the second was 186 other Beasts.

Those two summes I must write as you taught me, thus: Then if I put the two first Figures together, saying, 6 and 8, they make 14. That must I write under 6 and 8 thus:

841
186
14

Master. Not so: and here you are twice deceived. First, in going about to adde together two summes of sundry things, which you ought not to doe except you seek only the number of them, and care not for the things: For the summe that should result to that Addition, should be a summe neither of sheep nor of other beasts, but a confused summe of both. Howbeit sometimes ye shall have summes of divers Denominations to be added, of which I will tell you anon:

anon: but first I will shew you where you were deceived in another point, and that was in writing 14, which came of 6 and 8, under 6 and 8, which is unpossible: for how can two figures of two places be written under one figure and one place?

Scholar. Truth it is, but yet I did so understand you.

Master. I said indeed, that you should write that under them that did result of them both together: which saying is alwayes true, if that summe doe not exceed a Digit: But if it be a mixt number, then must you write the Digit of it under your figures, as you have said before: and if it be an Article, then write 0 under them, and in both sorts you shall keep the Article in your minde; and therefore when you have added your second Figures, which occupy the place of tens, you shall put that one thereto, which you kept in your minde; for though it were ten indeed, yet in that place it is but as one, because that every one of that place is ten, for that it is the place of tens. And in like manner, if you have in the second place so great a number that it amounteth above 9, then write the Digit, and reserve the Article in your minde ever adding it to the next place following, and so of all other places, how many soever you have. And if you have a mixt number when you have added your last Figures, then write the Digit under the last Figures, and the Article in the next place beyond them: so shall your number resulting of Addition, have one place more then the numbers which you shall adde together.

A place.

Scholar. Now do I perceive you, and the reason of this is, (as I understand) because that no

one

one place can contain above 9, which is the greatest Figure that is, and then all tens or Articles must be put to the next place following: for every place (as I may see) exceedeth the other place next before him by 10.

Now, if it please you, I will return to my example of Cattell. But I remember you said I might not adde summes of sundry things together, and that I may see by reason.

Master. Truth it is, if you seek the due summe of any thing, but if you only seek a bare summe, and have no respect to the thing, then were it better to name the summe only without any thing: as in saying 848 without naming sheep or any thing else. And likewise 186, naming nothing.

Now let me see how you can adde those two sumes.

Scholar. I must first set them so that the two first figures stand one over another, and the other each one over his fellow of the same place; then shall I draw a line under them both. And so likewise of other figures, setting alwayes the greatest number highest, thus as followeth.

Then must I adde 6 to 8, which make 14, that is a mixt number, therefore must I take the digit which is 4, and write it under 6, and 8, keeping the Article 1 in my minde thus:

848
186
4

Next that, I doe come to the second figures, adding them together, saying 8 and 4 make 12, to the which I put the one reserved in my minde, and that maketh 13, of which number I write the Digit 3 under 8 and 4, and keep the Article in my minde, thus:

848
186
34

Then come I to the third figures, saying, 1 and

and 8 make 9, and 1 in my mind maketh 10. Sir, shall I write the Cypher under 1 and 8?

Master. Yea.

Scholar. Then of 10 I write the Cypher under 1 and 8, and keep the Article in my mind.

Master. What needeth that, seeing there follow no more figures?

Scholar. Sir, I had forgotten, but I will remember better hereafter. Then seeing I am come to the last Figures I must write the Cypher under them and the Article in a further place after the Cypher thus:

848
186
1034

Master. So now you see, that of 848, and 186 added together, there amounteth 1034.

Scholar. Now I thinke I am perfect in Addition.

Master. That will I prove by this example. There are two Armies of Souldiers: in the one are 106800, and in the other 9400. How many are there in both Armies say you?

Scholar. First, I set them one over another, beginning with the first number on the right hand, thus:

106800
9400

But the nether number will not match the other number

Master. That forceth not.

Scholar Then doe I add 0 to 0, and there amounteth 0. that must I write under the first place thus:

106800
9400
0

Master. Well said.

Scholar. Then likewise in the second place I adde 0 to 0, & there ariseth 0, which I write under the second place thus:

106800
9400
00

Then

Then I come to the third place, saying, 4 and 8 make 12, of which I write the Digit 2, and keep the Article 1 in my mind, thus:

106800
9400
200

Then I adde 9 to 6, which make 15, to that I adde the Article 1 that was in my mind, and it is 16, I write 6 under 6 and 9, and keep 1 in mind, thus:

106800
9400
6200

Master. Why doe you not write both figures, seeing you are come to the last couple of numbers.

Scholar. Nay, reason sheweth me, that I must adde that Article that is in my mind unto the next Figure of the over summe, though there be no more in the nether summe.

Master. That is well considered: then do so.

Scholar. Then say I. 0 in the over summe, and 1 in my mind maketh 1: that write I under 0. Then followeth there yet one more in the over summe, which hath none to be added to it, for there is none in the nether summe, nor yet in my mind: therefore I think I must write that even as it is.

Master. Yea.

Scholar. Then doth my whole summe appear thus:

106800
9400
116200

Master. If you mark this, you have learned perfectly the common Addition of all summes which are of one Denomination: so that ye observe this also, that in Addition you must have two numbers at the least: or else how can you say that you doe adde? And ever let the greatest number be written highest, for that is the best way, though it be not necessary.

And forget not this, that (if you have may numbers

to adde together) you shall have oftentimes an Article of a greater value then 10, sometimes 20, sometimes 30, sometimes more, yea (peradventure) 100. Therefore as you did with the Article 10, so do with them, reserving them in your mind, and adding to the number next following so many, as their valuer or value certain is: that is to say, 2 for 20, 3 for 30, 5 for 50, 10 for 100, 12 for 120, and so forth of other like. So that if the Article be 100, then must you set down the 0 and keep 10 in mind, to be carried to the next row of Figures or place, if any such happen to come. For your better understanding take this example for all. I would adde these thirteen summes into one, which I set after this manner: then doe I beginne and gather the summe of the first row of Figures, which come to 107, (for I take 9 there tenne times, and that is 90) then 9 and 8 is 17, that is in all 107, of which summe I write the 7 under the first row of Figuers, and then for that 100 is tenne tennes, I keep tenne in mind, which ten I must adde unto the next row of Figures, which are in the second place: ————————

4889
4599
2290
3699
2299
4099
1099
3298
299
699
499
899
389

which second row of figures (when they are added together with that tenne that I had in my mind) make in all 125, of which summe write the Digit 5 under the second row, and then (for that 120 containeth twelve tennes) I keep twelve in mind to be added to the third place or row of figures; which being added together, make in all 60, the Cypher 0 I set down under the row of figures in the third place:

And

And the Figure 6 I keep in mind to be added to the row of Figures in the fourth place, which (when they are added together) make 29. The Figure or Digit 9 I ſet down under the fourth place. And becauſe it is my laſt worke, I ſet down the 2 alſo that I have in my mind to the 9 in the fifth place; ſo theſe ſummes doe make in all 29057.

¶ But (for your more eaſe in work) when you have an Addition of ſo many ſummes to be added together, you were beſt part that ſumme into two or thrée parts, and work them ſeverally, and ſo put their Additions together, and this were the beſt thing you could doe when over many ſummes fall to be added.

4889
4599
2290
3699
2299
4099
1099
3298
299
699
499
899
382
29057

Scholar. This ſéemeth ſomewhat hard, by the reaſon of ſo many numbers together.

Howbeit, I think (if I doe often prove, even with the ſame example, either by working of it alone, or elſe by parting it as you ſaid even now) that I ſhall be able to do ſo ſhortly with any other ſumme.

Maſter. So ſhall you. For it is often practice that maketh a man quick and ripe in all things: but becauſe, as well in great ſummes as in ſmall there may chance to be ſome errour, I will teach you how you ſhall prove whether you have done well or no.

Scholar. That were a great help and eaſe.

Maſter. Begin firſt with the higheſt number, and then to all the other orderly, and adde them together, not having regard to their places, but as

The proof of Addition.

though they were all Unites : and ſtill (as your number encreaſeth above 9) caſt away 9. Then go forth, ever caſting away 9 as often as it amounteth thereto : and ſo doe till you have gone over all the numbers that you intended firſt to adde ; and whatſoever remaineth after ſuch Addition and caſting away of 9, write it in ſome void place by the end of a line, for the better remembrance : and thus is the firſt place of your work proved. Then ſecondly, put together the Figures that reſult of the Addition under the line, ſtill caſting away 9 alſo. And then that that remaineth write at the other end of the line ; and if thoſe two Figures be like, than have you well done, but if they be unlike, then have you miſſed. As for example, in this preſent ſumme. The firſt Figure of the over line is 9, let him goe, then 8 and 8 is 16, take away 9, there reſteth 7, and adde that 7 to 4 that followeth and it maketh 11 from which if you take 9, there reſteth 2. Then come to the next row, whoſe firſt and ſecond numbers are 9, therefore overpaſs them both, and take the 5 to the 2 which did remain in the firſt row, that maketh 7, put thereto the 4 following, and that maketh 11, thence take 9, and there remaineth 2. Next unto that goe to the third line, whoſe two firſt numbers you may let paſs, becauſe they are nines : then take the two Figures of 2, which (with the other two that remained, in the ſecond row) maketh 6. Then goe to the fourth row, whoſe two firſt numbers let goe, and take the 6 to the 6 that remaineth, and that maketh 12 : take away 9, and there reſteth 3, which with the 3 that is next, maketh 6. And ſo goe through all the

other

other numbers, and you shall find that there remaineth 5, after you have cast away 9, as often as you can find it: therefore write 5, at the end of the line in a void place thus:

5 ———— ————

Then gather all the Figures of the Totall summe, which is under the lowest line, and cast away 9 as often as you can find it; as thus, 7 and 5 make 12, take away 9, there resteth 3, to that if you adde the 2 that is last, (for you may omit the 9) then doth it make 5, which 5 you must write at the other end of the line that you made in the void place, thus:

5 ———— ———— 5

And then you see that these two Figures be like, whereby you may know that you have done well, and so you may prove in any other.

Scholar. (If it please you) I will prove in another summe.

Master. With a good will.

Scholar. Then will I take one of your former examples, which was this.

First in the highest line 8 and 6 make 14, then 9 taken away, there remains 5. to which I adde the 1 that followeth, and that maketh 6: then come I to the second line, where I find first 4, which with 6 maketh 10, from that I take 9, and there resteth one, the next Figure is 9, and therefore I let him alone, so find I 1 remaining, which I set at the end of a line, thus:

106800
9400
116200

1 ———— ————

Then I come to the Totall summe, and there I find

find that all the Figures put together make 10, from which I take 9, and there resteth 1 also, which I put at the other end of the line thus:

1———————1

And because they be like, I know that I have well added.

Master. So you know now both how to adde two summes or more together, and also how to prove whether you have done well or no: and now I will teach you how to adde summes of divers Denominations together: which thing can never be but when the one Denomination is such that it containeth the other certain times. And yet you shall adde them to the other, not after this sort (as you did them that were of one Denomination) but after such a sort as I will now shew you, that is to say:

Addition of numbers of divers Denominations.

If you have a summe of divers Denominations, then look that you set every Denomination by himself, with some note or figure of his Denomination, as they are wont to be written. Then write your other summes so under that first, that every one bee set under the other of the same Denomination: As for Example, if your Denominations bee pounds, shillings, and pence, write pounds under pounds, shillings under shillings, and pence under pence: and not shillings under pence, nor pence under pounds.

Scholar. Now that you have spoken it, me thinketh it needeth not to warne me of it, for it were against reason so to confound summes: but yet if you had not spoken of it, peradventure I should have been deceived in it.

Master. If you doe say it is plain, I will speak

no

no more of it, but with an example make the matter to appear evidently.

First, one man oweth me 22 l. 6 s. 8 d. another oweth me 5 l. 16 s. 6 d. and another oweth me 4 l. 3 s. I would know what this is all together: Therefore must I first set down my great summe, and then the other, every one under his Denomination agreeing to the greatest summe, as here you see with a line under them.

li.	s.	d.
22	6	8
5	16	6
4	3	0

Then must I begin at the smallest numbers (which must alwayes be set next to the right) and adde them together: and if the summe will make 1 or 2 or 3 of the next Denomination, then must I keep it in my minde till I come to that place, and under that first place must I note the residue (if there remain any of the same Denomination) but if there remain none, then need I to write under it nothing. And this is all that you must marke in this Addition: for all other things are like to the manner of Addition before mentioned: *Therefore the chiefest point of this Addition is, to know the values of common Coines, and rated summes,* As how many shillings be in a pound, how many pence in a shilling, of which (and of other like things) I will instruct you hereafter in teaching of Reduction: But now I may not disturbe your wit from the thing that we are about.

Therefore let us return to that former example which I purposed of the Debtors: which summes when I had set orderly, they stood thus with a line under them.

Then to adde them into one ſumme, I muſt begin at the right hand where the ſmalleſt Denomination is, and adde them together, firſt ſaying 6 and 8 make 14. Now ſeeing theſe 14 are pence, which contain one ſhilling and 2 pence: the 2 pence I ſet down under the line of pence, and the one ſhilling I keep in my mind to carry to the next row being the place of ſhillings.

li.	s.	d.
22	6	8
5	16	6
4	3	0
		2

Then doe I adde the ſhillings together, ſaying, 1 in my mind and 3 make 4, and 6 make 10, and 6 make 16, and 1 in the ſecond place which ſtandeth for 10, make 26, which is 1 pound 6 s. The 6 s. I ſet down under the place of ſhillings, as appeareth in the example. And the 1 pound I keep to carry to the pounds.

li.	s.	d.
22	6	8
5	16	6
4	3	0
	6	2

Then come I to the pounds, adding them all together, ſaying, 1 that I kept and 4 make 5, and 5 make 10, and 2 make 12. The Figure or Digit 2 I ſet down right under that place or row of pounds where I gather them, and the Article 1 I keep to carry to the next place, ſaying, 1 in my mind and 2 is 3, which 3 I ſet down directly under the 2. And then appeareth my whole ſumme thus.

li.	s.	d.
22	6	8
5	16	6
4	3	0
32	6	2

And thus muſt you doe with any ſuch like ſums whatſoever, whether they be money, weight or meaſure, which (if you practiſe divers ſummes)

you ſhall be well acquainted with the feat of Addition.

But now can you tell how to proue this Addition, or ſuch other like of diuers Denominations, and to try whether you haue well done or no?

Scholar. I would I could.

Master. That ſhall you doe by this means: You muſt make a Croſs which ſhall haue as many lines as you haue ſundry Denominations in your Addition: As if you haue but two Denominations then you may take it thus: that the ouer part and nether part may ſerue for one Denomination.

Proof of Addition of divers Denominations.

And if you haue thrée Denominations (as pounds, ſhillings, and pence) then muſt you make thrée lines, thus: The upright line may ſerue for pounds, and the higheſt thwart line for ſhillings, and the loweſt for pence: as for example, the ſumme which we laſt wrought.

li.	s.	d.
22	6	8
5	16	6
4	3	0

5

6 — | — 6

2 — | — 2

5

For the proof of which, becauſe it containeth 3 Denominations, I muſt make a croſs of 3 lines, as in the example before. Then I reckon firſt at the right hand the pence, 6 and 8 make 14, from which I take 12 for the next Denomination, that is to ſay, a ſhilling, and there reſteth 2, which I muſt

write

write at one end of the nether thwart line.

After that I gather the summe of the shillings 3, 16, 6, which maketh 25, to whom I put 1 that I took of the pence and that maketh 26, from those I take 20, the quality of the next greater Denomination, that is to say, a pound, and there resteth 6, which I write at the end of the highest thwart line.

Thirdly, I adde together the pounds, 4, 5, and 2, which make 11. to them I adde the one that came of shillings, and they make 12, from whence I cast 9, and there resteth 3, that three I joyne to the 2 in the next place, and they make 5, which 5 I set at the Cross also. And thus is my first part of my work proved.

That done, I come to the totall summe under the line, and examine it, beginning at the pence, where I find but 2, and cannot take 9 from him: therefore I set him at the other end of the nether thwart line: then I come to the shillings, where I find only 6, which (because it is less then nine) I set it at the other end of the line of the shillings, that is, the overmost thwart line.

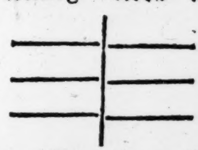
Last of all, of the 32 li. I take three times 9, which is 27, and there remaineth 5, which I write under the upright line: either else I may reckon them simply without any respect of their valuation or place: saying, 2 and 3 make 5, which because it is less then nine, I set under the upright line as before. Then I consider every number, comparing it to the number that is against it: and because I find them to be every one like his match, I know that I have well done.

Scholar. This Cross I perceive doth serve for these

theſe 3 Denominations, pounds, ſhillings, pence: but what if I had l.s. d.ob. and qd.

Maſter. Theſe lines, as I have ſaid, doe ſerve for 3 Denominations, ſuch as they be, as here 3 doe ſerve for pounds, ſhillings, and pence: but if you have no pounds in your ſumme, then may they ſerve for ſhillings, pence, and half penies: yea, for d. ob. and q. or in weight for C.q. and l. or in meaſure for Elles, Quarters, and Nailes, if you have no greater Denomination: ſo that you remember that the upright line ſerveth for the greateſt Denomination, and the higheſt thwart line for the next, and the loweſt for the leaſt.

And ſo if you have foure Denominations, you muſt make your croſs with ſo many lines: And if your ſumme be of more Denominations, make ſo many lines in your croſs. And thus will I make an end of Addition, ſaving that here (for the better underſtanding of this Rule) I have ſet you down certain examples both of money, weight, and meaſures with their works and proofs.



Examples of Addition.

li.	s.	d.	li.	s.	d.
23	10	4	130	17	10
45	6	8	28	6	8
37	2	9	13	13	4
25	13	6	120	0	0
131	13	3	292	17	10

The

5
4 —|— 4 The proofs. 8 —|— 8
3 —|— 3 1 —|— 1
5

4 (above the second cross) and 4 (below it)

C.	q.	li.	yards	q.	nayles.
34	1	3	17	3	3
12	2	2	35	2	1
7	3	4	26	1	3
13	0	13	54	2	0
67	2	23	134	1	3

3 8
2 —|— 2 1 —|— 1
4 —|— 4 3 —|— 3
3 8

Subtraction.

Then have I learned the two first kinds of Arithmetick: *now (as I remember) doth follow Subtraction, whose name (me thinketh) doth sound contrary to* Addition.

Subtra-ction. Master. So it is indeed: for as Addition increaseth one gross summe, by bringing many into one: so contrariwise, Subtraction diminisheth a gross summe by withdrawing of other from it: *So that*

that Subtraction *or Rebating is nothing elſe but an* Art *to withdraw and abate one ſumme from another, that the remainer may appear.*

Scholar. What doe you call the Remainer?

Maſter. That you may perceive by the name.

Scholar. So me thinketh: but yet it is góod to aske the truth of all ſuch things, leſt in truſting to mine own conjecture, I be deceived.

Maſter. So it is the ſureſt way. And, as I ſée cauſe, I will ſtill declare things unto you ſo plainly, that you ſhall not néed to doubt. Howbeit, if I doe overpaſs it ſometimes, (as the manner of men is to forget the ſmall knowledge of them to whom they ſpeak) then doe you put mée in remembrance your ſelf, and that way is ſureſt.

And as for this word that you laſt asked me, Remainer.
take you this deſcription: The Remainer is a ſum left after one Subtraction made, which declareth the exceſs or difference of the two other numbers, as if I would abate or ſubtract 14 out of 18, there ſhould remain 4, which is called the Remainer, and is the difference betwéen thoſe two numbers 14 and 18.

Scholar. I perceive then what Subtraction is: now reſteth to know the order how to work it.

Maſter. That ſhall you doe by this meanes. Firſt, you muſt conſider, that if you ſhould goe about to rebate, you muſt have two ſundry ſummes propoſed: the firſt, which is your groſs ſumme, (or ſumme totall) and it muſt be ſet higheſt: and then the rebatement (or ſumme to be withdrawn) which muſt be ſet under the firſt, (whether it be in one parcell or in many) and that in ſuch ſort, that the

the first figures be one just over another, and so the second and third, and all other following, as you did in Addition: then shall you draw under them a line, and so are your summes duely set to begin your working.

Then begin you at the right hand (as you did in Addition) and withdraw the nether number out of the higher, and if there remain any thing, write that right under them beneath the line: and if there remain nothing (by reason that the two Figures were equall) then write under them a Cypher of nought: And so doe you with all the other Figures, evermore abating the lower out of the higher, and write under them the Remainder still, till you come to the end: And so will there appear under the line what remaineth of your gross summe after you have deducted the other summe from it, as in this example.

I received of your Father 48 s, of which I have laid out for you 36 s. now would I know what doth remain: And therefore I set my number thus in order. First, I write the greatest summe, and under him the lesser, so that the Figures at the right side be even one under another, and so the other, thus.

48
36

Then doe I rebate 6 out of 8, and there resteth 2, which I write under them right beneath the line thus.

48
36
2

Then I goe to the second Figures, and do rebate 3 out of 4, where there remaineth 1, which I write under them right, and then the whole sum and operation appeareth thus.

48
36
12

Whereby

Whereby it appeareth, that if I withdraw 36 out of 48 there remaineth 12.

Scholar. Now I will prove in a greater summe, and I will subtract 2367924 out of 3468946, those summes I set in order thus:

3468946
2367924

Then doe I begin at the right side, and deduct 4 out of 6, and there resteth 2, which I write under them. Then go I to the second figures, and withdraw 2 out of 4, and there remaineth 2, which I set under them also, then I take 9 out of 9 and there resteth 0, which I write under them (for you say, that if the Figures be equall, so that nothing doth remain, I must write the Cypher 0 under them.)

Master. It was well remembred: now go forth.

Scholar. Then I come to the fourth place, and draw 7 out of 8, and there remaineth 1. which I write under them also. Then in the fifth place, I take 6 out of 6, and there resteth 0, (for it I write under them the Cypher 0.) Then in the sixth place 3 rebated from 4 there remaineth 1, which I write under them, and likewise in the seventh and last place, 2 taken from 3 there is left 1, which I write under them: so have I done my whole working, and my summes doe appeare thus. Whereby I see, that (if I doe rebate 2367924 out of 3468946) there remaineth 1101022.

3468946
2367924
1101022

Master. This is well done. And that you may be sure to perceive fully the Art of Subtraction, let mee see how you can subtract 52984732 out of 8250003456.

Scholar.

Scholar. Firſt, I ſet down the greateſt ſumme, and after that I will write under it the leſſer number, beginning at the right ſide, and then my Figures will ſtand thus:

8250003456
52984732
————————

Note. Then take I 2 from 6 and there reſteth 4, which I write under them. Then do I withdraw 3 from 5, and there remaines 2, which I write under them. Then take I 7 out of 4, but that I cannot, what ſhall I now do?

Maſter. Mark well what I ſhall tell you now, how you ſhall doe in this caſe, and in all other the like: If any figure of the nether ſumme be greater then the figure of the ſumme that is over him (ſo that it cannot be taken out of the figure over him) then muſt you put 10 to the over figure, and then conſider how much it is, and out of that whole ſumme withdraw the nether figure, and write the reſt under them. Can you remember this?

Scholar. Yes, that I truſt I ſhall. Now then in mine example where I ſhould have taken 7 out of 4, and could not, I put 10 to that 4, which maketh 14, from it I take away 7 and there reſteth 7 alſo, which I write under them.

Maſter. So have you done well: But now muſt you marke another thing alſo: that (whenſoever you do ſo put ten to any Figure of the over number) you muſt adde one ſtill to the figure or place that followeth next in the nether line: as in the example there followeth 4, to which you muſt put 1, and make him 5, and then go on as I have taught you.

8250003456
52984732
————————
018724

Scholar.

Scholar. Then shall I say, 4 and 1 (which I must put to him for the 10 that I added to 4 before) make 5, which I should take out of 3. but that cannot be; therefore I must put to it also 10, and then it will be 13, from which I take 5, and there resteth 8 to be written under them: and because of that 10 added to the 3, I must adde 1 to 8 that followeth in the nether line, and that maketh 9, which I should take out of 0 and cannot; therefore I put thereto 10, & that maketh 10, from 10 I take 9, and there remains 1, which I write under them.

Thus do I adde 1 likewise to the next figure beneath, which is 9, and that maketh 10, that 10 should I take out of the figure above, but I cannot, for it is 0, therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written under them.

Then come I to the next figure which is 2, and to him I doe adde 1, which maketh 3, that 3 I cannot take out of nought, therefore of that nought I make 10, and thence doe take 3. so there remaineth 7 to be written under them: likewise doe I put 1 to 5, which make 6, that 6 I cannot take out of 5, therefore I adde 10 to that 5 & make it 15, from which I rebate 6 there remaineth 9, which I write under them. Now have I spent all the nether Figures, and what shall I do more?

$$\begin{array}{r} 8250003456 \\ 52984732 \\ \hline 8197018724 \end{array}$$

Master. You should have added one to the next figure following (if there had been any) because you added 10 to the last figure before of the over line: but being there is no figure following, you

must adde that one to the place following, and then deduct that one from the number above.

Scholar. Then shall I say, because I borrowed 10 to the over 5, I must put 1 in the next place beneath that is under 2, then must I subtract that 1 from 2, and there resteth 1 to be written under that in the ninth place. Now I have no more to subtract, for there is not any figure remaining beneath, neither yet any unite to be added, because I borrowed not 10 to the figure last before: and yet is there 8 remaining in the over line, which I think (by reason) should be set at the end of the Figures in the lowest row, which is under the line, for because there was nothing taken from it.

Master. That is well considered, and reason teacheth so indeed.

Scholar. But Sir, I beseech you, shall I alwayes when any number so remaineth alone, as thus 8 did, write him under the line straight against his own place?

Master. Yea, what else? whether they be one or many: and this well remembered, you have sufficiently learned Subtraction; Howbeit, because of certain things that might deceive you, if you did not take good heed to your working, I will propose to you another example of many numbers to be subtracted, as thus: I received of a friend of mine to keep 2869 Crowns, of which at one time I delivered him again 500, at another time 368, at another time 440, at another time 80, and at another time 64, now would I know how many doe rest behind.

Therefore first I set down my gross summe

2869

2869 Crowns received, and underneath it I ſet all the parcells thus, and under them a double line.

500	
368	
440	*Delivered.*
80	
64	

Then firſt I begin at the firſt place, and gather together the ſum of all thoſe lines (ſave the overmoſt) in the firſt figures, & ſo I doe with all the figures of the ſecond place, & ſo forth, as I did in Addition, ſave that I leave out the higheſt row of numbers (as the line warneth me) that ſum ſo gathered betwéen the double line, is the ſum delivered in all: which ſumme I doe afterwards ſubtract out of the higheſt row of numbers, & the remainder doe I ſet under the nethermoſt line: as for example.

I ſet the ſummes as before: then do I gather the firſt figures of all the places delivered together: where I find but 4 and 8, that maketh 12, (for thrée Cyphers increaſe no ſum in Addition, as you learned before,) of the 12 therefore doe I write the Digit 2 betwéen the double line and kéep the Article in my mind, till I come to the ſecond place, where I find 6, 8, 4, 6, that maketh 24, to them I put the Article in my mind, and it is 25, 5 of which I write under the ſecond place, and kéep the Digit 2 in my mind for the third place, where I find 4, 3, 5, that makes 12, to the which I adde the 2 in my mind, and it maketh 14, thereof I write the 4

2869	*Crowns received.*
500	
368	
440	*Delivered.*
80	
64	
1452	*Delivered in all.*
1417	*Reſt behind.*

under the third place, and because there remains no more Figures to be added, I write the Digit in the fourth place, as you see in the Example, and so it appeareth I have delivered in all a thousand foure hundred fifty two Crowns.

Then come I to the subtracting of this summe between the lines, for by Addition it is equall to the five parcells over it, therefore I proceed to subtract it from the overmost summe, saying, 2 from 9 remains 7 to be written under them beneath the lowest line. Then in the second place I take 5 from 6 and there resteth 1 to be written under them. Then in the third place. 4 from 8, resteth 4. Last of all in the fourth place, 1 from 2, remaineth 1. And thus I see that after those five sums are subtracted from 2869, the Remainer is 1417.

Scholar. This I perceive: but is there no shorter way and more speedy?

An abridgment of the former manner of Subtraction.

Master. Yea, when you are a while exercised in it: for you may (as fast as you can gather the numbers together) withdraw them out of the highest sum. But if in quantity those numbers added together, exceed the highest sum, or upper number, then shall you (as before hath been taught) imagine to borrow 10, 20, or 30 moe, as need shall require, and put them to the upper number, to help to further the abatement, reserving or restoring the Articles that you borrowed to the next place again: and so still goe forward till you have ended your work: as for example. In the last summe proposed, I gather first in the first place 4 and 8 that maketh 12, which 12 I should deduct or take out of 9 in the upper number above the line,

line, but I cannot: and therefore I adde unto 9 an Article of 10, and make the upper number 19, from whence I take 12, then there reſteth 7, then for the Article 10 I adde to the next place of money delivered; ſaying, 1 that I bring and 6 make 7, & 8 make 15, & 4 make 19 & 6 make 25, which 25, I ſhould take out of 6 in the upper number, but I cannot. Therefore I adde 2 tens, or 20 unto 6 in the upper number, & that maketh 26, then 25 out of 26 reſteth 1; then the tens which I borrowed, or have in mind, I adde to the next row or ſum delivered; ſaying, 2 that I bring, and 4 make 6, and 3 make 9, & 5 make 14; then 14 out of 8 I cannot take, but 14 out of 18 reſteth 4. Now becauſe there are no more places to be added, the one that I borrowed, or have in mind, I rebate from 2 in the upper line, & there remaineth 1, which I ſet down in the remainer line: & ſo my ſum appeareth (as before) to be 1417 Crownes.

Loe thus have you now a ſhorter way.

Scholar. I like both wayes well: and I perceive both well: yet, as in one the working ſeemeth ſomewhat long, ſo in the other it leaveth very much (mee ſeemeth) to remembrance, and therefore may cauſe errour quickly, except a man have a quick and an exerciſed remembrance. But yet for the ſharpning of my wit by your patience (if you will give me leave) I will try what I can do in a like ſum, to work it the ſhorteſt way: whereupon I would ſubtract out of 40301964, theſe three parcels. Therefore I ſet them firſt in due order: then I gather the parcels of the firſt place, which are 8, 2, 1, that is 11,

40301964	*Charge.*
20003428	*Diſcharge.*
10002432	
10101461	
43	

which I should take or deduct out of 4, which is over him, but I cannot: therefore I adde an Article, or one tenne to 4 which maketh 14. then 11 out of 14, there resteth 3 to be written under the first place between the two lines.

Then come I to the second place, saying, 1 that I borrowed to have in my mind, and 6 make 7, and 3 make 10, and 2 make 12, which I cannot take from 6, therefore I adde 10 to 6, which maketh 16, and then 12 from 16, resteth 4, which I write under the second place between the two lines.

Then come I to the third place saying, 1 that I borrowed or have in mind and 4 make 5, and 4 is 9, and 4 make 13, which I should take out of 9 that is over them but I cannot: therefore I adde 10 to 9 which make 19, then 13 out of 19, rest 6.

Then come I to the fourth place, saying, 1 in mind and 1 is 2, and 2 is 4 and 3 make 7, which because it cannot be taken from 1, I take it from 11, and there resteth 4.

After that I come to the fifth place, where are only three Cyphers, which make nothing, unto which I adde 1 in mind, then should I take that (that is to say) 1 from the Figure over them, which is also a Cypher: therefore I say thus, I cannot take 1 from 0, but 1 from 10 remaineth 9: so must I write 9 under them. Then in the sixth place I find but 1, and 1 in mind make 2, which I take out of 3 over him, and the remainer is 1: that must be written between the two lines in the sixth place. So I goe to the seventh place, where I find onely Cyphers, and in the gross

summe

ſumme over them a Cypher alſo: therefore muſt I write the remainer (which is nothing) with a Cypher alſo. Then in the eight and laſt place I gather 1, 1, 2. that maketh 4, which if I take out of that 4 that is over them, there will nothing remaine. And that muſt be noted with a Cypher betwéen the two lines (as I have often ſaid) and ſo have I ended my work, and the figures ſtand as followeth.

Scholar. But Sir, I remember you taught me that Cyphers ſhould not come in the laſt place, for becauſe they ſerve only to increaſe the value of other Figures which follow them and ſerve not thoſe figures that goe before them: and now in my Example I have ſet two Cyphers in the two laſt places.

Maſter. I commend you for your remembrance. And truth it is, you ſhould not have ſet them here, but only becauſe that I would make you plainly to perceive the Art of Subtraction. Therefore ſéeing that you doe now perceive it, whenſoever you would write down a Cypher, look whether any other figures be yet behind: and if not, then let go the 0 alſo, for it néedeth not to write him in the latter places, where no other figure doth follow, except it be (as I did now ſuffer you) to teach the uſe of Subtraction the plainer.

40301964	*Charge.*
20003428	*Diſcha.*
10002432	
10101461	
0164643	*Reſt.*

Therefore your figures muſt ſtand thus when the work is ended.

Scholar. Sir, I do think with that you taught me before, and by theſe two ſummes that you taught me

laſt alſo, that now I could Subtract any ſumme.

Maſter. So may you, if you have marked what I have taught you. But becauſe this thing (as all other) muſt be learned ſurely by often practiſe, I will propound here two Examples to you: wherein if you often exerciſe your ſelf, you ſhall be ripe and perfect to ſubtract any other ſumme lightly; for in them is contained all the obſervances of whole numbers. And becauſe you ſhall perceive ſomewhat both how to doe it, and alſo whether it be well done when you have proved to doe it; therefore have I written under them both the Remainers.

30606.	*Lent.*	308964.	*Debt.*
10354 10249 163	*Paid.*	103145 102597 101024	*Paid.*
20766	*Paid in all.*	02198	*Reſt.*
9840	*Reſt to pay.*		

Scholar. Sir, I thank you: but I think I might the better doe it, if you did ſhew me the working of it.

Maſter. Yea, but you muſt prove your ſelf to do ſome things without my aid, or elſe you ſhall not be able to doe any more then you are taught: And that were rather to learne by wrote (as they call it) then by reaſon. And again, there is nothing in theſe examples, or any other of whole numbers,

but

but I have taught you the rules of them already.

Scholar. Then I truſt by practiſe to attain the uſe of it. And is this all that I ſhall learn of Subtraction?

Maſter. Yea, ſaving that (as you have ſeen in Addition) there are numbers of divers Denominations, in which the working is not much unlike: yet (without ſome inſtructions be given of it) it might ſeem to a learner more difficult then indeed it is. Therefore I will briefly ſhew you the uſe of it only by an example or two.

A certain man owed to mee 14 l. 12 s. 8 d. *of which he paid me at one time* 4 l. 6 s. 8 d. *at another time* 3 l. *at another* 2 l. 3 s. 4 d. *and laſt of all* 6 s. 8 d.

Now would I know what remaineth unpaid yet? therefore I ſet my ſums thus, every one in their due place: As pounds under pounds, ſhillings under ſhillings, pence under pence.

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
—	6	8

Scholar. Sir, I pray you why doe you write 2 l. for the common ſpeech uſeth rather to ſay, 40 s.

Maſter. We muſt here uſe the Denomination that is greateſt in any ſumme, ſo that we may not write according as we uſe to ſpeak, ſaying, 16 d. 18 d. or likewiſe 7 groats, 8 groats, 24 s. 40 s. 48 s. and ſuch other: but we muſt write every Denomination that is in any ſumme by it ſelf.

Note how the pen differeth from the common order of Counters.

Namely, ſhillings and pounds. So muſt we write for the laſt ſummes now named, 1 s. 4 d. 1 s. 6 d. 2 s. 4 d. 2 s. 8 d. 1 l. 4 s. 2 l. 8 s. and ſo forth of other like.

Scholar.

Scholar. So that we may not write in Arithmetick, pence, when the summe amounteth to shillings, nor shillings, when the summe maketh pounds, Now (if it please you) end your example.

Master. When my summes are so set as I shewed then (according to the rules of Addition) I gather all the particular summes, which be paid me into one totall summe, directly to be set under them betwéen the two lines, not medling with the 14 l. 12 s. 8 d. as the line warneth me: therefore must I beginne with the smallest Denomination, saying, 8, 4, 8. is 20, pence, which maketh one shilling and 8 pence, the 8 d. I set down under the place of pence, and the one shilling I kéep in mind to carry to the next Denomination of shillings; Then come I to the shillings, and say 1 that I bring or have in mind, and 6 is 7, and 3 is 10 and 6 makes 16, which because it containeth not one pound, I set directly under the place of shillings. Then come I to the pounds, whose parcels are 2, 3, 4. that is in all 9, that 9 doe I set down directly under the pounds: And so the totall or whole Addition of all the particulars paid, amounteth to 9 l. 16 s. 8 d.

li.	s.	d.	
14	12	8	
4	6	8	
3	0	0	
2	3	4	
	6	8	
9	16	8	
4	16	0	*Rest.*

Now for the work of Subtraction, I must rebate that totall summe of Addition out of the highest number, that is to say, from the 14 l. 12 s. 8 d.

There-

Therefore to perform the work I ſay, 8 d. out of 8 d. remaineth or reſteth nothing, therefore in the place of the reſt or remain, right under the Denomination, I ſet down o. Then coming to the ſhillings, where I find 16. which ſhould be taken out of 12, but I cannot: therefore I imagine to borrow 1 out of the next Denomination, that is, of the 14 l. and put that one pound ſo borrowed unto 12 s. that maketh 32 s.

Now 16 s. out of 32 s. reſteth 16 s. which 16 s. I ſet down directly under the place of the reſt.

Laſtly, comming to the pounds, ſaying, one pound in mind that I borrowed, and 9 make 10, then 10 out of 14, there reſteth 4.

So doth my whole reſt or remain appear to be 4 l. 16 s. o d.

This I account the eaſieſt way for a young beginner to practiſe, though it be ſomething long.

Scholar. Is there any ſhorter way for this work alſo?

Maſter. Yes, as in the laſt Example I will alſo ſhew you, for you may adde together the particular ſums as they are ſet in order, beginning with the pence, ſaying, 8, 4, 8, make 20 d. which 20 d. you ſhould take out of the 8 d. above the line, but you cannot, therefore ſhall you borrow 1 of the next Denomination, that is to ſay 1 of the ſhillings, and put it to the 8 d. that maketh 20 d.

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0

now 20, out of 20 d. reſteth o, which Cypher I ſet

down

down directly under them. Then one shilling that I borrowed or had in mind, and 6 make 7, and 3 make 10, and 6 make 16, the 16 out of 12 I cannot take, therefore of the next Denomination I doe borrow one l. and put it to 12 s. which maketh 32 s. then 16 s. out of 32 s. resteth 16 s.

Lastly I come to the pounds, saying, 1 l. in mind, or that I borrowed, and 2 make 3, and 3 is 6, and 4 is 10, then 10 out of 14, there resteth 4.

So doth my remainer or rest appear as before to be 4 l. 16 s. 0 d.

Scholar. Then doe I perceive very well: and if there be no other thing to be learned in Subtraction, then may I come to Multiplication, for that you reckoned to be next in order.

Master. We have done indeed with the Art of Subtraction, as touching the working.

Proof of Subtraction. But yet before we go to Multiplication, I will instruct you how to examine your work, whether it be well done or not. For the performance whereof, if you mark what I said in Addition, you may easily perceive what is to be done for the proof of Subtraction, which is best made by the aid of Addition thus.

Draw under the lowest number (which is your Remainer) a line, and then adde this Remainer and all the other that you did subtract before, together, and write that that amounteth under the lower line: and if the summe that cometh thereof be equall to the highest of the Subtraction, then is the Subtraction well wrought, or else not: As you may see for example in the summes set down before, and first in sums of one Denomination, whereof one was this.

Where

Where the Number 52984732 is subtracted from 8250003456. and the Remainer is 8197018724 Now to prove whether it be	8250003456 52984732 ——— 8197018724

truly wrought or not, I adde the Remainer and the number Subtracted together, beginning at the right hand; and first I say 4 and 2 is 6 which is set under the line:

Example in a sum of one Denomination.

The number given	8250003456
The number to Subtract	52984732
The Remainer	8197018724
The Proof	8250003456

Then again in the second place I say 2 and 3 is 5, which I write under, next that in the third place, 7 and 7 are 14, of which I write the Digit 4, and keep the Article 1 in my mind. Then in the fourth place 8 and 4 is 12 and 1 in my mind maketh 13, whereof I write down the Digit 3, and keep the Article 1 in my mind. Again in the fifth place, 1 and 8 is 9, and 1 in my mind is 10. Whereof I set down 0 and keep the 1 in my mind. And so going on to the rest (as it is taught in Addition) when I have made an end, I see that the lowest line of numbers and the highest be alike: wherefore I know that I have well done.

So likewise the Proof is to be made in numbers of divers Denominations: as for Example, in our summe of that kind which in the first form of working stood thus: (all the particular numbers to be subtracted, being drawn into one)

Where,

Example in a sum of divers Denominations.

Where, in the title of pence. I find 8 & 0: the 8 I set down directly under in that of pence.

Then in the place of shillings I find 16 and 16 which make 32 shillings, wherein is contained 1 l. and 12 s. the 12 s. I set down directly under them in the due place of shillings, & one pound I keep.

Then comming to the pounds, I say 1 that I keep, and 4 is 5, and 9, is 14, which 14 in due order I set down directly under them as this Figure sheweth. And the whole summe is 14 l. 12 s. 8 d. agreeing with the upper number above. So I find the work is good, and the Subtraction well wrought.

	li.	s.	d.
	14	12	8
	4	6	8
	3	0	0
	2	3	4
		6	8
Paid in all.	9	16	8
Rest.	4	16	0
Proof.	14	12	8

The same thing is to be done for the latter forme of Subtraction (where the particular summes are not gathered together into one gross.) For the Remainer and all the particular summes subtracted, being added together, if the summe that commeth thereof be equall to the highest number above, then is the Subtraction well wrought, or else not.

Example of a proof in the latter forme of Subtraction.

As for example also in the last sums which stood thus.

First in the title of pence, I adde, 8, 4, 8, that maketh 20 d. which containeth 1 shilling 8 pence.

The 8 I set down under the lowest line in the row or title

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0
14	12	8

title

title of pence, and that 1 shilling I keep to carry to the next Denomination or place of shillings.

Then returning to the shillings, saying: one in mind, or that I keep, and 16 make 17, and 6 make 23, and 3 make 26, and 6 make 32 shillings, which amounteth to one pound 12 s. the 12 s. I set down under the title of shillings: and 1 pound I keep or have in mind to carry to the next Denomination or place of pounds. Then come I to the pounds, saying, 1 that I bring and 4 make 5, and 2 make 7, and 3 is 10, and 4 make 14; then doe I write 14 under the pounds, and so have I ended the Addition: and I see that the lowest line is like unto the uppermost line in number, wherefore I know that I have well done.

And thus have I taught you the Art of Subtraction, and the means to prove whether it be well wrought or not. Therefore now will I make an end thereof, and will instruct you in Multiplication.

Multi-

Multiplication.

Multiplication what it is.

Multiplication is an operation whereby two sums produce the third: which third sum so many times shall contain the first, as there are Unites in the second. And it serveth in stead of many Additions. As for Example:

When I would know how many are 30 times 48. if I should adde 48 thirty times, it would be a long work. Therefore was Multiplication devised, which shall doe that at once that Addition should do at many times.

Scholar. I perceive the commodity of it partly, but I shall not see the full profit of it till I know the whole use of it. Therefore Sir, I beseech you, teach me the working of it.

Multiplication of Digits.

Master. So I judge it best; but because that great summes cannot be multiplied but by the Multiplication of Digits, therefore I think it best to shew you the way of multiplying them. As when I say, 9 times 8, or 8 times 9, &c. And as for the small Digits, under 5 it were but folly to teach any rule, seeing they are so easie that every child can do it: but for the Multiplication of the greater Digits, thus shall you do.

First, set your Digits one right over the other, then from the uppermost downwards, and from the nethermost upward, draw straight lines, so that they make a cross, commonly called Saint

Andrews

Andrews cross, as you see here. Then looke how many each of them lacketh of 10, and write that against each of them at the end of the lines, and that is called the difference: as if I would know how many are 7 times 8, I must write those Digits thus.

The difference.

Digit difference.

8

X

7

Then doe I look how much 8 doth differ from 10, and I find it to be 2: that 2 doe I write at the right hand of 8, at the end of the line thus.

Digit difference.

8 2

X

7

After that I take the difference of 7 likewise from 10 that is 3, and I write that at the right side of 7, as you see in this example.

Digit difference.

8 2

X

7 3

Then doe I draw a line under them, as in Addition, thus.

Last of all, I multiply the two differences, saying 2 times 3 make 6, that must I ever set under the differences, beneath the line: then must I take one of the differences (which I wil, for all is like) from the other Digit (not from his own) as the lines of the Cross warne me, & that is left must I write under the Digits. As in this example, if I take 2 from 7, or 3 from 8, there remaineth 5: that 5 must I write under the Digit, & then there appeareth the multiplication of 7 times 8 to be 56.

Digit difference.

8 2

X

7 3

5 6

And so likewise of any other Digits, if they be above 5, for if they be under 5, then will there difference be

greater then themſelves, ſo that they cannot be taken out of them. And again, ſuch little ſums every child can multiply, as to ſay 2 times 3, or 4 times 5, and ſuch like.

Scholar. Truth it is. And ſeeing me ſeemeth that I underſtand the multiplying of the greater Digits, I will prove by an example how I can do it. I would know how many are 9 times 6.

Maſter. It is all one in value to ſay 9 times 6, or 6 times 9: but yet the order is beſt to put the leſs ſum firſt, ſaying, 6 times 9, and ſo of all other ſums.

Scholar. Then would I know how many are 6 times 9: therefore I ſet the Digits thus and make the croſs, thus.

```
9
  X
6
```

Then doe I ſet their differences from 10 at the right ſide, the difference of 9, which is 1, againſt it, and the difference of 6, which is 4, againſt it alſo, as in this example.

```
9   1
  X
6   4
```

And under them draw a line, Then doe I multiply the differences together, ſaying: 1 time 4 maketh 4, that 4 doe I write under them thus.

```
9   1
  X
6   4
------
    4
```

Then take I one of the differences from the other Digit, as, 1 from 6, or elſe 4 from 9, and each wayes there reſteth 5, which I doe write under the Digit thus. And ſo appeareth the multiplication of 6 times 9, to be 54. Thus I ſee the feate of this manner of multiplication of Digits.

```
9   1
  X
6   4
------
  54
```

Maſter.

Master. Now might you go straight to the multiplication of great numbers, save that both for your ease and surety in working I will draw you here a Table, whereby shall appear the multiplication of all the Digits, and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would know the product in any multiplication of Digits, seek your first or last Digit in the greater figures; and from it go right forth towards the right hand, till you come under the number of your second Digit, which is in the highest row, and then the number that is in the meeting of the rows of little squares (which come directly from both your propounded Digits) is the Multiplication that amounteth of them. As if I would know by this Table the multiplication of 7 times 9, seek first 7 in the greater figures, and then go right forth toward the right hand, till you come under 9 of the highest row in which place where you so come under the

other Digit (as here for example you come under 9) is alwaies contained the off-come or product, which you seek, and that place we terme to be in the common angle, in respect of the two numbers so taken on the outsides: as here in that common angle, where the rowes of little squares directly proceeding from 7 and 9 do meet, you have 63, which 63 is the summe of the multiplication of 9 by 7.

To multiply greater summes.

Scholar. This is very good and ready. And so may I find the multiplication of any Digits: but now how shall I doe in greater summes?

Master. *when you would multiply any* summe *by another, you shall mark that it is the meetest order to set the greatest* number *highest, which is the place of the* number *that must be multiplied: and likewise the lesser* number *under it, for that is the place of the*

Multiplier.

Multiplier *or* Multiplicator *that is to say, the* number *by which the* Multiplication *is made, and is in* English *alwayes put before this word, Times: in such*

Times.

speaking as when I say 20 times 70. And the number *that followeth this word Times, is that which must be* Multiplied.

Therefore when I would multiply one number by another, I must write the greatest highest, and the lesser under it, as in Addition. And under them must I draw a line. As for example. If I would multiply 264 by 29, I must set them thus.

$$\begin{array}{r} 264 \\ 29 \\ \hline \end{array}$$

¶ Of which number thus set down to be multiplied, may be formed a question, as thus, There are 29 men, and each man hath 264 Lambes. The question is, how many Lambes they have in all.

To

To the performance whereof, I muſt multiply every figure of the high row by every figure of the nether row: and that that amounteth I muſt ſet under the line, as thus.

Firſt I do multiply 4 by 9, ſaying, 9 times 4 (or 4 times 9 which is all one) and that maketh 36, as the Table before of Digits doth declare, of that 36, I muſt write the 6, that is the Digit, under the 9, and the Article 3 I keep in mind to carry to the next place.

```
264
 29
———
  6
```

Then come I to the ſecond figure of the higher row, which is 6, and ſay 9 times 6 make 54, and with the 3 in my mind make 57, the 7 I ſet down under the 2, and 5 I keep in mind.

```
264
 26
———
 79
```

After that I come to the next figure, which is 2, and multiply it by 9, and that maketh 18, and with 5 that I have in mind, maketh 23: wherefore becauſe it is the laſt work of the Multiplier, I ſet it down in order as you ſee:

```
 264
  29
————
2376
```

And ſo have I ended the firſt figure of the Multiplier. Wherefore I give it now a fine daſh with my pen.

Then begin I with the next figure, & multiply it into all the higher figures as thus.

```
 264
  2~~9~~
————
2376
```

Firſt, 2 times 4 make 8, that 8 do I write under the ſecond place: for evermore the Digit or firſt figure of the Multiplication that amounteth of the figure of the higher number, muſt be ſet under the Multiplier of it, the other in their order toward the left hand.

Scholar. I underſtand you thus that the Digit of the ſumme amounting of the Multiplication of the firſt figure of the higher row, by the firſt figure of the lower row, or Multiplier, muſt be ſet under the firſt place: and that that amounteth of the ſame firſt figure by the ſecond Multiplier, muſt be ſet under the ſecond place, and ſo of the other, if there be more Multipliers.

Maſter. So mean I indeed: and if there amount but a Digit then muſt it be ſet under the Multiplier.

And now to goe forth; I multiply by the ſame 2, the ſecond figure of the higher row, which is 6, ſaying two times 6 make 12, wherefore I write the Digit 2 under the third place, and the Article 1 I keep in mind.

264
~~2~~8
2376
528

Then doe I multiply the laſt figure of the higher ſumme by that ſame 2, ſaying, two times 2 is 4, and with the 1 that I have in mind maketh 5, which I write under the fourth place. And ſo have I ended the whole Multiplication: wherefore I alſo give the 2 a daſh with my pen, thus: and ſo I doe ever as ſoon as I have diſpatched any Digit by which I multiply: and the ſums ſtand thus.

264
~~2~~~~8~~
2376
528

Then muſt I draw a line under all thoſe ſummes that mount of the multiplication, and muſt adde all them into one ſumme, as in the Example, you may ſee,

264
~~2~~~~8~~
2376
528
7656

Where

Where in the first place I find but 6, and therefore write I it under the line. Then in the second place 8 and 7 make 15, whereof I write 5, and keep one in my mind, and so forth as you learned in Addition. And so appeareth the whole summe to be 7656, which amounteth of the Multiplication of 264, by 29, and that is the just number of the Lambes that 29 men had.

Scholar. If there be no more to be observed in it, then can I doe it I suppose, as by this Example I shall prove.

¶ There is a peece of ground which containeth 1365 yards in length, and 236 yards in breadth: I would know how many yards square there is in all this peece of ground: which numbers I set down with the greater above, and the lesser under, as you see.

1365
236
———

Then doe I multiply 5 by 6, saying, 6 times 5 make 30, of which I write the Cypher in the first place, and the Article 3 I doe keep in mind to carry to the next place.

1365
236
———
0

Then do I by the same 6, multiply the second figure of the higher sum, which is 6, saying, 6 times 6 make 36 & 3 in my mind make 39, of which I write the 9 under the second place, & the Article 3 I keep in mind.

1365
236
———
90

Then doe I multiply the third figure, which is 3 by the same 6, and that maketh 18, and 3 in my mind make 21. The 1 I set down, and keep 2 in mind.

1365
236
———
190

Then come I to the laſt figure of the higher ſumme, and multiply it by 6, ſaying, 6 times 1 make 6, and 2 in my mind make 8, that 8 doe I write under the fourth place. And ſo have I ended the firſt Multiplier, and daſh him ſleightly with my Pen.

1365
236
8190

Then begin I with the ſecond Multiplier, and ſay firſt, 3 times 5 make 15, of which I ſet the 5 under the ſecond place, becauſe that the Multiplier is there, and the Article 1 I keep in mind.

Then come I to the ſecond Figure that is 6, and multiply it by 3, which maketh 18, and with 1 in mind maketh 19, the 9 I ſet down under the third place, and 1 I keep in mind.

1365
236
8190
95

Then come I to the third figure, which is 3, & multiply it by 3, ſaying, 3 times 3 make 9, and with 1 in mind, make 10, the Cypher I ſet under the fourth place, and the Article 1 I keep in mind.

1365
236
8190
095

And then coming to the laſt figure 1, I multiply it by 3, and it maketh 3, & with the one in mind, it maketh 4, which 4 I ſet in the fifth place and then have I ended two of the Multipliers, and the ſummes ſtand as you may ſee in the latter end of the page going before, and then I give 3 his daſh.

1365
236
1190
4095

Then

Then come I to the third Multiplier, and multiply it into every figure of the higher summe, and first I say, 2 times 5 make 10, of which I set the Cypher under the Multiplier in the third place, and the Article 1 I keep in mind.

1365
236
8190
4095
0

And so multiplying the second figure 6 by that same 2, there amounteth 12, and 1 in my mind maketh 13, whereof I write the Digit 3 under the fourth place, and the Article 1 I keep in mind:

1365
236
8190
4095
30

Then do I multiply the said 2 by the third figure of the higher summe, which is 3, and that maketh 6, and the one in mind make 7, which 7 I set down under the fifth place, as appeareth by the example.

1365
236
8190
4095
730

Then come I to the last place, and multiply that 1 by 2, and there amounteth 2, which I set in the sixth place, and then doth the summe stand thus.

1365
236
8190
4095
2730

And so have I ended the whole Multiplication.

But now (as you taught me) to know what this whole summe is, I must adde all those parcels together, and then under the line will appear, as you may see, the gross or totall summe is, 322140. Whereby I know there is so many yards square in that piece of ground.

1365
236
8190
4095
2730
322140

Master.

Master. This is well done.

Scholar. Then me thinketh I could call it well done, when I know, whether I had well done or no.

Master. It is to be proved by 9 as Addition was, but the surest proof is by Division, and therefore I will reserve that proof by Division, till you have learned the Art of Division. And anon I will shew you how it is commonly proved.

¶ *But first, for your further instruction in this exercise of* Multiplication, *I will with one example try your cunning, and so make an end: And the* question *is this. I would know how many daies it is since* the Nativity of our Lord and Saviour Jesus Christ, *unto this year* 1630. *Which to performe, you must* multiply *this present year* 1630 *by the daies in one whole year, which are* 365

Scholar. Now for that you have given me so much light into the question, you shall see I will handsomely finish the work, for according to your former instructions, I set them down with a line under them thus.

$$\begin{array}{r} 1630 \\ \underline{365} \end{array}$$

Then say I, 5 times 0 is 0, which I set down under the first place, as here appeareth. Then say I, 5 times 3 make 15, the Digit 5 I set down in the second place under 3, and the Article 1 I keep in mind to bee added to the next Multiplication. Then saying five times 6 make 30, and 1 in mind 31, the 1 I set down in the third place, and 3 I keep in mind. Then comming to the last Figure, I say once 5 is 5, and 3 in mind make 8, that 8 doe I set down under the fourth place: and thus have I ended my first Murtiplier, and

and therefore I give it a daſh with my pen.

Then come I to the ſecond Multiplier, which is 6, and do likewiſe multiply it into the upper number, ſaying; 6 times 0 is 0, which I ſet down in the ſecond place, right under his Multiplier: then ſay I, 6 times 3 make 18, the 8 I ſet down under the third place, and 1 I keep in mind. Then ſay I 6 times 6 make 36, and 1 I keep in mind make 37. the Digit 7 I ſet down in the the fourth place, and 3 I keep in mind: Then ſay I 6 times 1 is 6, or once 6 is 6, and 3 in mind make 9, which I ſet down next, and ſo have I ended two Multipliers: wherefore I daſh the 6 with my pen.

```
 1630
  365
 ----
 8150
9780
```

Then I begin to multiply the third Multiplier into the over number, ſaying, 3 times 0 is 0; the 0 I ſet down in the third place right under his Multiplier. Then ſay I 3 times 3 make 9, which I ſet down in order next: then ſay I, 3 times 6 is 18, the 8 I ſet down, and 1 I keep. Laſtly, I ſay, once 3 is 3, and 1 I keep is 4, which I ſet down orderly next: And ſo have I ended the Multiplication, and my figures ſtand thus.

```
                        or thus
    1630                  1630
     365                   365
  ------                ------
    8150                   815
   9780                   978
  4890                   489
  ------                ------
  594950                594950
```

Maſter. I commend you for your diligence, the work is very perfectly done; which parcels if you now adde together into one ſum it will be 594950,

which

which is the gross or totall summe of that Multiplication, and declareth the number of dayes since our Lord and Saviour his incarnation, unto the end of 1630 *yeares, besides* 407 *dayes, and twelve houres for leape yeares.*

Scholar. This is marvellous, me think, that such great matters may so easily be atchieved by this Art, which heretofore I ever thought had been impossible, as infinite sorts of people are of that mind.

Master. Truth it is, that knowledge hath no greater enemy then ignorance, for this is one of the least of ten thousand things that may be done by this Art, as hereafter you shall be able to justifie.

Scholar. The manner of Multiplication I perceive, if there be no more in it.

Master. *Yes, there are other formes and helps for ease and shorter labour of the work of* Multiplication, *but I will remit them till you have a little tasted* Division, *where also the like help into* Division *may be used: and so therefore under one example for both, will I shew you both ease in* Multiplication, *and also in* Division.

But sith the other formes and workings do nothing differ from these works in effect, but only in setting of the numbers, I will overpass them till a more meet place and time. And now will I instruct you in Division, so that you think your self sufficiently to perceive what I have taught you.

Scholar. Yes Sir, I thank you, but I doe not perceive how to examine my work to try whether I have well done or no: therefore as you

you promised me ere-while, I pray you first shew me how I shall prove it.

Master. *That is commonly used by the proofe of 9, as you learned before in* Addition, *saving that it differeth from that form in divers respects: As for example.*

First, must you make a cross after this manner. X

Then must you examine your summe that should be multiplied, and look what remaineth after casting away of 9, that set you at the one side of the cross, then examine the Multiplier, and whatsoever remaineth in it after casting away 9 so often as you can, write that at the other side of the cross: then must you multiply those two numbers together, and look what amounteth thereof, if it be under 9, write it at the higher part of the cross: but if it be above 9, then take thence 9 as often as ye can, and write the rest at the head of the cross: As for example put forth of the piece of ground that contained 1365 yards in length and 236 yards in breadth. Proof of Multiplication.

Therefore first I cast away all the nines from the summe to be multiplied, saying, 5 and 6 make 11, cast away 9 rest 2: then 3 and 2 makes 5, and 1 is 6, that 6 I write at one side of the cross thus.

X
6

Then do I examine the Multiplier which is 236; wherein when

the

the 9 is cast out, there remaineth 2, that 2 therefore I set at the other side of the cross.

2 6

Then doe I multiply 6 by 2, and it maketh 12, from which 12 I withdraw 9, then resteth 3, which 3 doe I set at the head of the cross. Then doe I examine the gross summe, amounting of the Multiplication, which is 322140, where I find 9 once, and 3 remaining; that 3 I set at the foot of the cross, and then I see it to agree with the other 3 at the toppe of the cross, and so know I that I have done well: for if they two did differ, then were my work vaine, and the Multiplication false.

3

2 6

3

A sure proof of Multiplication.

This is the common proof: but the most certain proof is by Division of which I will anon instruct you.

Schol. Sir, what is the chief use of Multiplication.

Master. The use of it is greater then you can yet understand: howbeit, these plain commodities it hath, that if you would resolve any great and whole value into many small and less proportions, as if you would change pounds into shillings and pence or any other greater or smaller parcels, by Multiplication ye shall do it speedily and easily. Also if you should need to adde one sum to it self, or to any other often times you shall doe it by Multiplication much more speedily, readily, easiely, and surely, then by often and sundry Additions. Take you these commodities grosly shewed for an answer at this time, and hereafter I will more abundantly make you to perceive the use of it.

Division

Diviſion.

Scholar.

WEll Sir, then in Diviſion *I pray you to inſtruct me. But me thinketh by the name of it, that it ſhould be all one with* Multiplication : *for I call that* Diviſion, *when any thing is parted into diverſe and many parts.*

Maſter. You take it as it is taken commonly : howbeit, if you marke well, you ſhall perceive that it is quite contrary to Multiplication, and doth not part one thing or few things into many, but contrarywayes, it bringeth many parcells into few, but yet ſo, that theſe few taken together, are equall in value to the other many : for by Diviſion pence are turned into ſhillings, and ſhillings into pounds: As for example, of 120 ſhillings, it maketh 6 pounds, ſo are 120 turned into 6, which is a ſmaller number: but then if you conſider the Denominators, you ſhall ſee that they are ſuch, that one of the latter is equall to 20 of the firſt, and ſo in value the ſummes are one, though in number they do differ, and the latter ſumme is the leſſer, and ſo it is alwayes in Diviſion, howbeit, yet in the working the ſumme is parted by another, and thereof doth it take the name.

Scholar.

Scholar. I think I shall better understand the reason of the name when I know the use of the work, therefore now would I gladly learn that.

Division what it is.

Master. *Division is a distributing of a greater summe by the unites of a lesser:* Or, *Division is an Arithmeticall producing of a third number, in respect of two propounded numbers; which third number shall so often contain an unite, as the greater of the two propounded numbers can contain the lesser.* So that as Multiplication did seem to serve in stead of many Additions, so Division may seem to be in place of many Subtractions: Because that third number briefly expresseth how many times the lesser of your two propounded numbers may be subtracted from the greater: as in practise will more largely appear. Therefore (as you may perceive) unto Division are required three numbers: The first, which should be divided, and that must (generally) be the greater: and the second, by which the other must be divided, and that is (generally) the lesser, and is called the Division: And the third, which answereth to the question (How many times?) and therefore is called the Quotient.

A generall rule for placing the figures.

The first must be first written, and the second so set under it, that the last figure of the lower number be right under the last of the higher, contrariwise to the work of other kinds of Arithmetick: for in them the two first figures were set ever meet one under the other; but in Division, the last figures must be set meet, except it chance so that the last figure of the Divisor be greater then the last of the higher number, for then you shall set the last of the Divisor under the last save one of the higher

higher number, as for example:

If you ſhould divide 365 (which are the ſumme of the dayes of a year) by 28, which are the dayes of a common Month, then ſhould you ſet them thus.

365
28

But if you ſhould divide thoſe 365 dayes by 52, which is the number of weeks in one year, then ſhould you ſet them thus.

365
52

Likewiſe if I would divide the ſame 365 by 4, which is the ſumme of the quarters of years, then muſt I ſet them thus.

365
4

Scholar. Sir, this doe I underſtand, but now how ſhould I doe to divide the one by the other?

Maſter. You muſt beginne with the laſt Figure next the left hand, and ſee how many times the firſt Figure of the Diviſor may bee taken out of the laſt Figure of the other Number, and that ſhall you note within a crooked line toward your right hand: As for example, I would divide 365 by 28, then ſet I thoſe two ſummes thus.

365 (
28

And I looke how many times I may find 2 (which is the laſt figure of the Diviſor) in 3, (which is the laſt of the number to be divided) and conſidering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I ſet 1, and that is called the Quotient number, as I told you. Quotient numbers.

Then becauſe that when 2 is taken out of 3, there remaineth 1, I muſt write that 1 over 3, & deface or cancell the 3 & the 2, then will the figures ſtand thus.

1
~~3~~65 (1
~~2~~8

Then come I to the next figure of the Divisor, and take it likewise so many times out of the figures that be over it, and look what doth remain, that I must write over them, and cancell them, as in this example.

Therefore now do I take once 8 out of 16, and there remaineth 8, which I must set over the 6, and cancell or cross out the 16, and the 8 of the Divisor: and then will the figures stand thus. And so I have once wrought.

~~1~~8
~~3~~~~6~~5 (1
~~2~~~~8~~

Scholar. So I perceive that you take the nether figure, not only out of the other that is right over him, but out of that with the other also that remaineth before, and are written toward the left hand.

Master. So must you doe: for you must so take the Divisor out of the over number, that there remain not over it so great a summe as it self is: for then were your work in vains.

But yet again here must you mark, that when you seek how many times the last figure of the Divisor may be found in the number over him, that you looke also whether you may as often find all the figures following in those that are above them (considering all the remainers, if there be any) if not, take your Quotient less by one, and then prove again, and so till you find a meet Quotient: and by that meet Quotient must you alwayes multiply your Divisor, and set the product under your Divisor, so that the first figure stand under the first figure of your Devisor, and the second under the second, and so forth; and then subtract that product from the number to be divided that standeth di-

rectly over it, as you have seen me doe.

When you have thus wrought once, then must you begin again, and write your Divisor anew, neerer toward the right hand by one place, as in this example, you shall set 2 under 8, and 8 under 5, thus.

~~2~~8
~~3~~~~6~~5 (13
~~2~~88

Then (as before) seek how many times you may take your Divisor out of the number over him now.

Scholar. That may I doe here 4 times.

Master. Truth it is that you may find 2 foure times in 8: but then marke whether you can find the figure following so many times in the other that is over him. Can you find 8 foure times in 5?

Scholar. No, neither yet once.

Master. Therefore take 2 out of 8 once less.

Mark how to consider this kinde of Remainer.

Scholar. That is three times.

Master. Well then, 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the 5 following make 25, in which summe I find 8 three times also, and therefore I take 3 as a true Quotient, and write it within the crooked line of the Quotient before the 1, thus:

2
~~2~~8
~~3~~~~6~~5 (13
~~2~~88

Then say I, 3 times 2 make 6, then 6 out of 8 resteth 2, therefore I cancell the 8, and write over it the 2 that doth remain, thus:

2

Then doe I take 8 as many times out of 25, saying, 3 times 8 make 24, and if I take 24 out of 25, there remaineth 1, so then I cancell 25 and 8, and over the 5 set 1 thus:

Or you might (after you find 3 to be a fit Quo-

tient) straight way have multiplied the whole Divisor 28, by that at once: which giveth 84, which being set under 28, and duely subtracted from 85, of the number divided, giveth 1, the remainer of the whole division, as before you had. Work which way you list, here you may see also the form.

2
x81
385 (13
288
2

And now have I done with the dividing; for I cannot find my Divisor 28 any more in the over summe.

Scholar. No; except you would part the 1 that remaineth into 28 parts.

Master. That is well said, and so must we doe in such cases, when there remaineth any thing: but I will let that pass now, and will make you perfect in division of whole numbers, and will hereafter teach you particularly of broken numbers called Fractions. Now if you doe perceive the order of division, then doe you divide this summe 136280 by 452.

Scholar. First I set down the number that should be divided; then doe I set the Divisor under the last figure of the over number. Then will it be thus.

136280
452

Master. Can you take the last of your divisor (which is 4) out of 1 which is the last of the over number?

Scholar. I had forgotten, because the last of the divisor cannot be taken out of the last of the over number, in so much as it is the greater, therefore must I set the divisor one place more forward toward the right hand, thus.

136280
 452

And

And then must I look how often I may find the last figure of the divisor (that is 4) in 13, which I may doe 3 times, therefore doe I say, 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then do I make at the right hand of my summes a crooked line, and write before it my Quotient 3, and I cancell 13 and 4, and over the 3 I set the 1 that remaineth, and then the figures stand thus.

1
~~1~~~~3~~6280 (3
~~4~~52

Then I multiply the same Quotient into every figure of the Divisor, and withdraw the summe that amounteth out of the numbers over them, as first I say, 3 times 5 make 15, which I take from 16, and there resteth 1, I cancell therefore 16 and 5, and write over the 6 that 1 that remaineth, thus.

~~1~~1
~~1~~~~3~~~~6~~280 (3
~~4~~~~5~~2

Then doe I say likewise, 3 times 2 make 6, which I take out of 12 and there resteth 6, therefore I cancell the 12 and the 2 over, and then I write the 6 that remaineth, thus.

~~1~~~~1~~6
~~1~~~~3~~~~6~~~~2~~80 (3
~~4~~~~5~~~~2~~

Then should I set forward the Divisor into the next place toward the right hand thus.

~~1~~~~1~~6
~~1~~~~3~~6~~2~~80 (3
~~4~~~~5~~~~2~~2
45

Master. But you may see that over the 4 is no figure, therefore I must set the divisor yet forwarder by another place.

And marke, whensoever it chanceth so, that you should set forward the divisor, and that it cannot stand here, because there is no number over the

the laſt place, or if there be any, it is leſſer then the laſt figure of the diviſor, then muſt you remove the diviſor yet once again: and becauſe that his firſt place of removing ſerved not to ſubtract him ſo much as once, therefore you ſhall write in the Quotient a Cypher, and if you ſhould by chance need to do ſo oft times, for every time write a Cypher in the Quotient. The reaſon of this will I ſhew you hereafter.

Scholar. Then muſt I ſet my ſums thus.

$\cancel{1}\cancel{1}\cancel{6}$
$\cancel{1}\cancel{3}\cancel{6}\cancel{2}80(30$
$\cancel{4}\cancel{5}\cancel{2}\cancel{2}$
$\cancel{4}\cancel{5}$

And becauſe I removed the diviſor, ſo that I overſkipped one place, I muſt write a Cypher in the Quotient: and then muſt I ſeek a new Quotient, as in this example I muſt ſay, How many times 4 is there in 6? (and ſith it can be but once) therefore doe I write 1 in the Quotient: and then ſay I, 1 time 4 taken out of 6, remaineth 2, I cancell the 6 and the 4, and write 2 over them, thus.

2
$\cancel{1}\cancel{1}\cancel{6}3$
$\cancel{1}\cancel{3}\cancel{6}\cancel{2}80\ (301$
$\cancel{4}\cancel{5}\cancel{2}\cancel{2}2$
$\cancel{4}\cancel{5}\cancel{5}$
$\cancel{4}$

Then ſay I again, once 5 out of 28, remaineth 23: I let the 2 ſtand as it did, and over that 8 I ſet 3, cancelling the 8 and the 5 under it, thus.

2
$\cancel{1}\cancel{1}\cancel{6}3$
$\cancel{1}\cancel{3}\cancel{6}\cancel{2}\cancel{8}0(301$
$\cancel{4}\cancel{5}\cancel{2}\cancel{2}2$
$\cancel{4}\cancel{5}\cancel{5}$
$\cancel{4}$

Maſter. You might as well have ſaid, once 5 out of 8, and ſo remaineth 3, but now go forward.

Scholar. Then, once 1 out of 0 cannot be: What ſhall I now do?

Maſter, Borrow of the next number that is behind

hind (for there is 230) and do as you learned in Subtraction in like caſe.

Scholar. Then muſt I borrow 1 out of the 3 comming behind next, and make that 0 to be 10 and then take I 2 out of 10, and there reſteth 8: and becauſe I borrowed one of the 3, I muſt cancell the 3, and write 2 over it, then doth the figure ſtand thus.

```
     22
    xx638
   x36280(301
    45222
     455
      4
```

Maſter. Now have you done, and yet remaineth 228. and your Quotient ſheweth you, that if you divide 136280 by 452, you ſhall find your Diviſor in your greater number 301, that is CCC times and once, and 228 remaining.

And in the other example (where I divided 365 by 28, the Quotient was 13, and 1 remained: whereby I knew that in a year (which containeth 365 dayes) there are 13 months, reckoning 28 dayes (or 4 weeks) iuſt to a month, and 1 day more.

Scholar. Why then do we call a year but 12 moneths?

Maſter. Of that at a more convenient time will I fully inſtruct you: but now it is not convenient to intangle your mind with other things then do directly pertain to your matter. Therefore if you remember what you have heard, you have learned a ſhort manner of Diviſion, which I would have you often to practiſe, ſo that you may be perfect in it, and hereafter I will ſhew you certain other proper points touching it.

Scholar. Then I pray you tell me how I ſhall

examine and try my work, whether I have done well or no, that though no man be by to tell me, yet I may perceive it my self.

Proof of Division.

Master. Some men (yea and commonly most) doe try it by the rule of 9, as in all the other kinds, save that their order is; First, they cast away 9 as often as they can out of the Divisor, and that that remaineth they set at one side of a Cross, as in our first example the Divisor was 28, from which you may take 9 three times, and 1 remaineth: which they set by a Cross, thus.

X 1

Then they likewise examine the Quotient, (which in our example is 13) and from thence they cast away 9 as oft as they can, and the remainder they set at the other side of the Cross, and then they multiply together those two remainers: and to it that amounteth they adde the remainer of the Division, if there were any, from that whole summe they withdraw 9 as oft as they can, and the rest they set at the head of the Cross, as in our example, the Quotient is 13. from which take 9, and there remaineth onely 4, and therefore must you set 4 at the other side of the Cross, thus:

4 X 1

Then multiply 4 by 1, and it yeeldeth but 4, thereto adde the remainer of the Division (which was 1) and it will be 5, which summe doth not amount to 9, and therefore must be set wholly at the head of the Cross, as you see here.

5
4 X 1

And this nnmber on the head of the Cross is the first proof, to which if you find another like in the

number

number that was divided, then you have done well.

Therefore now ſhall you likewiſe examine the whole ſumme that was divided, and take away 9 as often as you can, and that that remaineth, ſet at the foot of the Croſs: and if it be equall to that in the head of the Croſs, then have you done well, elſe not.

As in our example the whole ſumme was 365, which maketh 14, from that take 9, and there reſteth 5, which ſet at the foot of the Croſs, thus.

5
4 X 1
5

And you ſhall ſee that they agree: therefore have you well done.

Now will I likewiſe examine our ſecond example, where the Diviſor was 452, which maketh 11; from thence I take 9, and the 2 that remaineth I ſet at the right ſide of the Croſs, thus.

X 2

Then examine I the Quotient, which was 301, where I find but only 4: that I ſet at the other ſide of the Croſs, thus:

4 X 2

Then I multiply 4 by 2, and it maketh 8: to that I adde the remainer of the Diviſion (which was 228, and it maketh 12) and they two make 20, wherein I find twice 9, and 2 remaining: that 2 muſt I ſet at the head of the Croſs thus:

2
4 X 2

Then I examine the whole number to be divided, which was 136280, where I find twice 9 and 2 remaining, which I ſet at the foot of the Croſs, thus:

2
4 X 2
2

And becauſe it doth agree with the

figure

figure at the head of the Cross, I know that the Division was well wrought.

The Proof of Division more certain by Multiplication.

Master. This is the common proof: Howbeit, the more certain working is by the contrary kind: as to prove Division by Multiplication, thus:

Multiply the quotient by the Divisor, and if the sum that amounteth be equall to the sum that should be divided, then have you well divided: else not.

Howbeit, this must you marke, that if there remained any thing after the Division, that must you adde to the summe that amounteth of the Multiplication. As in our first Example our Quotient was 13, and the Divisor was 28: Now multiply the one by the other, and the summe will be 364: to that if you adde the 1 that remained after the Division; then will it be 365, which was the sum that should be divided: and therefore I know that I have well done.

Scholar. Now will I prove the same in the second example, whose Divisor was 452, and the Quotient 301: these doe I multiply together, and there amounteth 136052: to which if I adde the 228 that remained, then will it be 136280, which was the whole summe to be divided: and therefore I perceive that I have well done.

Master. This is the surest way to examine Division by Multiplication: and contrariwise, the surest proof of Multiplication is by Division.

Proof of Multiplication by Division.

And therefore (according to my promise) now will I shew you how you may prove Multiplication by Division.

When you have ended Multiplication, and would know whether you have well done or not, set the

gross

gross summe that amounteth of the Multiplication overmost, and divide it by the Multiplier: and if the Quotient be the same number that should be multiplied, then have you well wrought, else not, as in that example where we multiplied 264 by 29, the gross summe was 7656.

Now if you will know whether that Multiplication be true, you shall divide that 7656 by the multiplier 29, and you shall perceive that the Quotient will be 264, and that is a token that you have well wrought.

Scholar. By your patience I will prove that, and first set down the gross summe and the multiplier, not after the rule of Multiplication, but after the rule of Division, for now that number is become the Divisor, that was before the Multiplier: I should set them therefore thus.

7656
29

Then shall I seek how many times 2 in 7, that may be three times, & one remaineth: but then may not 9 be found so often in 16: therefore must I take a lesser Quotient, that is to say 2: then say I, twice 2 maketh 4, which I take out of 7, and there remaineth 3, then do I cancell 7 and 2, and over 7 I write 3, and in the Quotient I set 2: so the figures stand thus.

3
~~7~~656(2
~~2~~9

Then say I further, two times 9 make 18, which I abate out of 36, and there resteth 18: then cancell I 3, and over him set 1, and likewise I cancell 6 and 9, and over them I set 8: so that thus stand the Figures.

1
~~3~~8
~~76~~56(2
~~29~~

Then I set forward the Divisor by one place, and

and ſeek a new Quotient, that is to ſay, how many times 2 are in 18, which I find to be 9 times: but then can I not find 9 ſo many times in 5: but therefore I take a leſſer Quotient, as to ſay 8; but yet that is too great: for if I take 8 times 2 out of 18, there remaineth but 2, & I cannot find 8 times 9 in 25: therefore yet I take a leſs Quotient, that is 7, which is alſo too great; for if I take 7 times 2 out of 18, there reſteth 4, but now I cannot take 7 times 9 out of 45, therefore yet I ſeek a leſſer quotient, as to ſay 6, then ſay I 6 times 2 make 12, that I take out of 18, and there remaineth 6, ſo I cancell 18, and the 2 and write 6 over 8, thus:

$$\begin{array}{l} \cancel{1}6 \\ \cancel{2}8 \\ \cancel{7}\cancel{6}56\ (26 \\ \cancel{2}\cancel{9}9 \\ \ \cancel{2} \end{array}$$

Then ſay I forth, 6 times 9 maketh 54, that take I out of 65, and there remaineth 11, and the figures ſtand thus:

$$\begin{array}{l} \ \ 1 \\ \cancel{1}\cancel{6} \\ \cancel{2}\cancel{8}1 \\ \cancel{7}\cancel{6}\cancel{5}6\ (26. \\ \cancel{2}\cancel{9}\cancel{9} \\ \ \cancel{2} \end{array}$$

Then muſt I ſet forth the Diviſor again, and ſeek a new Quotient, which will be 4: for though I may find 2 in 11, 5 times, and 1 remain, yet I cannot find 9 ſo often in 6, therefore I ſet the figures thus:

$$\begin{array}{l} \ \ 1 \\ \cancel{2}\cancel{8}1 \\ \cancel{7}\cancel{6}\cancel{5}6\ (264 \\ \cancel{2}\cancel{9}\cancel{9}9 \\ \ \cancel{2}\cancel{2} \end{array}$$

And the 4 in the quotient I multiply into the figures of the Diviſor, ſaying, four times 2 makes 8, which I take out of 11, and there reſts 3, therefore I cancell the 11, and the 2, and ſet 3 over the firſt place of 11, thus:

$$\begin{array}{l} \ \cancel{1} \\ \cancel{1}\cancel{6}3 \\ \cancel{2}8\cancel{1} \\ \cancel{7}\cancel{6}\cancel{5}\cancel{6}\ (264 \\ \cancel{2}\cancel{9}\cancel{9}\cancel{9} \\ \ \cancel{2}\cancel{2} \end{array}$$

And then doe I ſay forth, 4 times

9 maketh

9 maketh 36, which I take from 36, and there remaineth nothing, so that the Quotient of this Division also (where 7656 is divided by 29) is 264: Which doth declare, that if 264 be multiplyed by 29, the summe will be 7656. And thus I perceive now how both Multiplication is proved by Division, and Division also by Multiplication.

Master. Now have I ended the five common kinds of Arithmetick: For (as touching Mediation, Duplation, Triplation, and such other) they are no severall kinds of Arithmetick, but are contained under the other. For Mediation is contained under Division, and is nothing else but dividing by 2: and so are Duplation and Triplation contained under Multiplication: for Duplation is nothing else but multiplying by 2, and Triplation is multiplying by 3, of which I will only propose an example, for the Rules you have heard already.

If you would mediate, or divide into 2, this sum 4531010, you shall set 2 for the Divisor, and work as you learned before, as thus: An example of Mediation.

4531010 (
2

Then I find 2 in 4 two times, therefore my Quotient must be 2: so I cancell 4 and 2, and remove the Divisor forward thus, as the work requireth, and as before in Division hath been declared.

4531010 (2265505
$\cancel{2}\cancel{2}\cancel{2}\cancel{2}\cancel{2}\cancel{2}\cancel{2}$

Which mediation or division by 2 being finished, you shall have for your Quotient 2265505, which is the half of 4531010, as you may try by Duplation; for double that Quotient, or multiply it by 2, and the same number will amount.

I

I will no longer tarry about these, seeing they are but members of the other kindes. But here now (according to my promise) I will teach you certain easie formes both of Multiplication and of Division. And first of Multiplication.

Easie formes of Multiplication.

If you would therefore multiply any summe by 10, you shall need to do no more but adde a Cypher before his first place; as for example, 36 multiplied by 10, make 360.

Likewise if you would multiply any summe by 100, put two Cyphers at his beginning. So if you would multiply any summe by 1000, adde three Cyphers to the beginning of it.

Scholar. This doe I well perceive, and also the reason of it.

Master. I will omit all reasons till our next meeting, when I shall tell you the reason of all other parts of Arithmetick also: and as to our matter now, look, as I have told you, that you both remember it, and also often practise it.

And now you have learned how to multiply easily by 10, 100 1000: and of like manner may you do with any other of like sort.

But now if you will multiply by 20, 30, 40, and so forth, or by 200, 3000, and such like, where there is one Cypher in the first place, or many orderly in the first places, you shall take away those Cyphers, and multiply the summe only by the other figure or figures (if they be many) and then at the beginning of the summe that amounteth, you shall set so many Cyphers as you took away.

Example of 2873, which I would multiply by 300. First, I omit the 2 Cyphers from the Multiplier,

plier, and I multiply the ſumme by thrée onely that is left, and it amounteth to 8619: before which I put the two Cyphers that I before omitted or took away, and then is it 861900. And that is the ſumme that amounteth when 2873 is multiplied by 300.

Scholar. And if there were two or more figures beſide the Cyphers, I muſt only take away the Cyphers, and multiply by the other figures, as I learned before: As if I would multiply 93648 by 25000, I ſhould take away the thrée Cyphers, and multiply the ſame by 25, & then at the beginning of that totall ſum ſhould I adde the 3 Cyphers again.

Maſter. Even ſo: but if it chance the number that ſhould bée multiplied, or both the ſummes, as well the number that ſhould be multiplied, as the multiplier, to have Cyphers in their firſt places, evermore omit the Cyphers and work by the reſt. But remember to reſtore as many Cyphers to the amounting ſumme as you abated before. As in this example: 30200 ſhall bée multiplyed by 206, I ſhall only take away two Cyphers from the greater number, and then multiply 302 by 306, and afterward adde the two Cyphers again. But if I would multiply the ſame 30200, by 2060 I ſhall not only take away the two Cyphers from the number that ſhould be multiplied, but alſo I may take away the one Cypher from the Multiplier, and then muſt I adde thrée Cyphers to the ſumme that amounteth: but take héed that you take away no Cypher that commeth after any ſignifying figure, as in the laſt example, you muſt not take away that in the fourth place of the higher number,

neither

neither that in the third place of the multiplier: howbeit yet thus you may do: If one Cypher or more come in the last of your summes, you may multiply the other figures, and overskip them: but so that you give every figure his due place: as thus, I will multiply 3026 by 2004, therefore I set them thus.

3026
2004
12104
52

And thus doe I multiply them. First 4 times 6 make 24, I set 4 under the first place, and keep the two still in my mind. Then say I again, 4 times 2 make 8, and the 2 that is in my mind maketh 10, I set down the Cypher 0, and keep the Article 1 in my mind: Then 4 times 0 is 0, and the 1 in my mind maketh 1, I set down the figure 1, and say again, 4 times 3 is 12, I set down 2, and keeping the 1 still in my mind (having no more places of the upper number to multiply it withall) I put it down next 2 in the fifth place.

But now when I come to the next place (being a Cypher 0) I let it go, because it multiplieth nothing, and likewise the second Cypher.

But then when I come to the 2, and multiply it into the 6 of the over number, you must take heed (according as I taught you in Multiplication) that the first number amounting of the multiplication be set right under the multiplier, and the other orderly toward the left hand, according as you may see in this example, which being finished, with the addition thereof gathered together, will stand as in this Example sheweth.

3026
2004
12104
6052
6064104

Which

Which is indeed wrought so much the sooner and shorter by overskipping of the two Cyphers: which otherwise, (if the same example were wrought at length) it would have had two workings more, as by the same example here also set down doth appear.	3026 2004 ——— 12104 0000 0000 6052 ——— 6064104

Scholar. Sir I thank you, for I see great ease in this way of Multiplication: and (if you can) shew me such like in Division you shall greatly further me.

Master. Yes, I will teach you some easie wayes in Division also, and the first this: If you would divide any summe by 10, you shall onely with your pen make a square line between the first figure of your summe and the second, and then have you done: for the whole number that followeth the line, standeth for the Quotient, and the figure that is before the line, is the remainer: As for example, 3648 divided by 10. Where 364 is the Quotient, and betokeneth that so many times are 10, in 3648, and the 8 after the line is the remainer, which cannot be divided into 10, but by breaking it into Fractions, wherewith I will not meddle yet. Easie forms of division.

364 | 8

And so likewise if you would divide any summe by 100, with your Pen you shall cut away the two first figures, and if you would divide by 1000, you must cut away the three first figures, and so of any other divisor, whose last figure is 1, and the other Cyphers; look how many Cyphers the divisor hath, and so many figures at the beginning shall

you cut away with the square line, and they stand alwaies for the remainer, because they are lesse then the divisor, and cannot be divided by it, and the other figures that are behinde the line stand for the Quotient.

But now if your divisor have any other figure in his last place then 1, and in all his other places have Cyphers, looke how many Cyphers they be, cut away so many of the first figures of the number that should be divided, and divide the rest that followeth the line by that figure that is in the last place, as if it were the whole divisor.

Example of 64284, which I would divide by 300, here must I cut away the two first figures, (for so many cyphers my divisor hath) and must divide the rest by 3, which is the figure in the last place of the divisor. First therefore I part away the two first figures, and the summe standeth thus:

642	84	(2
3	00	

Then do I divide 642 by 3, and the Quotient is 214: for in 6 I finde twice 3, and in 4 once, and 1 remaining, which 1 with the 2 next before, doth make 12, wherein I finde 3 four times: And this is a ready way to turn shillings into pounds: for sith one pound doth contain 20 shillings, I must divide the whole number of shillings by 20. Therefore easily do I see that my divisor hath one cypher, and therefore I cut away one figure from the beginning of the whole summe of shillings, and then do I mediate or divide by the 2 other figures or summe that followeth.

Scholar. I will put an example.

If you would divide 64287 shillings, by 20: that

that is to say, If I would turn so many shillings into pounds, I must cut away the first figure, that is 7, and divide the rest, that 6428 by 2, so shall the Quotient be 3214, whereby I know that 64287 shillings make 3214 pounds. and 7 shillings remaining.

Master. Now prove by Multiplication whether you have well done or no.

Scholar. The Quotient is 3214. which I do multiply by the divisor 2, and it doth amount to 6428.

Master. Hereby you may perceive not onely that you have well done, but also how by Division you may turn shillings easily into pounds: and contrariwise by Multiplication you may turn pounds into shillings.

But here shall you see amongst divers men divers forms of such division: but if you marke what I have told you, you shall perceive easily all the wayes. For some men do not cut away so many of the first figures of the summe that they would divide, as there are Cyphers in the first place of the divisor: but they set all their Cyphers orderly under the first places of the Number that they would divide: and then with the other figure or figures, (if there be many) they divide the rest of their summe.

Example, If they would divide 725931 by 3400, they doe set their summs thus:

7259	31
34	00

And then doe they divide orderly till they come to the Cyphers: for there they stay and end their worke, as in this example.

They seek how often 3 may be found in 7, which

you cut away with the square line, and they stand alwaies for the remainer, because they are lesse then the divisor, and cannot be divided by it, and the other figures that are behinde the line stand for the Quotient.

But now if your divisor have any other figure in his last place then 1, and in all his other places have Cyphers, looke how many Cyphers they be, cut away so many of the first figures of the number that should be divided, and divide the rest that followeth the line by that figure that is in the last place, as if it were the whole divisor.

Example of 64284, which I would divide by 300, here must I cut away the two first figures, (for so many cyphers my divisor hath) and must divide the rest by 3, which is the figure in the last place of the divisor. First therefore I part away the two first figures, and the summe standeth thus:

642	84	(2
3	00	

Then do I divide 642 by 3, and the Quotient is 214: for in 6 I finde twice 3, and in 4 once, and 1 remaining, which 1 with the 2 next before, doth make 12, wherein I finde 3 four times: And this is a ready way to turn shillings into pounds: for sith one pound doth contain 20 shillings, I must divide the whole number of shillings by 20. Therefore easily do I see that my divisor hath one cypher, and therefore I cut away one figure from the beginning of the whole summe of shillings, and then do I mediate or divide by the 2 other figures or summe that followeth.

Scholar. I will put an example.

If you would divide 64287 shillings, by 20: that

that is to say, If I would turn so many shillings into pounds, I must cut away the first figure, that is 7, and divide the rest, that 6428 by 2, so shall the Quotient be 3214, whereby I know that 64287 shillings make 3214 pounds. and 7 shillings remaining.

Master. Now prove by Multiplication whether you have well done or no.

Scholar. The Quotient is 3214. which I do multiply by the divisor 2, and it doth amount to 6428.

Master. Hereby you may perceive not onely that you have well done, but also how by Division you may turn shillings easily into pounds: and contrariwise by Multiplication you may turn pounds into shillings.

But here shall you see amongst divers men divers forms of such division: but if you marke what I have told you, you shall perceive easily all the wayes. For some men do not cut away so many of the first figures of the summe that they would divide, as there are Cyphers in the first place of the divisor: but they set all their Cyphers orderly under the first places of the Number that they would divide: and then with the other figure or figures, (if there be many) they divide the rest of their summe.

Example, If they would divide 725931 by 3400, they doe set their summs thus:

72593	1
34	00

And then doe they divide orderly till they come to the Cyphers: for there they stay and end their worke, as in this example.

They seek how often 3 may be found in 7, which

is two times, and 1 remaining: therefore they set 2 in the quotient, and cancell 3 & 7, and over 7 they set the 1 that remaineth, thus:

```
 1
725931 (2
34  00
```

Then doe I goe forth, saying, two times 4 make 8, which they take out of 12, & there remaineth 4, thus:

```
14
725931 (2
34  00
```

Then remove they the divisor forward, and seek how often 3 may be found in 4, which is but once, and 1 remaineth, then set they 1 in the Quotient, and cancell 3 & 4 & over them they set that 1, thus:

```
  1
 14
725931 (21
344 00
  3
```

Then take they once 4 out of 15, & there resteth 11. Or else more easily: Take once 4 out of 5, and there resteth 1: so they cancell the 4 and 5, and set 1 over them, thus:

```
  1
 141
725921 (21
344 00
  3
```

Then set they forth the divisor again, and seek how many times 3 are in 11, which the find three times, and 2 remaining: so they set 3 in the Quotient, and cancell 11 & 3, and over them set 2, thus:

```
  12
 141
725931(123
344400
  33
```

Then doe they multiply 4 by 3, which maketh 12, that withdraw they out of 29 & there resteth 17, of which the 7 must be set over the 9, and the 1 over the 2 thus:

```
   1
  12
 1417
725931(213
344400
  33
```

And now are the two Cyphers next ensuing, so that the divisor can no more be set forward, and therefore is the division ended, and the remainer is 1731. Now the Quotient which is 213, doth declare that if you

divide 725931 by 3400, you shall find it therein 213 times, and there remaineth 1731: so shall you find it, if you work as I taught you, by cutting away the two first figures, because of the two Cyphers. But this must you mark (as you may perceive by this last example) that if there be left any other Remainer in the sum that was behinde the square line that the Remainer must be set to the latter end of the first remainer, which was cut away with the square line: as if you would divide 725931 by 3400, after the forme that I taught you, then would you sums appear, thus: Note.

```
      | 1
   ------
    12 |
  1417 |
  725931  (213
  3444
    33
```

So that 17 which remaineth after the line, must be set to the 31 (that was cut away with the line) in higher places, as you see here: where that 17 with the 31, do make 1731.

Scholar. Sir, is there no other form of division in practise but this?

Master. Yes verily, there are other forms in practise, but because I love brevity, I will declare only one, which I first learned of, and is practised by that worthy Mathematician, my ancient and especiall loving friend Master Henry Bridges, wherein not any one figure is defaced or cancelled. As if I should divide 72 by 6, first place them thus. 6)72

Then if you please you may write the divisor in a loose paper that it may more easily without cancelling or defacing of the work be applied to, and removed from the dividend at pleasure; then apply your divisor 6 to 7, the first figure of the Write the Divisor in a loose paper, to remove at pleasure.

dividend, and inquire how oft it may be had in 7, and seeing 6 is but once in 7, set 1 in the Quotient line, thus: 6) 72 (1

Then multiply the divisor 6, by the quotient 1, and set the Product 6 under 7, thus: 6) 72 (1
6

Then draw a line under 6, and subduct 6 out of 7, setting the remainer 1 under 6, thus: 6) 72 (1
6
———
1

Then bring down the next figure of the dividend, and set it with the Remainer 1 under the line, thus: 6) 72 (1
6
12

And bring the moveable divisor 6 under the 2, and as before enquire how oft 6 is in 12, and finding it to be twice in 12, set 2 in the Quotient, thus: 6) 72 (12
6
———
12
12
0

And multiply 6 by that new Quotient 2, setting the Product 12, under the other 12, and subducting it out of the upper number, there resteth nothing.

And since the unites of this Product do stand under the unites of the dividend, the division is ended, otherwise you should proceed as before, bringing down the next figure; removing the divisor, dividing, multiplying, subducting, &c.

Schol. This is very easie; but if there be greater numbers propounded, is the operation the same?

Master. If the numbers be never so great, the work is the same without any difference, as shall appear by this example.

Divide 7890 by 33. 33) 7890 (2

First set them thus: then bring the divisor under 78, and see how oft it is there found, which is twice, and therefore set 2 in the Quotient, by which multiply

multiply the diviſor 33, and ſet the Product 66 under 78, and ſubduct it out of it thus.

Then bring the next figure 9 down, and ſet it with the Remainer 12, it maketh 129, and removing the diviſor 33 thereto, enquire how often 33 is contained in 129, and I find it but thrice,(though at the firſt it made a ſhew of more) therefore ſet 3 in the Quotient, and multiplying 33 by 3, ſet the Product under 129, ſubducting that product out of the number above, and proceed as before.

33)7890(239 $\frac{3}{33}$
66
129
99
300
297
3

Then ſhall you find the Diviſor 9 times in the Remainer, therefore ſeeing 9 in the Quotient, multiply, and ſubduct as before, and at the laſt you ſhall find onely 3 remaining, which muſt be ſet above a line after the Quotient, and the Diviſor under, as above appeareth.

Scholar. Is there no more difficulty in the whole Rule?

Maſter. Not any, although your Number be never ſo great, as before I have ſaid.

¶ And here will I make an end of Diviſion, (ſaving that I doe requeſt you to exerciſe your ſelf well herein by many ſummes, till you have attained ſome expertneſs therein.)

For the reaſons & concluſions thereof are ſo many, and ſo available for all ſorts of men whatſoever; that if I ſhould ſpeak of the infinite uſes thereof, I ſhould rather lack words then matter. And therefore recommending it to your judgement hereafter, upon your further travell into the Art, I will here end

this Treatise, representing unto you one example, or simple question of Division and Multiplication, in stead of many, which is this.

A question of shooting in Ordinance.

There are foure brass Peeces: The first of them at a shot spendeth 9 pounds of powder, the second spendeth 5 pounds, the third 4 pounds, and the fourth 2 pounds. They are all appointed against the battery of a Hold, and there is allowed by the Master Gunner, 700 pounds of powder to be spent by these four Peeces, in this assault. The question is twofold. The first, how many shot each Peece shall justly make about with this 700 pounds of powder? And lastly, how many pounds of powder ought justly to be allowed to each Peece for his true proportion?

Scholar. Why Sir, you make me smile, to beare me in hand, that these two demands may be simply resolved be Multiplication and Division.

Master. Truely that they may, and that you may by and by work your self with a little labour: First, adde together their quantities of powder, that is 9 pounds, 5 pounds, 4 pounds, and 2 pounds, all which make 20: Divide the 700 pounds of powder by that 20, and your Quotient giveth 35, as here appeareth, which sheweth for most certainty that they shall make just 35 shootes about.

1
700
202(35
2

Scholar. Sir, all this have I done, and I see it is so, but whether it be true or not, I cannot tell.

Master. To try the truth of the same, multiply the first peece that spends 9 pounds by 35, and you shall see his allowance, which is 315 pounds of powder. Multiply also the second peece the

spends

spends 5 pounds by 35, and you shall find 175 pounds his allowance: then 4 by 35, and you shall find 140 pounds his allowance. Lastly, multiply 2 by 35 and you shall find 70 pounds his allowance. All which four particular summes you shall adde together by Addition, as here appeareth, and it maketh just 700 pounds, and so is the question truely absolved.

315
175
140
70
700

Scholar. Truely Sir, these excellent conclusions doe wonderfully more and more make me in love with the Art.

Master. It is an Art, that the further you travell, the more you thirst to goe on forward. Such a Fountaine, that the more you draw, the more it springs: and to speak absolutely, in a word (excepting the study of Divinity, which is the salvation of our Soules) there is no study in the world comparable to this, for delight in wonderfull and godly exercise; for the skill hereof is well known immediately to have flowed from the wisdom of God into the heart of man, whom he hath created the chief image and instrument of his praise and glory.

Scholar. The desire of knowledg doth greatly incourage me to be studious herein: and therefore I pray you cease not to instruct me further in the use hereof.

Master. With a good will. And now therefore for the further use of these two latter, that is, Multiplication and Division, I will briefly shew you the feat of Reduction.

Reduction.

Reduction what it is.

REduction is by which all summes of gross Denomination may be turned into summes of more subtile Denomination. And contrariwise all summes of subtile Denomination may be brought to summes of grosser Denomination.

Gross Denomination.

Scholar. What call you gross Denomination, and subtile Denomination?

Master. That I call a gross Denomination, which doth contain under it many other subtiler or smaller: as a pound (in respect to shillings) is a gross Denomination: for it is greater then shillings, and containeth many of them. And shillings (in comparison to pounds) are a subtile Denomination, for because they are lesser then pounds, and many of them are contained in one of the other: and so likewise of other things: whatsoever thing is compared to other, if it be greater and containeth many of them, it is a gross Denomination, but if it be lesser (so that many of them are in the other) then are they called the subtile Denominations: whereby you may perceive that one Denomination may be called a gross Denomination, and also a subtile (that is to say a great, and small) in diverse comparisons. For shillings compared to pounds, are a subtile or small Denominotion: but compared to pence they are a gross, or great Denomination.

Subtile denomination.

Scholar. Now I understand the name, I pray you teach me the use.

Master, The use is easily learned, if you remember

member what you have learned before. For if you will reduce any summe of a gross Denomination into a summe of a smaller or subtiler Denomination, you must consider how many of that subtiler Denomination do make one of the grosser Denomination, and by that number or numerator doe ye multiply the summe. As if you would reduce 20 pounds into shillings, you must consider that in a pound are included 20 shillings, therefore multiply the one 20 by the other 20, and there will amount 400, whereby you may know that in 20 pounds are contained 400 shillings. Likewise, if you would reduce 30 shillings into pence, considering that in a shilling are 12 pence, you must multiply 30 by 12, and it will be 360, whereby you may find that in 30 shillings are contained 360 pence. And thus may you reduce any gross Denomination into a more subtiler, by Multiplication, if you know how many of the lesser doe make the greater: of which thing I will anon give you a brief Table for the most accustomed kinds of Money, Weights, Measures, and Time, and such like: whereby you may know how often each subtile Denomination is contained in the grosser, when you shall need it for the foresaid kind of Reduction. And also the same shall serve you, if you would reduce any summe of a subtiler Denomination, into a summe of a greater Denomination. For in such Reduction you must consider (as in the other form) how many of the smaller doe make the greater: and by that number you must divide the other summe, and the Quotient will declare how many of the greater Denomination are comprehended in that summe:

To reduce gross denomination to subtile.

as

as for example; If you would know how many shillings are contained in 3240 d. consider that 12 pence doe make 1 s. you must divide that 3240 by 12, and your Quotient will be 270, whereby you may know that so many shillings are in 3240 d. But if you would know further how many pounds are in these 370 shillings seeing that every pound containeth 20 shillings: divide that 270 by 20, and it will be 13 and 10 remaining, whereby you may know, that in 3240 d. (or 270 shillings) are 13 pounds and 10 shillings. For evermore the Remainer must be named by the name, or Denomination of that summe that was divided, which in this place were shillings. And thus may you do with any other kinds of Denominations.

¶ Wherefore, to the intent you may have certain light or knowledge in most common Coynes, Weights and Measures, (which is the chief and principallest thing in traffick to be known) I have in each Reduction, as they come in order, set down certain instructions incident thereunto. And first I have hereunto added this Table, wherein is comprehended, not onely our currant and common coynes, but also the most part of the usuall coynes of Christendome, with their just weights and value currant in the Realme of England, intending at the latter end of my Addition to this Book, to write of the ordinary Money used in divers places, and their common values currant for traffick, with the manner of their exchanges from place to place, &c.

A Table of the names, and now valuation of the most usuall Gold-coyns throughout Christendom, with their several weight of pence and Grains: and what they are worth of currant English money, this present year 1630.

The names & titles of the Golde.	The weight in Pence.	Granes.	The value in Shil.	Peuce.
Great Soverain,	10	0	33	0
Double Sover. K.H.	8	1	22	0
Double Sov. of Q.E.	7	7	22	0
Royall.	4	23	16	6
Half Royall.	2	1d.	8	3
Old Noble.	4	6	14	8
Half Noble.	2	3	7	4
Angell.	3	8	11	0
Half Angell.	1	16	5	6
Salute.	2	5	6	11 ob.
2 parts of Salute.	1	11	4	7
George Noble.	3	0	9	9 ob.
Half George Noble.	1	11	4	11 q.
First Crown K. H.	2	9	6	1.1 ob.
Base Crown K. H.	2	0	5	6
Sover. K.H. *best.*	2	14	11	8 ob.q.
Soverain K. H.	4	0	11	0
Edward Sover.	3	15d.	11	0
Elizabeth Sover.	3	15d.	11	0
Elizabeth Crown.	1	9	5	6
Half Crown.	0	19	2	9
Unite.	0	12	22	0
Double Crown.	3	6	11	0
Britain Crown.	1	1	5	6
Thistle Crown.	1	7	4	4 ob.q.

Half

The names & titles of the Gold.	The weight in Pence.	Grains.	The value in Shil.	Pence.
Half Crown.	0	19d.	2	9
Crosse Dagger.	3	6 d.	11	0
Half Crosse Dagger.	1	15	5	6
Rose Royall.	0	21	33	0
Spur Royall.	4	10d.	16	0
The Angell.	2	23d.	11	0
Half Angell.	1	11d.	5	6

All the severall pieces of gold heretofore mentioned, are set down according to their valuation by the Kings Majesties Proclamation for Gold, dated the 23 *of November,* 1611.

A Table of forain Gold coyn, according to their ancient valuation and several weight in Pence and Grains.

The names & titles of the Gold.	The weight in Pence.	Graines.	The value in Shil.	Pen ce.
Vnicorn of Scot.	2	10	6	0
Scottish Crown.	2	5	6	0
French Noble.	4	16	13	4
All sorts of Fren. Crowns	2	5	6	0
Flanders Riders.	2	6	6	6
Gelders Rid rs.	2	2	3	6
Philips Royall.	2	10	10	0
Philips Crown.	2	5	5	0
Collen Gilden.	2	2	4	8
New And. Gild.	2	2	5	0

Flanders

Flanders noble.	4	10	12	0
Half Flan. Noble.	2	6	6	0
Flan. Angel best.	3	6	3	0
Flan. Royall or &c.	3	10	10	0
Carolus Gilden.	0	12	3	6
Flanders Royall.	2	6	5	0
Saron Gilden.	2	2	4	8
Flanders Crown.	2	5	6	0
Philips Gilden.	2	3	4	2
Half Phil. Gilden.	1	1	2	1
Golden Lion.	1	16	7	8
3 parts of goldẽ Liõ	0	21	2	5
⅓ parts of gol. Lion	1	19	4	11
Davids Gilden.	2	2	4	0
Horne Gilden.	1	12	4	11
Old under Gilden.	2	3	4	10
Crusa. long Crosse.	2	6	6	0
Crusa. short crosse.	2	6	6	2
Milreys.	4	20	1	4
Half Milreys.	2	10	6	8
Portague 1 ounce	2	16	68	0
Golden Castile.	2	23	8	10
Ducket of Aragon.	2	6	6	6
Hungary Ducket.	2	7	6	4
Double Pistolet.	4	9	11	8
Single Pistolet.	2	4 d.	5	10
Ducket of Floren.	2	5	6	4
Double Ducket.	4	11	13	0
Single Ducket.	2	6	6	6
Double duc. of Rome.	4	13	12	8

It is to be understood (gentle Reader) that whereas in these Tables, the weight is called by the name of a penny

penny, it is not ment a penny of silver money, but a penny of Gold-smiths weight, which containeth 24 Barley-corns. Concerning which see Troy weight in folio 133.

So if a man have not the weight wherewith to weigh any peece of gold, he may do it with barley-corns, being dry, and as it is said, folio 133.

The prices of Gold which the bringers in of forrain Gold shall receive at the Mint, according to the Kings Majesties Proclamation Dated the 14 of *May Anno* 1612.

For an ounce of *French* crowns being 22 Karacts fine.	3 li.	6 s.	
For every ounce of *Spanish* Pistolets, being 21 Karacts, 3 grains and a half fine.	3 li.	6 s.	
For Duckets of *Spaine*, being 23 Karacts, 1 grain fine at least the ounce.	3 li.	8 s.	8 d.
For Milreas Crusado long cross. Crusado short cross, the ounce.	3 li.	6 s.	2 d.
For *Hungary* Duckets being 23 Karacts, 1 grain fine at least the ounce.	3 li.	9 s.	2 d.
For the Checkeen of *Venice*, being 23 Karacts, 1 graine fine at least the ounce.	3 li.	10 s.	
For *Barbary* Gold being 23 Karacts & di. grain fine, at the least the ounce.	3 li.	9 s.	

¶ *And if the said Barbary Gold be of less fineness, abatement to be made according to the rate.*

For

For Sultaines being 23 Karects, 1 Graine fine at least the ounce.	3 li. 8 s. 8 d.
For all other Gold, being 22 Karects fine the ounce.	3 li. 6 s.

¶ *And being finer, a greater price according to that rate; and being courser a less, so that the bringer in supply the less fine with the more fine, in such sort, that in the totall it makes good the same rate of 22 Karects fine.*

The Price of Silver, which the bringers in of forain Silver shall receive at the Mint, according to the Kings Majesties aforesaid *Proclamation.*

FOr the ounce of *Spanish* Silver money of *Sevill.*	5 s.
For the ounce of *Mexico* money.	4 s. 10 d.
For Ingots of Silver, being 11 ounces, 2 d. weight fine according to the Standard of *England*, the ounce.	5 s.

¶ *And for other Silver of more fineness, a better price according to that rate, and for courser a less: so that the bringer in supply the less fine with the more fine, in such sort, that in the totall it makes good the said rate of* 11 *ounces,* 2 *penie weight fine, according to the Standard of England.*

Of Silver Coynes currant in this Realme.

The *Edward* crown of 5 s.

The *Edward* half-crown of 2 s. 6 d.

The *Edward* shilling, half-shilling, and the three pence.

Philip and *Maries* shilling, and half shilling.

The *Mary* groat, and *Mary* two pence.

Queen *Elizabeths* shilling. 9 d. 6 d. 4 d. 3 d. 2d. 1 d. three farthings, and half penny.

HEre would I now express the values of sundry other Coynes of divers Countries, but for

three causes I now refrain. The first and chiefest is, because they are not currant by the statutes of this Realm. Another cause is, by reason they are so uncertain, that they be never long at one rate. And again, they are so different in so many places, that it were matter enough for a great Book to speak sufficiently of them all. Howbeit, because you shall not be altogether ignorant of them, I will shew you the values of some that are most in use, and first of France.

French Coynes. The most common Money are Deniers, Soulx, and Franks: 12 Deniers make 1 shilling, 20 Soulx make 1 Frank: so that you may see these three kinds are like in the rate to pence, shillings and pounds with us; but that this is the difference, that their Denier is but the ninth part of our penny, and to their Soulx (commonly called Souses) goe 9 to our shilling, and 9 of their Franks to an English pound of money. So that three of their Franks make a Noble. And by those three you may practise how to reduce French money into English money, according as I have set forth here following.

2160	Deniers make	240 d. or 20 s.
3240		360 d. or 30 s.
8352		928 d. or 3 li. 17 s. 4 d.

2160 Soulx make 240 shillings. And so of other in like late. As for the rest of their Coynes I omit them till hereafter that you have some understanding in broken numbres.

Flanders Coynes. But now as for the Coynes of Flanders, they be so changeable, that you must know them from time to time, else you cannot reduce them into our money certainly; but yet that you may have an example

ample of their money to exercise you withall, you shall take those that be most common: as Stivers both single and double, Groates Flemish, Carolus and Gyldens. A Flemish Groat is a little above 3 farthings English. A single Stiver is 1 d. ob. q. half farthing. The double Stiver Carolus is 4 d. ob. half farthing. Then there is also the Carolus Gylden which is worth 10 Stivers. And the Flemish Noble is worth 3 Carolus Gyldens, and 12 Stivers.

So that if you would convert Flemish money or any other kind of money whatsoever it be, justly into Sterling, you must reduce it first into the smallest part of English money that is in that Coyne. As for example: If I would reduce 368 double Stivers into English money (considering that a double Stiver containeth 3 d. farthing) you shall first look how many farthings be in the double Stiver, and you shall find them 13, therefore multiply the summe of the Stivers by 13, and then have you their value in farthings, which is 4784. Now if you divide that by 4, then there will appear the number of pence: but better it were to divide it by 48 (for so many farthings are in one shilling) and then will the Quotient declare the summe of 4 li. 19 s. 8 d.

Likewise if you would reduce any summe of single Stivers into English money, you must multiply the summe first by 13, and then have you reduced them into a certain summe, that is to wit, half-farthings, which summe if you divide by 8, then will amount the summe of pence: or if you divide it by 96, the summe of shillings will appear.

But marke this in all Division: when ye do reduce to bring one Denomination into another, if there be any Remainer after the Division, that must be named by the Denomination of the gross sum that was divided. As for example, I would bring 254 farthings into pence, therefore I divide that 254 by 4 (for so many farthings make a penny) and the Quotient in 63, which is the summe of the pence, and then remaineth yet 2, which are farthings still, as one may prove by dividing. And this must be marked in all Division, namely, when it is done for Reduction,

Note well.

¶ Touching Danks Mony, they have their Soulx, whereof 20 is a Liver which is 2 shillings sterling. They have also their Grash whereof 80 make a Gilden, which is 4 shillings sterling. They have also Dollors, and their common or old Dollor is 35 Grash. New Dollors they have which be divers, some valued at 24 Grash, some at 26, and some at 30. And thus much I thought good to adde to the Author, touching Danks Mony.

Danks Mony.

Concerning Spanish Mony, whereof the most common are Cornadoes, Marveides, Marveide, 4 Marveides make a Ryall, and 11 Ryalls make one Ducket, so the Ducket containeth 374 Marveides, which is about 5 shillings 10 pence sterling. Therefore if you would convert 124 li. 5 s. sterling into Duckets, consider that pence is the last value or Denomination named in this question: therefore reduce 124 li. 5 s. into pence, and it maketh 29820 pence: which if you divide by pence that a Ducket is worth, (which is 70) you shall have for your Quotient 426 Duckets your desire.

Spanish Mony.

In

In Venice they have Bettes, Souldyes, Lieure. 5 Bettes make an English penny, 60 Bettes a shilling, which is 2 Souldyes, and 20 Souldyes a Lieure of Venice which is a pound sterling. Venice Money.

Thus much have I said of Money: Now will I shew you in the like sort the distinction of weights. Weights.

After a Statute made anno 11 *H.* 7. *there ought to be but one sort of weight, as* 24 *Barley-cornes dry and taken out of the midst of the Ear, do make a penny weight,* 20 *of these penny weights make an Ounce;* 12 *Ounces a pound of Troy weight, by which is weighed Bread, Gold, Silver, Pearle, Silke, and such like.* But commonly there is used another weight called Haberdupoise; in which 16 Ounces make a pound. Therefore when you would reduce Ounces into pounds, you must consider whether your weight be Troy weight or Haberdupoise: and if it be Troy weight you must divide your Ounces by 12, to bring them to pounds, but if it be Haberdupoise, you must divide them by 16. Now again, there be greater weights, which are called a hundred, half a hundred, and a quarterne, and also a half quarterne, &c.

Troy weight. A penny weight. An ounce. A pound Troy. Haberdupoise weight. A hundred weight.

Scholar. Why? so there may be reckoned 20 pound, 40 pound, 200 pound, and such innumerable.

Master. All these are numbers of weight, but they have not common weights made to their rate as the other have. And again, these that I did name are not just in number as they seem by their name: for an hundred is not just 100, but is 112 pound. And so the half hundred is 56, the quarterne 28, and the half quarter 14.

And these be the common weights used in most things that are sold by weight.

Wooll Weights. Todde. Stone.

Howbeit there are in some things other names, as in Wool 28 pound is not called a quarterne: but a Todde: and 14 pound is not named half a quartern, but a Stone, and the 7 pound half a Stone. Other names because they differ in many places, and agree in few, I let them pass.

Sack of Wooll.

But a sack of Wooll by the Statutes is limited to be 26 Stone.

Cheese weights.

¶ Now in Cheese, though it be sold by the hundred, and by the Stone in some places, yet the very weights of it are Cloves, and Weyes. So that a Clove containeth 8 pound, and a VVeye 32 Cloves, which is 256 pound, that is 12 score and 16 pound, and so much weigheth the VVeye of Suffolk Cheese, and the like is or should be the Barrel of Suffolk Butter.

The VVeye of Essex Cheese containeth six score and sixteen pound: and so much is also the Barrel of Essex Butter.

Moreover this weight is used by the Apothecaries in their Physicall composition, and mixture in medicine, wherein the least is a grain,

The Apothecaries weights.

And	20 *Goaines*	make	*A Scruple*	thus charactered.	℈
	3 *Scruples*		*A Drachme or Dragme*		ʒ
	8 *Dragmes*		*An Ounce*		℥
	16 *Ounces*		*A Pound.*		lb.

Measures for liquor. A pint. Gallon.

Now of weights are made other measures both for grain and liquor. For a pound in Troy weight maketh a pint in measure, so that 8 pound or 8 pints doe make a gallon: half a gallon is named a pottle,

and

and half a pottle is called a quart which containeth two pints: Now above a gallon the next measure is a Firkin: then the Tertian or Kilderkin or half a Barrell, and a Barrell: And by these measures are sold commonly Ale, Beere, Wine, and Oyle, Butter and Soap, Salmon, Herrings, and Eeles.

Pottle. Quart. Ferkin. Tertian. Kilderkin. Barrell.

But as these be unlike things, so the measures of their vessels doe differ, for the measures of them all are as followeth.

Of Ale	*The firkin*	*containeth*	8	*Gallons.*
	The kilderkin		16	
	The barrell		32	

Ale measures.

Of Beer	*The firkin*	*containeth*	9	*Gallons.*
	The kilderkin		18	
	The barrell		36	

Beer measures.

Soap measures, both Firkin, Kilderkin, and Barrell should be equall to Ale measure.

Soap measures.

Moreover the Statutes do limit the weight of every of those three vessels being empty.

A barrell	*to weigh empty*	26	*Pounds.*
Half a barrell		13	
A firkin		$6\frac{1}{2}$	

Herrings also sold by the same measures that Ale and Soap be sold by.

Herrings.

Herrings are sold by the tale, 120 to the hundred, ten thousand to the Last.

Salmon and Eeles have a greater measure.

Salmon & Eeles	*The butt*	*holdeth*	84	*Gallons.*
	The barrell		42	
	Half barrell		21	
	The firkin		$10\frac{1}{2}$	

Salmon & Eeles.

Howbeit, some Statutes did limit Eele vessels equall with Herring vessels.

Wine measures.

Now as for wine veſſels they are ſeldome ſmaller then Hogſheads which are of 63 Gallons: Every Hogſhead, is two Barrels: yet there are many other wine veſſels, but of them all ſee this Table; and marke the meaſures one by another.

Of wine and oyle	*The runlet*	*hold-eth*	$18\frac{1}{2}$	*Gallons.*
	The barrell		$31\frac{1}{2}$	
	The hogſhead		63	
	The tertian		84	
	The pipe		126	
	The tunne		252	

Tertians.

But you ſhall mark that there be other kinds of Tertians: for there be Tertians, (that is to ſay) Thirds of pipes, of hogſheads, and of barrels, as well of other things as of wine.

A Butte.

Alſo Malmſeyes, and Sacke, &c. the half Tun is not called a Pipe, but rather a Butt.

And thus much have I thought meet to tell you at this time.

Scholar. And is that alwayes true?

Maſter. I have told you how it ſhould be, but how it is, I may not ſay: how they doe differ daily from their juſt meaſure, that Gaugiers can tell you better then I. But I will let this paſs now, and ſpeak briefly of the other meaſure.

Drie meaſures.

And as of weights there did ſpring the liquid meaſures (whereof I ſpake laſt) ſo of the ſame ſpringeth dry meaſures, as Pecks, Buſhels, Quarters, and ſuch like, whereby are meaſured corne and like grains, alſo Salt, Lime, Coals, and other like. And this is the order and quantity of them.

A pecke is the meaſure of two Gallons.

A

A bushell } containeth { *foure pecks.* — A Bushell
A quarter } containeth { *eight bushels.* — A Quarter.
A wey } containeth { *six quarters.* — A Wey.

These are the common names and measures, but in divers places there be divers sorts.

The Bushell in many places is two bushels, but then is that bushell there called a strike: and in some places half a quarter is called a Cornock. But those diversities are too many to tell you briefly them all, and again, sith they are against the Law and Statutes, I count them unmeet to be used.

Strike. Cornock.

But now remaineth yet another kind of measures, whereby men mete length, breadth and thickness, and those are, an Inch, a Foot, and such other, whose names and quantities this Table sheweth.

Measure to mete length, breadth, & thicknes.

3 *Grains of barly in length make an inch.* — An Inch.
12 *Inches* } *make* { *a foot.* — Foot.
3 *foot* } *make* { *a yard.*
3 *foot and 9 Inches* } *make* { *an ell.* — Yard.
5 *Yards and a half* } *make* { *a pearch.* — Elle.

1 *Pearch in breadth, 40 in length, doe make a rodde of land, which some call a rood, some a yard land, some a farthendele.* — Pearch.

2 *Farthendels* } *make* { *half an acre of ground.*
4 *Farthendels* } *make* { *an acre.* — Acre.

¶ *More, 40 rods in length do make a furlong, 8 furlongs make an English mile, which containeth 320 Perches.*

So that an English mile, grounded upon the Statute, is in lenguh 1760 yards, 5280 foot, and 63360 Inches.

Somewhat greater then the Italian mile of 1000 paces, and 5 foot to a pace.

Here

Here might I tell you many things else touching measures, and also how to reduce strange measures to our measures, but because it cannot be well done without the knowledge of Fractions, which as yet you have not learned, I will let them pass till another time, that I have taught you the knowledge of broken numbers.

The parts of time.

Scholar. But yet Sir of the parts of time, I pray you tell me somewhat.

A day. An houre, Week, Moneth, Year.

Master. You know that a naturall day hath 24 houres, and every houre hath 60 minutes. It needeth not to tell you, that 7 dayes make a week, and 4 weeks make a common moneth, and 13 months make a year, lacking one day and certain houres and minutes: but of that I shall instruct you hereafter,

Here will I make an end of Reduction *for this time, which though it be counted no kind severall of* Arithmetick, *you see it is no less needfull to be known, or easier to be done, then any of the other.*

Scholar. Marry sir, it seemeth unto me much harder then any other sort, for it requireth the knowledge of so many things: but now Sir when you see time I am ready to learn forth, as much of Reduction as you have taught me, I remember; but and if I doe at any time forget, I shall have recourse to the Tables which you set forth for me.

Master. So doe you: for it will not be remembred without exercise. But in as much as you understand so much as we have intreated of, I will now instruct you in Progression.

Progression.

Progreſſion.

Although untill this day the moſt part of Writers have defined Progreſſion *as a compendious kind of* Addition, *yet truely it is not ſo : for* Progreſſion *(as the very nature of the word doth inform any man) is a going forward and proceeding in numbers, and that regularly and orderly, whoſe place is aptly choſen to be very neere, or rather next after* the expoſition of the foure principall parts of Arithmetick : *for in it after a moſt eaſie manner, are all the foure former parts exercised and practiſed : and not onely* Addition, *as cuſtomably is done. Which cuſtome hath been the cauſe, why it hath ſo ſpecially been named a kind of* Addition, *and defined to be a quick and brief* Addition *of divers* ſummes, *proceeding by ſome certain and reaſonable order.* What Progreſſion is.

You ſhall alſo underſtand there are infinite kinds of Progreſſions, but for you (as yet) two are ſufficent to be exerciſed in, of which the one I call Arithmeticall and the other Geometricall.

Arithmeticall progreſſion *is a rehearſing or placeing down of many* numbers, number *after* number, *in ſuch ſort that between every two next* numbers *rehearſed or placed down, the difference, diverſity, or exceſs, be equall and alike.* Arithmeticall Progreſſion.

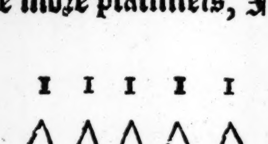
Scholar. Sir I thank you for that you have both opened unto me what Progreſſion is truly, and alſo why it is here placed.

But I pray you with an example make plain your definition.

Maſter.

Maſter. Examples cannot want, ſeeing all reaſonable creatures naturally uſe the order of one kind of Arithmeticall progreſſion (which therefore is alſo named naturall) whenſoever they diſtinctly do count or number any multitude by one, ſaying, 1, 2, 3, 4, 5, 6, whereby the proceeding from number to nnmber and every one ſurmounting and exceeding his fellow next before by a like quantity (which here is 1) declareth the ſame to be Arithmeticall progreſſion. And for the more plainneſs, I ſet it down in this manner.

The common exceſs.



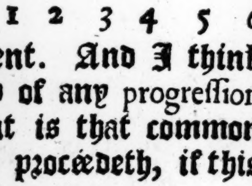
The Progreſſion.

Scholar. This is moſt evident. And I think that I am able to tell you now of any progreſſion Arithmeticall propounded, what is that common exceſs or difference whereby it proceedeth, if this order be kept in it.

Maſter. What ſay you of 3,6,9 12, 15?

Scholar. They exceed each other by 3: And that may I ſet down in ſuch evident order, as you did your example of naturall progreſſion, in this wiſe.

The common exceſs.



The progreſſion.

Maſter. And doe you not alſo now perceive, that the whole Table of Multiplication may be made by the other of progreſſion Arithmeticall?

either

either if you will begin at the firſt number of any of them on the left hand, and ſo procéed right overthwart: or any of the firſt number of the upper row, and go directly downward.

Scholar. I pray you let me conſider the thing a little, and I will anſwer you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceive it now very well, for the common exceſs or difference betwéen any two next, is continually as much as the firſt number of every row, either from the left hand overthwart taken, or from any of the uppermoſt overthwart rowes downward.

Maſter. Now then, if of any ſuch progreſſion, you would ſpéedily know the totall ſumme much quicklier then by common Rules of Addition: firſt tell how many numbers there are (which numbers here we call places or parcels) and if they be odde, write their ſumme down by it ſelf: as in this example, 2, 4, 6, 8, 10, 12, 14. where the numbers are 7, as you may ſée: therefore ſet down 7 in a place alone,

To know the totall ſumme of an Arithmeticall progreſſion.

alone, then adde together the first number and the last, as in this Example: Adde 2 to 14 and that maketh 16, take half of it, and multiply by the 7 which you noted for the number of the same places, and the summe that amounteth, is the summe of all those figures added together; as in this example 8 multiplied by 7 maketh 56, and that is the summe of all those figures.

Scholar. That will I work by another example. I would know how much this summe is, 5, 8, 11, 14, 17, 20, 23, 26, 29. I tell the places and there are 9, that I note. Then I put the first number 5; and the last 29, together, and they make 34. I take half of it, that is 17, and multiply by 9, and it maketh 153. That you say is the summe of all the numbers.

Master. So shall you find it if you try it.

Scholar. How shall I try it?

Master. By your common Addition, for if you adde all the parcels together, you shall see the same summe amount, if you did work well. And that manner of Addition trieth all kinds of summing any progression.

Scholar. Then can I summe any progression, if the number of the parcels be odde. But what if they be even, as in this example, 1, 2, 3, 4, 5, 6, 7, 8?

Master. When the number of the parcels is even, then note that also as you did before, and likewise adde the first summe to the last, and by the half of the number of the places, do you multiply it: as in our example, the parcels are 8, that I note, then adding the first summe to the last,

last, there amounteth 9, that doe I multiply by half of the parcels, that is, by 4, and it maketh 36, which is the summe of the parcels.

But if you will take one Rule for these both, doe thus: Multiply the half of the one by the other whole, and the summe will amount all one. For sometime it chanceth that the number of the parcels be odde, so their half cannot be taken: and that some time it chanceth the Addition of the first number and the last, doe bring forth an odde number, so that half of it cannot be taken: but they will never be both odde. A generall rule.

Scholar. Then I perceive this, if there be no more belonging to it.

Master. As accustomably it hath been taught, this hath been the chief and onely exercise in Progression used. But that you may perceive how divers wayes and to how great profit so simple a thing (as this Arithmeticall Progression is) may be considered and used, I will here propound you six Propositions, of which foure of them were invented by a friend of mine, and never before this published: and the two first were never to my knowledge written of but by three men.

Scholar. This doth greatly encourage me to be attentive unto your words, seeing I shall not onely be instructed at your hands in the common known Rules of this excellent Art but besides that so aboundantly in other new Rules enformed, as my very entrance shall seem to pass a great many mens further studie, and longer continuance. Therefore Sir, I beseech you let me know your six Propositions.

Master.

Maſter. These they are.

1 *To know the laſt* number *without proceeding by continuall* Addition, *till you come unto it, ſo that the common exceſs, the firſt* number, *and the* number *of the places be known.*

2 *The firſt* number *of the* Progreſſion *and the laſt being known, with the common exceſs to find the* number *of the places.*

3 *The exceſs being given, and the* firſt *or* laſt, *to know the quantity of any middle* number, *whoſe place is given from the* firſt *or* laſt.

4 *The* totall ſumme *being given, and the* firſt *and* laſt, *to find out the* number *of the places.*

5 *The* totall ſumme *of any* Arithmeticall Progreſſion *being given, and the* firſt *and* laſt, *to find out the common exceſs.*

The totall ſumme *being given, and the mutuall exceſs with the* number *of the plases, to give the* firſt *or* laſt number *of the ſame* Progreſſion.

Many moe conſiderations could I propound you in theſe *Arithmeticall Progreſſions*, but theſe are ſufficient, to give you occaſion to think, that Rules of knowledge and Arts are infinitely capable of enlargment.

Scholar. Happy were I if I did but well underſtand that which is already invented and written: And yet in my ſimple fantaſie, theſe things offer themſelves (in manner) to be ſtudied for about Progreſſion, therefore I pray you to procœd to the Rules anſwering to theſe propoſitions.

Maſter. I will orderly for every of theſe 6 propoſitions give you Rules, and with every one an example, unleſs the plainneſs and eaſineſs nœd no further exemplifying.

For

For the Solution of the firſt, multiply the exceſs by a number *leſs by* 1 *then the* number *of the places, and the off-come adde to the firſt* number, *ſo you ſhall have the laſt* number, *which is ſought for.* 1 Propoſition.

As for example. If there were ſeven places in a progreſſion Arithmeticall, whoſe continuall encreaſe or mutuall exceſs were 4, and the firſt number were 5, and I would know what the laſt and ſeventh number is: I multiply 6 which is one leſs then 7, (the number of the places) by 4, thereof commeth 24, which I adde to 5, that maketh 29, and that is the laſt number which I deſire to know. And this you may ſtraightway prove by continuall procéeding from 5, till the ſeventh place, encreaſing every one by 4, as thus.

5, 9, 13, 17, 21, 25, 29.

Loe here the laſt, being alſo the ſeventh, is 29.

Scholar. I perceive already one good property in this Rule, which in all works is to be deſired: that is, it will eaſe one from great labour, if a progreſſion were propounded of a hundred or two hundred places, or moe: And alſo it is very eaſie to work, and moſt neceſſary for the totall ſumme finding, in a very long progreſſion.

M. It is true, and therefore now let me ſée if you can anſwer me this queſtion by this propoſition.

A Merchant buyeth 50 pounds of Spices, and agreeth to pay for the firſt pound 4 pence, for the ſecond 7 pence, for the third 10 pence, for the fourth 13 pence, &c.

The queſtion is, how much he muſt pay for the laſt pound, and then how much the 50 pound commeth to?

Scholar. According to the proposition, I multiply 49 (which is less by one then the number of the places) by the excess, which is 3: to the product 147, I adde the first number which is 4, it maketh 151 pence, the price of the last pound. Now I adde 4, the price of the first pound, to 151 the price of the last pound, it maketh 155, which I multiply by half the number of the places, which is 25 the Product, 3875 pence is the totall sum or price of the 50 pounds of Spices, as appeareth.

49 places 1 less	151 last
3 excess	4 first
147	155
4 first	25 half places
151 the last	775
	310
	3875 totall summe which amounteth to
	li 16 — s 2 — d 11

Master. It is truly wrought.

Scholar. Then I intreat you to proceed to your second proposition.

2 Proposition. Master. *The second* Rule *is this. From the last subtract the first, the* remainder *divide by the common excess, to the* Quotient *adde* 1, *and you have the* number *of the places, which you would know, as in this* Progression.

6. 11. 16. 21. 26. 31.

If I know only 6 and 31, and that they encrease by 5, then according to the Rule, from 31 I subtract

subtract 6, there remaineth 25, which 25 I divide by 5 (the common excess) the Quotient commeth forth 5, to which I adde 1 that maketh 6: and so many are the places as you see.

Scholar. This Rule is so easie, that I were much to blame, if I could not remember it.

Master. *The third* Proposition *may alwayes thus be solved. Multiply the* excess *by a* number *less by one, then the distance of the place is from the* first, *or the* last number given: *the offcome adde to the* first, *if the* distance *be reckoned from the* first, *and the* first *also known; or subtract from the* last, *if the distance be from the* last counted, *and the* last given *also, and that which commeth forth, either in that* Addition *to the* first *or* Subtraction form *the* last, *is the* number sought. As for Example, *I propound you this* Progression. 3 Proposition.

8 15 22 29 36 43 50 57

And for the apt considering the manner of this question, I will note over every place his distance from the first, and under every place his distance inclusively from the last, thus.

1	2	3	4	5	6	7	8
8	15	22	29	36	43	50	57
8	7	6	5	4	3	2	1

Now if the excess whereby this progression standeth, be known to be 7, and the first numbers given, being 8, I would know what number standeth under 4, that is to say, in the fourth place. I multiply 7 by 3 (which is less by 1 then the number of the place proposed) that yeeldeth 21, to which I adde 8, the first number, so commeth 29: which I say to belong to the fourth place, as you see in the example: or if in the

third place from the laſt, you would know what number in this example ſhould ſtand, the laſt number being known to be 57, and the common exceſs 7, then by 2 which is leſs by 1 then the place propounded, I multiply 7, that giveth 14, which appertaineth to the third place incluſively reckoned from the laſt, and ſo my example giveth you.

Scholar. I perceive right good uſe of this rule: for if I had forgotten what the firſt number were, and remember ſtill but the laſt, the common exceſs, and the number of the places, then might I come by the knowledge of my firſt number again.

And me thinketh, that it differeth not much from the firſt propoſition, ſaving that which you make here a middle number, there was made the laſt: and alſo in this point it differeth, that in it the laſt was only ſought, and no conſideration had in numbring the places from the laſt, as here I marke in your numbers noted under your Progreſſion.

Maſter. And think you not, the middle numbers of progreſſion ſtanding off a hundred or three hundred places or moe; may as much cumber a man to come to the knowledge of them by continuall encreaſing from the firſt by the common exceſs, or abating from the laſt continually the common exceſs as the very ſmall Numbers in a ſhorter progreſſion would doe?

Scholar. Yes ſir, that I think right well, and therefore I am glad of this new framed propoſition, and the manner of the working of it.

4 Propoſition. Maſter. *The rule of the fourth is this. Add the firſt and the laſt together, and by the off-come divide the totall*

totall summe. *Double the* Quotient, *and that will be the* number *of the places.*

Scholar. Then if in a progression, whose summe were 207, and the first number 12, and the last 57, if I adde 57 and 12 together, that maketh 69, and by it I divide 207, the Quotient will be 3, which I double; and so I have 6, and so many must be the number of the places that this progression standeth on.

Master. Whether it be so or no, how will you trie?

Scholar. Half 6, which is 3, being multiplied by 69, must make 207, the totall summe, if 6 be the number of the places. For so the whole work of your Rule in summing any Arithmeticall progression did enforme me. I will then multiply 69 by 3, thus.

It commeth forth justly.

69
3
207

Master. I must much herein commend your promptness both in memory and in well applying your Rule: although in manifest words it did contain no such matter.

Scholar. Sir, I pray you hear me frame one example or more.

Master. I am well pleased so that ye be short, for you make me stay longer here then willingly I would have been: but I cannot perceive how I could have omitted any thing as yet, without your great lack thereof.

Scholar. If I had received 85 pounds of certaine men, but of how many I have forgotten, yet I remember that the first gave me 7 pound, and the last 27 pound, and every payment after other did rise by

A question of money.

by a like ſumme. And the man for whom I received this money conditioned with me, that of every Payment I ſhould have twelve pence for my labour: now unleſſe I can by Art find the truth of this caſe, I am like to loſe the moſt part of my reward.

Maſter. I perceive you can handſomely frame an example, which ſhould concern your owne gaine: I pray you let mee ſee how you would do juſtice in this point.

Scholar. I adde the firſt and the laſt together, that maketh 34: by which I divide 85, thus:

1		
27		17
85	(2	
34		34

Why how now? Sir, here is a remnant of 17, in which 34 cannot be had: ſo that now I am in the briers for doubling of my Quotient, and farewell then both my Juſtice, and a good lump of my gains.

Maſter. Ye are never the farther from the matter, though it fall into a Fraction. For you ſhall underſtand that the fraction which of any ſuch work proceedeth, is every half of one ſuch, as the units of the Quotient before are. And that you may try, if you double that which ſo remaineth, for then it will be equall to your diviſor, as if ye double 17, (the remnant) it maketh 34, and your diviſor alſo was 34, this noteth the remainder to be half of one.

Scholar. Now I am glad of this hard Example: For with it I have a generall rule for the Fraction that may hap in this work. So that the Quotient being 2 and a half I double that, it maketh 5, therefore ſhould my gain be 5 ſhillings. And to be ſure (by your leave) I will try it, for I will multiply

half

half of 34 (which is the firſt and laſt number joyned together) by 5, thus.

17
5
—
85

It is moſt true (I ſee) that I ſhould leeſe nothing by the former working.

Maſter. *The fifth propoſition hath this rule appertaining unto it: By the fourth rule find the* number *of the places, that being done, from the laſt ſubtract the firſt, and the reſidue divide by a* number *leſs by* 1 *then the* number *of the places, and the* quotient *will ſhew the exceſs, which is ſought for.* 5 Propoſition.

An example hereof ſhall be this. If ye had diſburſed 685 pounds to a certain number of men, you neither can tell how many they were, or how much the ones money exceeded his next before, but you are ſure that the exceſs was equall between every two next: and alſo you remember that the firſt had 19, and the laſt 118 pounds, how would you find the number of the name and the exceſs continually obſerved in the ſucceſſion of their payments? Example.

Scholar. Your Rule doth plainly bid, firſt to find the number of the places, which I will do according to the fourth rule: I adde 19, and 118 together, thus.

118
19
—
137

By this 137, divide 685 thus.

Seeing there is no Fraction but a whole number, being 5, I double that, and then muſt the number of the places be 10. Now from the laſt I ſubtract the firſt, as 19 from 118, thus: And ſo remaineth 99.

~~1~~~~3~~
~~685~~ (5
~~137~~

118
19
—
99

This 99 I divide by a number leſs by one then the number of the places, and ſeeing the places where 10, I divide

99 by 9, thus : 99

The quotient is 11, and so was the excess, if I have followed your rule right. 99 (11

Master. You have wrought every part of this question both well in order, and truly in the practise of your Rules.

Scholar. I will then set it down also formably, so that the number of the places, the excess, and the totall summe may straight appear, as your first example stood.

The common excess.	11	11	11	11	11	11	11	11	11	11
The progression.	19	30	41	52	63	74	85	96	107	118

That the places be 10, and that from the first to the last, the common excess is 11, I perceive most evidently: but whether the totall sum be 685, I have not yet proved, which I will now doe: I adde 19 & 118 together, that maketh 137: I multiply that by half the number of the places, thus.

137
5
685

All things agrée most exactly, so that I am perfect enough in these Rules if I forget them not again.

Master. Use maketh all things perfect.

6 Proposition. *Your sixth rule is this. By the number of the places divide the totall summe, double the quotient, and that will be the first and last joyned in one summe. Then by a number less by 1 then the number of the places, multiply the excess, that off-come subtract from the first doubled quotient, and the half of the residue is the first number. The last number you may diversly find out, as by the first of our 6* Rules, *or by subtracting this first number from the* summe *which here contained both the first and last joyntly, (or thirdly) by continuall adding the excess.*

Scholar.

Scholar. I pray you make this ſomewhat more plain with an example.

Maſter. If every month in the year (counting them now as 13) you gained clearly 40 ſhillings more then you did the month next going before, and at the years end you find the whole gain 5720 ſhillings, but ye remember not how much either the gaine of the firſt month or the laſt was, by this Rule it may be tried out. Example of gaine.

Scholar. So that here ye ſeem to apply the 13 months to thirteen places, the 40 ſhillings every one more then the other next before it to be the common exceſs, and 5720 ſhillings to the totall ſumme.

Maſter. It is true: by 13 then I divide 5720 in this manner.

~~1~~
~~15~~
~~5720~~(440
~~1333~~
~~11~~

I double this Quotient, ſo have I 880 for the firſt and the laſt ſumme joyned together, by 12 which is leſs by one then the number of the places; I multiply 40 (the common exceſs) ſo commeth 480.

40
12
80
4
480

This 480 I ſubtract from 880, ſo remaineth 400: half whereof is the firſt number which we deſired to know: that is 200

And as for the laſt number, I can give you it three ways, As by the firſt of my ſix rules I multiply the exceſs, by a number leſs by 1 then the number of the places, as 40 by 12 that giveth 480, which I adde to the firſt, being 200, ſo ſhall the laſt be 680.

The ſame ſumme commeth forth if ye ſubtract 200 from 880.

And thirdly, If I begin at 200, and ſo proceed, encreaſing

encreaſing by 40, I ſhall at the thirteenth place have 680, as thus:

200 240 280 320 360 400 440
480 520 560 600 640 680

Scholar. I thank you moſt heartily for theſe ſix Rules. Now if it be your pleaſure I would hear and learn ſomewhat of progreſſion Geometricall.

Maſter. There are yet very many Rules and propoſitions which fall into this Arithmeticall progreſſion.

And for the uſe and practiſe of them, I will propoſe unto you certain pleaſant and neceſſary Queſtions of Arithmeticall progreſſion, and to the performance of their workings, ſuch neceſſary rules and documents, as are requiſite for the better underſtanding of them, or any ſuch like.

A queſtion of Velvet.

A certain Mercer *ſold* 20 *yards of Velvet to* 6
be paid in 12 *weeks by* Arithmeticall proporti- 12
on; *that is to wit, to receive the firſt* 18
week 6 ſhillings, *the ſecond week* 12 ſhillings, 24
the third week 18, *and ſo forth, increaſing the* 30
number *of weeks by* 6 ſhillings, *till the twelfth* 36
and laſt week were expired. The queſtion is 42
how may pounds he had for 20 yards of Vel- 48
vet. 54

To the performance of this queſtion, 60
and ſuch other the like, I ſet forth the 12 66
payments in ſuch ſort, as for example here 72
appeareth.

Then touching the adding together of theſe

ſumm s

ſummes, without the aid of Addition, according to the rules I taught you in progreſſion Arithmeticall, I note the number of the places, which are 12, then adding the laſt number of the progreſſion, which is 72, and the firſt number together make 78; and multiplying 78 by half the number of the places, which is 6, amounteth to 468 ſhillings, and in pounds maketh 23 li. 8 s. And ſo much hath the Mercer for his 20 yards of Velvet, which is nigh about 23 ſhillings 5 pence a yard.

Scholar. I underſtand this work very well, but is there any proof for the juſtifying hereof, as you have of other works?

Maſter. The work of it ſelf (being ſo perfectly wrought) that in your proceeding and going forward from number to number, each number exceeding his fellow by an equall or like quantity, is all that is demanded for juſtifying of the ſame: yet notwithſtanding, because your requeſt is reaſonable, I will propoſe an example for the proof hereof.

A certain man is bound to pay for 20 yards of Velvet, the ſumme *of* 23 pound 8 ſhillings, *and it is to be paid weekly in* 12 *weeks or termes by* Arithmeticall progreſſion. The queſtion is therefore to know with what number the ſame progreſſion is to be begun and continued in ſuch equall proportion Arithmeticall, that in 12 weeks the ſame may be juſtly accompliſhed. The proof of the laſt queſtion.

For the reſolution whereof, and of all ſuch other like, reduce 23 pound 8 ſhillings, all into ſhillings which maketh 468 ſhillings. A generall rule.

Then adde 1 unto 12, the number of the termes,

it

it maketh 13 which 13 you shall multiply by half the number of the termes which is 6, it maketh 78; then divide 468 by 78, and you shall find 6, in the Quotient, which is the true number that shall begin and continue the said Progression. That is to say, the first week 6 shillings, the second 12 shillings, and the third week 6 shillings more, which is 18 shillings, and so every week as they rise, 6 shillings more then the week before, as is manifest in the question aforesaid.

A question of a Farm. *A Farm is to be sold to be paid by the weeks in a year; the first week to pay 4 shillings, the second week 8 shillings, the third week 12 shillings, and so forth, increasing each number by 4, till the number of 52 (which are the number of weeks in a year) expired.* The question is, what the price of the Farm commeth to?

Scholar. I doubt not, but by that you have already taught me, to end this question very well; wherefore I set forth the Progression with his excess 52 times.

Master. Nay stay a while: And here for your further ease, (to abridge you of great labour that appeareth to fall out in this question, and so may doe in any other the like) If a question were proposed of 100 or 200 places, or moe, and that this question, or any other the like cannot be ended, unless you may know absolutely what the last number of the Progression of the 52 place is, (or ought to be) I will give you a generall rule how to know the last number of any Progression Arithmeticall, as well as if you had ordinarily proceeded by continuall Addition, till you had come to the last work which is this.

Mul-

Multiply the exceſs by a number leſs by one then the number of the places, and thereto put the firſt number of the Progreſſion, and you ſhall have your deſire.

A generall rule.

Scholar. This Rule is well worth the noting: for if I underſtand you aright, I conſider that my exceſs is 4, which I multiply by 51, which is one leſs then the number of the places, and it maketh 204, whereunto I adde the firſt number of the Progreſſion, which is 4, and then it is 208, which you ſay is or ſhould be the laſt number of the Progreſſion.

Maſter. This is a moſt approved truth, if there were never ſo many places.

Scholar. This Rule is ſo eaſie, that I were much to blame, if I doe not remember it. For by the benefit hereof, I have ſuch an eaſe and light into this excellent Art, that my firſt entrance doth ſeem to paſs a great many mens further ſtudy, and longer continuance.

Maſter. Many moe Conſiderations could I propound you in theſe *Arithmeticall Progreſſions*; but theſe are ſufficient for a taſte, to give you occaſion to think that *Rules* of Knowledge and *Arts*, are infinitely capable of enlargement.

Scholar. Happy were I, if I did but well underſtand that which is already invented and written. But theſe things in my ſimple fantaſie, offer themſelves to be greatly beneficiall unto the aide of Progreſſion. Therefore now I will goe forward with your Queſtion.

Now conſidering that the 52 and laſt place is 208, I adde thereunto the firſt number of the Progreſſion

it maketh 13 which 13 you ſhall multiply by half the number of the termes which is 6, it maketh 78; then divide 468 by 78, and you ſhall find 6, in the Quotient, which is the true number that ſhall begin and continue the ſaid Progreſſion. That is to ſay, the firſt week 6 ſhillings, the ſecond 12 ſhillings, and the third week 6 ſhillings more, which is 18 ſhillings, and ſo every week as they riſe, 6 ſhillings more then the week before, as is manifeſt in the queſtion aforeſaid.

A queſtion of a Farm. *A Farm is to be ſold to be paid by the weeks in a year; the firſt week to pay 4 ſhillings, the ſecond week 8 ſhillings, the third week 12 ſhillings, and ſo forth, increaſing each number by 4, till the number of 52 (which are the number of weeks in a year) expired.* The queſtion is, what the price of the Farm commeth to?

Scholar. I doubt not, but by that you have already taught me, to end this queſtion very well; wherefore I ſet forth the Progreſſion with his exceſs 52 times.

Maſter. Nay ſtay a while: And here for your further eaſe, (to abridge you of great labour that appeareth to fall out in this queſtion, and ſo may doe in any other the like) If a queſtion were propoſed of 100 or 200 places, or moe, and that this queſtion, or any other the like cannot be ended, unleſs you may know abſolutely what the laſt number of the Progreſſion of the 52 place is, (or ought to be) I will give you a generall rule how to know the laſt number of any Progreſſion Arithmeticall, as well as if you had ordinarily proceeded by continuall Addition, till you had come to the laſt work which is this.

Mul-

Multiply the exceſs by a number leſs by one then the number of the places, and thereto put the firſt number of the Progreſſion, and you ſhall have your deſire. A generall rule.

Scholar. This Rule is well worth the noting: for if I underſtand you aright, I conſider that my exceſs is 4, which I multiply by 51, which is one leſs then the number of the places, and it maketh 204, whereunto I adde the firſt number of the Progreſſion, which is 4, and then it is 208, which you ſay is or ſhould be the laſt number of the Progreſſion.

Maſter. This is a moſt approved truth, if there were never ſo many places.

Scholar. This Rule is ſo eaſie, that I were much to blame, if I doe not remember it. For by the benefit hereof, I have ſuch an eaſe and light into this excellent Art, that my firſt entrance doth ſeem to paſs a great many mens further ſtudy, and longer continuance.

Maſter. Many moe Conſiderations could I propound you in theſe *Arithmeticall Progreſſions*; but theſe are ſufficient for a taſte, to give you occaſion to think that *Rules* of Knowledge and *Arts*, are infinitely capable of enlargement.

Scholar. Happy were I, if I did but well underſtand that which is already invented and written. But theſe things in my ſimple fantaſie, offer themſelves to be greatly beneficiall unto the aide of Progreſſion. Therefore now I will goe forward with your Queſtion.

Now conſidering that the 52 and laſt place is 208, I adde thereunto the firſt number of the Progreſſion

greſſion, which is 4, it maketh 212, which I multiply by half the number of the places, which is 26, and it amounteth to 5512 ſhillings. And ſo much is the totall ſumme or addition of this progreſſion: which maketh 275 pound 12 ſhillings, as appeareth here by my Tables.

Maſter. I like well your labour, and commend you for your diligence: I will here propoſe one example more, and therewithall for this time will end progreſſion Arithmeticall.

A queſtion of Holland. *A certain man bought* 20 Ells *of* Holland, *to be paid in* 17 *weeks, or terms, by* progreſſion Arithmeticall. *And the firſt week to pay* 1 ſhilling 8 pence, *the ſecond week* 3 ſhillings 4 pence, *the third week* 5 ſhillings, *the fourth week* 6 ſhillings 8 pence, *and ſo forth, each week ſucceeding* 20 pence *more then the week before.* The queſtion is, what the ſumme of his 20 Elles cometh to.

Scholar. Becauſe here is mention made both of ſhillings and pence, I feare there is ſome harder matter contained herein, then in the other before: therefore I pray you work it your ſelf, and I will diligently mark your labour.

Maſter. There is no more to be done in this, then in the other before; but becauſe your requeſt is ſo reaſonable, be attentive unto me.

Firſt, by the generall Rules, I ſeek to find out the laſt Number of the 17 place, what this progreſſion ought to be. Therefore here in my Tables multiplying the exceſs 20 by 16, which is one leſs then the number of the termes or places, and it commeth to be 320; and thereunto adding the firſt number of the progreſſion, which is 20 pence,

pence, all is 340 pence, or 28 ſhillings 4 pence, for ſo much ought the laſt number of the payments to be.

Then finally, to know what the whole 17 places amount unto, I adde the firſt number of the progreſſion and the laſt together, which make 360. Now becauſe 17 is an odde number, whoſe half cannot be taken, I take the half of 360, which is 180, and multiplying 180 by 17, commeth to 3060 pence, which maketh as you ſée by Diviſion 12 pound 15 ſhillings. And ſo much is the buyer to pay for his 20 Elles of Holland. Which 3060 pence if you divide by 20, the number of Elles that was bought, you ſhall find 12 ſhillings 9 pence, and ſo much payed hée for an Elle one with another.

The Proof.

A certain man doth owe 12 pound 15 ſhillings, *to be paid in* 17 weeks *or* termes *by* Arithmeticall progreſſion. *The queſtion is, to know with what* number *he ſhall begin and continue the* progreſſion, *in ſuch equall proportion, as the ſame may be truly paid and ſatisfied in* 17 weeks.

A queſtion of debt.

The Anſwer.

Firſt I reduce 12 pounds 15 ſhillings, all into pence, which as you ſée here in my Tables, make 3060 pence, that I let ſtand by a while.

Then I adde 1 to 17, the number of the places or termes, which maketh 18, which I ſhould multiply by half the number of the weeks or termes,

which

which is 8 ½ which 8 ½ multiplied by 18 cannot well be done, unless you were acquainted with Fractions or broken Numbers, therefore you shall let that pass and multiply 17 by the half of 18, which is 9 (for that is all one with the multiplication of 8 ½) and the multiplication of 9 into 17 maketh as you see 153, with which number you shall divide the 3060 pence beforesaid, and the Quotient bringeth forth 20 pence, which is the first number or payment to beginne the progression withall: and so each week succeeding to rise 20 pence more then the week before, and thereby in 17 weeks shall 12 pounds 15 shillings be payed: as before was sufficiently declared. Thus much for progression Arithmeticall.

Scholar. Certainly Sir, I know not how to render you condigne thanks for these benefits shewed me, which me thinketh are so easie, delightfull, and pleasant, that I count my self happy to be in your company.

Master. I am glad you delight so well herein, which is an Art of wonderfull dexterity to all sorts of men, of what degree or profession soever they be. And now will I proceed to progression Geometricall, wherein I will be more brief, both because I have been so long in this part of Arithmeticall progression, and also for that it would require the knowledge of Roots, and surd numbers, (whereof ye have learned nothing) if I should frame the like propositions in them as I have done in these. Therefore I will onely teach you to practice about it, and so end the considerations and works of these progressions.

Progression

PRogression Geometricall *is when the numbers increase by a like proportion, that is, if the second number containe the first, 2, 3, or 4 times and so forth, then the third containeth the second so many times also: and so the fourth the third, and the fifth the fourth; wherefore I set these 3 examples.* Progression Geometricall.

3	6	12	24	48
1	3	9	27	81
2	10		50	250

Here in the first Example you see that every number containeth the other (that goeth next before him) two times; and in the second example three times, and in the third example five times. Now if you will know how to find easily the summe of any such number, doe thus: Consider by what numbers they be multiplied, whether by 2, 3, 4, 5, or any other, and by the same number multiply the last summe in the Progression.

Scholar. I pray you work it by this example, 2, 8, 32, 128, 512, 2048, which I have framed by proceeding from 2, and continually multiply by 4.

Master. Then must I multiply the last summe (which is 2048) by 4 also, and it will be 8192, Now must I abate from this summe the first number of the progression, which here is 2, then resteth 8190, which summe I must divide by 1 less then was the Number that I multiplyed by. Seeing then I multiply by 4. I must divide by 3, so dividing 8190 by 3, the Quotient will bee 2730, which is the summe of all the progression. And now to prove whether you can doe the same, I give you these Numbers to adde by this Rule 3, 15, 75, 375, 1875, 9375, 46875. To find the totall summe in any Geometricall Progression.

L Scholar.

Scholar. I cannot well tell by what number this progression doth increase.

Master. In any such doubt doe thus: Divide the second number by the first, and the Quotient will shew you the number that engendreth the progression.

Scholar. Then is that number in this example 5, for so many times is 3 in 15.

Master. So is it. Now work as I taught.

Scholar. The last number is 46875, which I multiply by 5, and it yeeldeth 234375, from which I abate the last number of the progression, that is 3, and there resteth 234372, which I divide by 4, for that is one less then 5, and the Quotient is 58593, which is the whole summe of the progression.

Master. If you remember well this, you have learned the Art of progression both Arithmeticall, and also Geometricall, which you may prove either by subtracting of each number alone from the sum, and so will there nothing remain: or else by adding together of all the parcels, for so will the same summe amount.

A question of Satten.

A Mercer hath 12 yards of Satten, which he valueth at 16 shillings the yard, and selleth the same 12 yards to another man to be paid as followeth: That is to wit, for the first yard to have one shilling, for the second yard two shillings, for the third yard foure shillings, for the fourth yard eight shillings, &c. Doubling each number following, till the twelfth and last yard. The question is, who hath made the better bargain of the buyer or seller.

First you may set down 12, the number of the yards as you see here in this Example. And against each number the number of shillings due to be paid, as

as the order of Duplation or Multiplication by two teacheth.

Then reſorting to the adding up or ſumming of this Progreſſion, where I conſider that the increaſe of this ſumme procéeded by the Multiplication of 2, and therefore after I have drawn a line under the 12, I work and multiply the laſt ſumme by 2 alſo, and it yéeldeth 4096: from whence I abate the firſt number of the progreſſion, which is 1, and then reſteth 4095: which I ſhould divide by one leſs then I did multiply by, but ſéeing it is 1, I néed not to divide it: for 1, (as I have ſaid before) doth neither multiply nor divide, therefore I take that ſumme 4095 for the whole ſumme of the ſhillings, which by Reduction amounteth to 304 pounds 15 ſhillings, and ſo much hath the Mercer for his twelve yards of Satten: which is 17 pound, 1 ſhilling, 3 pence a yard. But I think you will buy none ſo deare.

s	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4 6	

Scholar. No Sir, by the grace of God this year.

Maſter. *Then what ſay you to this* queſtion; *If I ſold unto you an horſe having 4 ſhooes, and in every ſhooe 6 nailes, with this condition, that you ſhall pay for the firſt nayl one* ob. *for the ſecond nayl two* ob. *for the third nayl foure* ob. *and ſo forth, doubling untill the end of all the nayls.* Now I aske you how much would the price of the horſe come unto? A queſtion of an horſe.

1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	22
4194304	23
8388608	24
16777216	

Scholar. Firſt, to know the number of the nayles, I muſt multiply 6 by 4, and it maketh 24. Then will I doe thus: I will write the number of the nayles every one in order from 1 to 24, and againſt each number of the nayles the ſumme of half pence duly, as the order of Duplation or Multiplication by 2 teacheth, and as in the next figure following appeareth.

Then do I reſort to the Rule of ſumming up the progreſſion, where I conſider that the increaſe of this ſumme proceedeth by the multiplication of 2, as the laſt Example did. And therefore multiplying the laſt ſumme by 2 alſo, and it yeeldeth 16777216, from which I abate the firſt Number which is 1, and then reſteth 16777215, which I ſhould divide by one leſs then I did multiply: but ſeeing that it is 1, I need not to divide it, for 1 (as you have before ſaid) doth neither multiply nor divide, therefore I doe take the number, 16777215 for the whole ſumme of the half pence, which by Reduction I find to be 699050 ſhillings and 7 pence halfpenie, that is 34952 pounds 10 ſhillings 7 pence, ob.

Maſter. That is well done, but I think you will buy no horſe of the price.

Scholar.

Scholar. No Sir, if I be wise.

Master. Well then answer me to this Question. *A Lord delivered to a Bricklayer a certain number of loads of Brick, whereof he willed him to make twelve walls, of such sort, that the first wall should receive two thirdells of the whole number, and the second two thirdells of that which was left; and so every other two thirdells of that that remained: and so did the Bricklayer: and when the* 12 *walls were made, there remained one load of Brick.*

Now I ask you, how many load went to each wall, and how many load was in the whole?

Scholar. Why Sir, it is impossible for me to tell.

Master. Nay, it is very easie if you marke it well. Marke well that I said, that every wall should receive two thirdels of the summe that was left. Now take away two thirdels from any summe, & you must needs grant that that which remaineth, is one thirdel of the summe last before, Example of 9. from which if you take 2 thirdels, there will remain 3, which is 1 thirdel of 9. Likewise from three bate two thirdels, & there remain 1.

Scholar. This is true, and now I perceive the least wall had but two load of brick.

Master. And by the same reason may you know how many load every wall had, according as this figure following doth shew, and likewise what the whole summe of bricks was, for if you make 12 summes, multiplying by 3, still from the last remainer, as you may see here on the left side of the Table, there will appeare all the remainers of the whole wall: and if you multiply the last of those 12 summes by 2 also, then will that be the summe of

the loads which was delivered to the Bricklayer.

	1	12	2	
The remainer after every wall	3	11	6	*Loads due to each wall,*
	9	10	18	
	27	9	54	
	81	8	162	
	243	7	486	
	729	6	1458	
	2187	5	4374	
	6561	4	13122	
	19683	3	39366	
	59049	2	118098	
	177147	1	354294	

Summe of the loads delivered 531440

Again if you double every Remainer, as you may see at the right side of this Table, those numbers will shew the summe of loads that went to each wall, whereby you may perceive that each wall was three times so great as the next less.

Scholar. Lo now it appeareth easie enough. Now surely I see that Arithmetick is a right excellent Art.

Master. You will say so when you know more of the use of it: For this is nothing in comparison to other points that may be wrought by it.

Scholar. Then I beseech you cease not to instruct me further in this wonderfull cunning.

The

The Golden Rule, or Rule of Proportion direct, called *the Rule of Three.*

Master.

BY *order of the Science (as Men have taught it) there should follow next the* Extraction of Roots of Number, *which because it is somewhat hard for you yet, I will let it pass for a while, and will teach you the feat* of the Rule of Proportion, *which for his excellency is* called the Golden Rule. Whose use is, by three numbers known to find out any other unknown, which you desire to know, as thus. — The Rule of Proportion. — The Golden Rule.

If you pay for your board for three moneths sixteen shillings, *how much shall you pay for* eight moneths? — Question of boarding.

To know this and all such like questions you shall consider which two of your numbers be of one Denomination, and set those two one over the other, so that the undermost be it that the question is of: as in my question 3 and 8 be both of one Denomination, for they both be moneths; and because 8 is the number that the question is asked of, I set the one over the other, and 8 undermost thus, with such a crooked draught of lines.

3 Z
8 Z

Then doe I set the other number which is 16, against 3 at the right side of the line, thus.

3 Z 16
8 Z

And now to know my question, this must I do: I must multiply the lowermost on the left side, by that on the right side: the sum that amounteth, I — Note.

must divide by the highest on the left side: or in plainer words, thus, I shall multiply the number of which the question is asked (which is called the third number) by the number of another Denomination (which is called the second) & the sum that amounteth, must I divide by the summe of like Denomination (which is called the first) Then for the knowledge of this question, I multiply 8 into 16, and there amounteth 128, which I divide by 3, and it yeildeth 42 shillings, and 2 shillings remaineth, which I turne into pence, and they be 24 pence, of which the third part is 8 pence, so the third part of 128 shillings, is 42 shillings 8 pence, which summe I write at the right hand of the figure against 8 thus.

The third number.
The second number.
The first number.

3 Z 16 shillings
8 42 shil. 8 pence.

Hereby I know that if three moneths boarding, doe come to 16 shillings, that 8 moneths boarding will come to 42 shillings 8 pence, and likewise of any other like question.

But here must you marke, that the first number and the third be of one Denomination, and also the second and the fourth, for which you seek: or else be of such Denominations, that you in working may bring them into one; As if a man should aske me this question.

Question of expence.

Twelve weeks journeying cost me 14 *French Crownes at* 6 *shillings the peece, how many pounds is that in one year?* Here you see no two numbers of one Denomination, but yet in working you may turn them into like Denomination: as thus; turn the one year into 52 weeks and the fourth summe will be French Crowns, by the order of the working.

Then

Then to know this queſtion multiply the third ſumme 52, by the ſecond 14, and the ſumme will be 728: that divide by your firſt number 12, and the Quotient will be 60 Crownes, and 8 Crownes remaining: which if you turn into ſhillings, they will be 48 ſhillings, which if you divide by your firſt number 12, the Quotient will be 4 which ſignifieth 4 ſhillings; put thoſe 60 French Crownes, which make 18 pounds with the 4 ſhillings for the ſumme that anſwereth to the queſtion, and it is the juſt expences of a year: And the work will be thus.

W C
12 Z 14
52 60 $\frac{2}{3}$

And take this evermore for a generall rule touching this whole art, that the doubtfull or unknown number that you would be reſolved of, ſhall alwayes be ſet in the third place. Note alſo, the firſt number and the third muſt ever be of one nature and denomination, or elſe muſt in working be brought to like Denomination, and then of neceſſity muſt the other number be in the ſecond place. A generall Rule.

Remember alſo, that the place of the firſt number is higheſt on the left ſide, and the place of the ſecond, right againſt it on the right ſide; the place of the third number is under the firſt as by thoſe examples you have ſeen.

Scholar. This I truſt I can do.

Maſter. But and if the queſtion be asked thus: *In 8 weeks I ſpend 40 ſhillings, how long will 105 ſhillings ſerve me?* Here you ſee that 8 weeks anſwers himſelf and ſaith 40 ſhillings.

But how long time 105 ſhillings will ſerve you know not. Therefore you ſhall ſet 105 in the third place, according as I told you even now. And the firſt

firſt place muſt alwayes be of the ſame nature or Denomination that the third is of, which here is 40. Then muſt 8 needs be that other: Now multiply 105 by 8 and it will be 840, which if you divide by 40, it will yeeld 21, which is the fourth number, and ſheweth how many weeks 105 ſhillings will ſerve if you ſpend 40 ſhillings in 8 weeks.

The figure of this queſtion is this: as if you ſhould ſay: if 40 ſhillings ſerve for 8 weeks: 105 will ſerve for 21 weeks.

Shillings. weeks.

40 Z 8

105 21

Other diverſities there be of working by this Rule, but I had rather that you would learn this one well, then at the beginning to trouble your mind with many formes of working, ſith this way can doe as much as all the other, and hereafter you ſhall learn the other more conveniently.

Note. ¶ And for your further aide and inſtruction, to make you better acquainted with this Golden Rule, I have here propoſed ſix queſtions, and their anſwers, which I think moſt convenient and meet to preferre the deſirous to perfect underſtanding. The firſt foure are all branches of one queſtion ſprung out of the beſt tree (for a young learner to taſt of) that groweth in this *Ground* of *Arts*: for that no manner of queſtion in the Rule of three whatſoever it be, can be propoſed, but it muſt be comprehended under the reaſon or ſtile of one of theſe foure.

The

The Questions.

If 15 *Elles of Cloth cost* 7 pound 10 shillings, *what comes* 27 *Elles to at that rate?* Answer. 13 pounds 10 shillings.

If 27 *Elles cost* 13 pound 10 shillings, *what are* 15 *Elles worth?* Answer; 7 pound 10 shillings.

If 27 *Elles cost* 13 pound 10 shillings: *how many Elles shall I have for* 7 pound 10 shillings? Answer; 15 *Elles.*

If I sell 15 *Elles for* 7 pound 10 shillings: *how many Elles are to be delivered for* 13 pounds 10 shillings? Answer; 27 *Elles.*

If 8 *pound of any thing cost* 16 shillings 6 pence: *what money is to be received for* 49 pound? Answer: 5 pound, 1 shilling $c\frac{3}{4}$.

If 4 pound *of any thing cost* 7 pence: *what money will* 8765 *pound of that commodity cost?* Answer; 63 pound, 18 shillings, 2 d $\frac{5}{4}$.

Of all which questions, **I omit the work of purpose, that you shall whet your wit thereby at convenient leisure, to clime each branch, and gather the fruit of them, and doe mind now, before we make an end of this** Rule, **to give you some instructions of the** backer Rule of three, **whose order is quite contrary to this that you have learned.**

Scholar. I thank you heartily for the six Questions, which I will (God willing) practise at convenient times; I pray you proceed therefore to the Backer or Reverse Rule.

The

The Golden Rule, or Rule of Proportion Backward, or Reverſe.

Maſter.

Note thir well.

IN the former evermore look how much the third number is greater then the firſt, ſo much the fourth number is greater then the ſecond. And contrariwiſe, look how much the firſt ſumme is greater then the third (if it doe chance ſo) ſo much is the ſecond ſumme greater then the fourth.

The backer or reverſe rule of three

But in this Rule, there is a contrary order, as this: That the greater the third ſumme is above the firſt, the leſſer the fourth ſumme is beneath the ſecond: and this Rule therefore you may call the Backer or Reverſe Rule, as in example.

A queſtion of cloth.

If I have bought 30 yards of cloth of two yards breadth, and would have Canvas of three yards broad to line it withall, how many yards ſhall I need?

Scholar. Why there is none ſo broad.

Maſter. I doe not care for that, I doe put this Example onely for your eaſie underſtanding: for if I ſhould put the Example in other meaſures, it would be harder to underſtand. But now to the matter: If you would know this queſtion, ſet your numbers as you did before: but you ſhall multiply now the firſt number by the ſecond, and that ariſeth thereof, you ſhall divide by the third: which thing if you doe here, I mean if you multiply 30 by 2 it will

will be 60: which summe if you divide by 3 there will appear 20. Whereby I know, that if 30 yards of cloth of two yards broad, should be lined with Canvas of three yards broad, 20 yards of Canvas would suffice, as this figure sheweth.

Breadth.	*Length.*
2	30
3	20

And now because ye found fault with my Example, how say you, perceive you this?

Scholar. Yes Sir, I suppose.

Master. *Then answer me to this question: how many Elles of Canvas of Ell breadth, will serve to line twenty yards of Say, of three quarters broad?*

Scholar. In good faith Sir I cannot tell, for I know not how to bring the summes to like Denominations.

Master. Then will I tell you: sith there is mention here of quarters, and again every one of the measures both Elles and yards may be parted into quarters, part them so both in the breadth and length, and then put forth the questions by quarters.

Scholar. Then shall I say thus. How many quarters of Canvas of 5 quarters broad will line 80 quarters of 3 quarters broad?

Master. Now answer to the question.

Scholar. First, I will set them down in their forme thus: for 5 is joyned with the question and is therefore the third number, then is 3 the number of the same Denomination, I meane because they be both referred to breadth. Now I multiply 80 by 3, & it is 240, which I divide by 5, & it yeildeth 48.

Breadth.	*Length.*
3	80
5	

Then

Then ſay I that 48 quarters of 5 quarters broad will ſuffice to line 80 quarters of three quarters broad.

Maſter. Turne the quarters again into Ells and yards.

Scholar. Then I ſay, that 9 Ells and three quarters of a yard of ellbroad, will ſerve to line 20 yards of three quarters broad, as this figure ſheweth.

Breadth. *Length.*

3 Z 80
5 Z 48

¶ Maſter. *Now what ſay you to this queſtion? I lent my friend* 400 pound *for* 7 *months, how much money ought he to lend me again for* 12 *months to recompence my courteſie ſhewed him? can you anſwer to this?*

Scholar. Yes Sir, I ſuppoſe, for I will ſet down my Numbers thus: where I multiply 7 into 400, and it maketh 2800, which I divide by 12, and it yeeldeth 233 pound, and there is 4 pound remaining of my Diviſion, what ſhall I do therewith?

Months. *ounds*

7 Z 400
12

Maſters. Turne the ſame 4 pound into ſhillings, and then divide it by 12 as you did before.

Scholar. Well Sir, it ſhall be done:. ſo have I 6 ſhillings for my Quotient, and yet remaineth 8 ſhillings upon my Diviſion.

Maſter. You muſt alſo reduce that 8 ſhillings into pence, which maketh 96, and divide that alſo by your Diviſor.

Sholar. So have I done, and I find 8 pence for my Quotient, and nothing is left.

Maſter.

Master. This must you always doe when any thing remaineth upon your Division, whether it be money, weight, measure, or any kind of thing whatsoever. This Rule is so profitable for all estates of man, that for this Rule onely (if there were no more but it) all men were bound highly to esteem Arithmetick.

By this rule may a Captain in war, work many things, as Master Digges in his Stratiocos doth declare. Onely now in this my simple addition, for a tast and incouragement, I will inlarge the Author with a question or two more, wishing you and every my Countrymen or Gentlemen whatsoever, that by nature be any thing given to Military affaires, to be familiar and acquainted with this Excellent Art, the which he shall find not onely at the Sea, but also in the Campe and Field-service, aboundantly to aid him, either in fortification, paying of Souldiers wages, charges of Ordnance, Powder, Shot, Munitions, and Instruments whatsoever, as for example.

If it should chance a Captain which hath 40000 *souldiers to be inclosed with his enemy, that he could have no fresh purveyance of victuals, and that the victuals he had would serve that Army but onely three months, how many men should he dismiss to make the victuall to suffice the residue* 8 *moneths ?* Question of an Armie.

Scholar. As you taught me, I set the numbers thus, saying, If three months suffice 40000, to how many will 8 suffice.

Months.		*Men.*
3	Z	40000
8		

To know this; I multiply the first number 3 into

the second 40000, and it yeildeth 120000, which summe I divide by 8, and there will be in the quotient 15000, which if I doe subtract from 40000, the remainer will declare that he must dismiss 25000 as this figure sheweth.

Moneths.	*Men.*
3	40000
8	15000

A question of a Fort.

Master. *Now answer me to this question: If 136 masons in a moneth be able to build a Fort to preserve the souldiers from the Enemy; and such expedition requireth that I would have the same finished in eight dayes: how many workemen say you is there to be appointed?*

Scholar. As you taught me, I set the numbers thus, saying: If 28 dayes require 136 Masons, what number of men by the like proportion will 8 dayes require?

28	136
8	

To know this I multiply the first number 28 into 136, and it yeildeth me 3808: which I divide by 8, and my Quotient is 476: which is the just number of Masons that shall supply this work. And now me think these questions are very easie.

Master. Truly if you take delectation herein, you shall find this Art not onely easie, but wonderfull pleasant and profitable. Now therefore one question more I will propose, and so leave off this Rule in whole numbers untill we come to the use of it in broken numbers: for had you the understanding of broken numbers perfectly, not onely in this Rule, but in all other, the question that in the sight or appearance seemeth to be 100 times harder to resolve, may thereby be wrought as soon or sooner then this.

Scholar

Scholar. Your words doe greatly incourage me to be ſtudious to attain whole numbers: but might I once attain to be a Practitioner in broken numbers, I ſhould think my ſelf happy.

Maſter. *What ſay you then to this queſtion? If* 48 *Joyners in two dayes make* 200 *light horſmen ſtaves (eſteeming they work but* 12 *houres a day) and ſuch need requireth that* 384 *Joyners are ſet to the finiſhing of thoſe* 200 *ſtaves; in what time, ſay you, will they make them up?*

Scholar. I ſee here that I muſt turn my 2 dayes into houres. And ſo doing, I ſet my numbers thus:

48 Z 24
384

Saying, if 48 men are 24 houres, 384 men will make an end quickly. For it is grounded upon an old Proverb, *many hands make light work.*

I multiply 48 into 24, and it amounteth to 1152, which I divide by 384, and my quotient is 3 houres which is my deſire.

I take this for a note worthy the marking, either in the Rule of Three, forward, or backward, when the two numbers are multiplied together, the product is of the ſame nature and determination that the ſecond number is of. Note.

The double Rule of Proportion direct.

Master.

The double Rule.

WEll, sith you perceive now the use of this rule, I will shew other which insue of the same, and first the *double Rule*, which is so called, because there is in it double working, by which thing onely it differeth from this.

Scholar. Then by an example I shall understand it well enough.

Question of carriage.

Master. *So shall you, and let this be the example: If the carriage of* 100 *weight (that is* 112 *pound)* 30 *miles doe cost* 12 *pence, how much will the carriage of* 500 *weight cost being carried* 100 *miles?*

Scholar. I pray you shew me the working of it.

Master. You must make two workings of it: the first thus: If C weight cost 12 pence, how much will five hundred weight cost? Set your figures thus:

C Weight.		*Pence.*
1	Z	12
5		

And multiply 5 by 12, and thereof amounteth 60, which if you divide by one, the Quotient will be still 60, that is the price of 500 weight for 30 miles.

Then begin the second work, saying: If 30 miles cost 60 pence, how much will 100 miles cost? Set your figures thus.

Miles.		*Pence.*
30	Z	60
100		

Then multiply 100 by 60 whereof amounteth 6000, which being divided by 30, will yeild 200 pence. Then you must say, that so many pence shall cost the carriage of 500 pound weight 100 miles, after the rate of 12 pence for the 100 carried 30 miles.

Scholar. Now I perceive it also.

¶ Master. These and such other like questions of the double Rule of three, are to be answered much sooner at one onely working by the Rule of proportion composed of five numbers, which anon I will shew you, and then when you have the use thereof, you may use it which way you think good.

Scholar. Sir, I thank you much for your courtesie. And I long now till this Rule be ended, that I may see how I may behave my self with that new Rule of five numbers: for that I have ever since you taught me hitherto in the Golden Rule, both forward and backward, wrought but with three numbers onely.

Master. But yet awhile we will go on forward with this Rule of Three, therefore answer to this question.

Thirty bushels of wheat sowed, yeilded in one yeare 360, how many will 80 bushels yeild in 7 yeares? I mean sowing every yeare of those seven, still four-score bushels? Question of sowing.

Scholar. First I say, that if 30 bushels will yeild 360 in one yeare, then 80 bushels will yeild 960 in one yeare. Then for the second work, I say. If one yeare yeild 960, then 7 yeares will yeild 6720; as these figures doe shew.

Seed.	*Encreaſe.*	*Year.*	*Encreaſe.*
30	360	1	960
80	960	7	6720

A queſtion of Corn.

But now Sir if I ſet forth 30 buſhels of Corne to another man for 7 years, agreeing ſo that he ſhall ſow every year the whole increaſe of the Corn, and I at the end of theſe 7 years to have the half of the whole increaſe: I would know how many buſhels will there amount to my part, ſuppoſing the increaſe to be after the rate of the laſt queſtion, for 30 buſhels in one year to yeild 360?

Maſter. In ſuch a queſtion you muſt have ſo many ſeverall workings as there be years: As for Example, in the firſt year thirty buſhels yeild 360, then to know the yeilding of the ſecond year, I muſt ſay, If thirty yeild 360, how many yeildeth 360? Work by your Rule, and you ſhall find 4320, Then ſay for the third year, If thirty yeild 360, How many will 4320, yeild? you ſhall have 51840, and ſo every year multiplying the whole increaſe by 360, and dividing it by 30, the increaſe of the next year will amount, as theſe 7 figures following do orderly declare: where I have ſet 7 letters for the 7 years, of which the firſt is ſet without Art, becauſe that is the increaſe which you doe preſuppoſe: and the laſt number of each other doth ſhew the increaſe of that year that it ſtandeth for, which the letters doe declare, ſo that the increaſe of the ſeventh year is 1074954240 buſhels: how many quarters that is, and alſo how many weyes, you may by Reduction ſoon find.

a

a

30 —— 360

b.

30 Z 360
360 Z 4320

c

30 Z 360
4320 Z 51840

d

30 Z 360
51840 Z 622080

e

30 Z 360
622080 Z 7464960

f

30 Z 360
7464960 Z 89579520

g

30 Z 360
89579520 Z 1074954240

Now with one question more I will prove you. *If six Mowers doe mow 45 Acres in 5 dayes, how many mowers will mow 300 Acres in 6 dayes.* Question of mowing

Scholar. If 45 Acres require 6 Mowers, then 300 Acres require 40. Now again, if 5 dayes require 40 mowers, then 6 dayes need but 33 mowers.

Master. Why doe you not make mention of the 2 that remaineth in the last Division? for the last part of the question in wrought by the Backer Rule, where the first number 5 is multiplied into the second, that is 40, whereof amounteth 200, which if you divide by the third number 6, the Quotient will be 33, as you said: but then will there remain 2, which cannot well be divided into 6 parts: howbeit you may understand by the 6 part of 2, the third part of one mans work, which you must put to

to the 33; or else you must say that 33 Workmen will end all the 300 Acres in 6 dayes save 2 mens work for one day, or two dayes work for one man. But such broken numbers called Fractions, you shall hereafter better perceive, when I shall wholly instruct you of them.

Master. *Yet one question more of field matters I will propose, and so I will make an end of this double Rule of Three.*

Scholar. With all my heart Sir, I Thank you, and I will dispatch it as soon as I can, because I would faine see the order of the next Rule of 5 numbers.

Question of entrenchings.

Master. *If a Captain over a band of men did set* 300 *Pioners a work, which in eight houres did cast a trench of* 200 *Rods: I demand how many labourers will be able with a like treuch in three houres, to intrench a camp of* 3400 *Rods.*

Scholar. I think I am now in the Backehouse ditch: for I know not well which way to go about it. And besides that, truly I think I shall never come to preferment that way, my growth is so small.

Master. You know not how God may raise you hereafter by knowledge and service into the favour of your Prince, for the avail of your Countrey.

Example for Navigation: Sir *Francis Drake*, a man greatly honoured for his knowledge, was not the tallest man, and yet hath made as great an adventure for the honour of his Prince and Countrey, as ever Englishman did.

Scholar. Sir, I thanke you for your good incouragement, My minde, though I be little,

is as desirous of knowledge, as any other: I have pondred now a little of it, and thus I set forth the work.

Rod.		Men.
200	Z	300
3400		

Saying. If 200 Rod require 300 men, what shall 3400 rods require? I multiply 3400 by 300, and it yeildeth 1020000, which I divide by 200. and my quotient is 5100 men.

Then must I say for the second work, if in 8 houres 5100 men, be able to discharge it, how many shall performe the same in 3 houres? Now if I would worke by the Golden Rule of proportion forward, I should finde a lesse number of men: because 3 houres is less then 8 houres: but because reason teacheth me, that the lesser the time is wherein the trench must be made, the more Labourers I ought to have, thereupon I use now the Backer Rule, as in example. And I have in my Quotient 13600. So many Pioners must I have to intrench the Campe in 3 houres.

Master. You have answered the question very artificially: And truly I commend you for your diligence and apt understanding: and now according to my promise, I will (in whole numbers) give you a little tast of the Rule of Proportion, compounded in 5 numbers.

The Rule of Proportion, composed of 5 Numbers.

The first part of the Rule of Proportion compound, direct.

THe Rule of proportion composed, is distinct for most needfull questions into severall parts or workings: And there belongeth unto it alwaies five numbers, whereof in this Rule being the first part, the second number and the fifth, are alwaies of one nature and like denomination, which Rule is to be wrought thus: you must multiply the first number by the second, and that shall be your divisor: Then again, multiply the other 3 numbers, the one by the other, and their product shall be your dividend.

And now according to my promise, we will first work the question of weight and carriage, which I delivered you in the double Rule of Three, to be absolved by this Rule, which was this.

If the carriage of 1 *C. weight* 30 *miles cost* 12 *pence, what will the carriage of* 5 *C. weight stand me in being carried* 100 *miles?*

C. weight.	*Miles.*	*Pence.*	*C. weight.*	*Miles.*
1	30	12	5	100

Now marke well how these five numbers stand: Then multiply the first number by the second, as 30 by 1. which maketh but 30, that number keep for your divisor, Then multiply the other 3 numbers, the one into the other, that is to wit, 12 by 5 which maketh 60: Lastly 60 by 100, which as you

you ſee here in our Tables, ariſeth to 6000 which 6000 you ſhall divide by the Product of the two firſt numbers, which here is 30. And you ſee there is found 200 pence, which is the duty that you ought to pay for the carraige of 500 weight 100 miles, after the rate of 12 pence a hundred, and agreeth with the concluſion of the double Rule of Three.

Scholar. Sir I thank you, it is even ſo.

Maſter. *Yet note this in a generality in this Rule, look what nature of denomination your middle number is of (which here are pence) and of the like denomination or nature is alwayes your quotient.* Note.

Scholar. Well now and if it pleaſe you, by your patience, I will ſee how I can end the queſtion next following of 30 Buſhels of wheat ſowed, which in one year yeildeth 360, how many then will 80 Buſhels yeild in 7 year following every year of thoſe 7 ſtill 80 buſhels, and according to your reaſons I ſet my numbers thus.

Buſh.	*Year.*	*Buſh.*	*Buſh.*	*Year.*
30	1	360	80	7
		80		
		28800		
		7		
		201600		

There I multiply 30 by 1, and it maketh 30 my Diviſor; then multiplying the other 3 numbers the one into the other, as here appeareth in my Tables, they make 201600, which I divide by 30: and my Quotient is 6720 buſhels, my deſire; for ſo much alſo it came to at two workings by the Rule of Three.

Maſter,

Master. Yet one question more I will propound unto you, and so leave this Rule, till it please God hereafter, that I may make you worke it in broken numbers.

Question of Interest. *What comes the interest of* 258 pound, *for* 5 *moneths to, after the rate of* 8 pound, *taken in the* 100 pound *for* 12 *moneths ?*

Scholar. Sir, this is yet within the compass of some reasonable usance. Therefore to minister equitie in this case, I will see how I can worke the same, which I set down thus, praying you if I have not done well, to shew me mine errour.

li. *moneths*. li. li. *mon.*
100---12-8-258--5.

Master. Proceed, you have done very well.

Scholar. Then I doubt not by the grace of God but to end it : I multiply 100 by 12 it yeildeth 1200, and the three other numbers multiplyed together produce 10320, which I divide by 1200 : and my Quotient is eight pounds. Then according as you have taught me heretofore, I turne the 720 pound that I left, into shillings : and dividing it by the first number, my Quotient is 12 shillings. So I answer, that the loans of 258 pounds for 5 moneths ,after the rate of 8 pound in the 100 pound for a yeare, comes to 8 pound 12 shillings.

Master. You say true, I commend your diligence : now behold the manner of the second part of this Rule.

The

The backer Rule, or the ſecond part of the Rule of Proportion compound.

Maſter.

IN *the ſecond part of this Rule of Proportion compoſed, the third number is like unto the firſt. And the Rule is to be wrought thus: you ſhall now, contrary to the laſt Rule, multiply the third number and the fourth together, and that Product ſhall be your Diviſor. Then multiply the fifth by the ſecond, and the Product thereof by the firſt: and that is the number that ſhall be divided.* For example I propound this queſtion for a proof of my laſt queſtion of Intereſt.

A Merchant hath received 8 pound 12 ſhillings,*for intereſt of certain money for* 5 *moneths tearm, which he received after the rate of eight* pound *in the* 100 *for a year. The queſtion is now, how much money was delivered to raiſe this intereſt.* The proof of the laſt queſtion.

Behold therefore the manner, how the queſtion is ſet forth.

li.	moneths.	li.	moneths.	li s.
100	12	8	5	8-12

Scholar. Sir I perceive it very well: and according to the doctrine which you preſcribed for the working thereof, if pleaſe you now it is ſet down, I think I can follow the work.

Maſter.

Master. Nay, stay a while, and before you work, marke well how I deliver a reason for the perfect understanding of this Rule, which is thus: *If* 8 *pound in* 12 *moneths doe yeeld me* 100 *pound, to take* 8 *pound* 12 *shillings for five moneths, must needs yeeld a great deale more.*

Note. So upon the knowledge that I have in this Art, The first part of this Rule is answerable to the Rule of Three forward: and this latter part accordeth to the Rule of Three backward.

Scholar. Sir, I yeild you most hearty thanks for these your last instructions, they have given me great light into these two rules, whereby I may the better by deliberation conceive how to use them hereafter when occasion shall require.

Note. Master. You say well, goe to now if you will, and try your cunning in the question: But this note take with you by the way, in as much as here is mention made of shillings: turn all your money as you work into shillings, for your more ease in working.

Scholar. If it please you to behold me a little, I will quickly end it: for I have but my first, my second, and my last number to be multiplied together for my Dividend: And my third into my fourth for my Divisor.

li.	Moneths.	Moneths.	li.	s.

100——————12 8——5——8——12

20	20	20	
2000	160	172	
12	5		
4000	800		
2000			
24000			
172			

48000 4128000(
168000 8 00
24000
——————4128000

Which 4128000 I divide by 800, and my Quotient is 5160 shillings, which in pounds yeildeth 258 my desire.

Master. I will here for this time in whole numbers, end this Rule, and I will instruct you in the Rules of fellowship. You may at your convenient leasure for your exercise work the same by the Rule of Three at twice. And for your aide and incouragement therein, I set down here a proffer how to apply it.

	A			B
Moneths.	li.	s.	*pounds.*	li.
5	8	12		100
12	412 4/5		412 4/5	258 li.

The

Master. Nay, stay a while, and before you work, marke well how I deliver a reason for the perfect understanding of this Rule, which is thus: *If* 8 *pound in* 12 *moneths doe yeeld me* 100 *pound, to take* 8 *pound* 12 *shillings for five moneths, must needs yeeld a great deale more.*

Note.

So upon the knowledge that I have in this Art, The first part of this Rule is answerable to the Rule of Three forward: and this latter part accordeth to the Rule of Three backward.

Scholar. Sir, I yeild you most hearty thanks for these your last instructions, they have given me great light into these two rules, whereby I may the better by deliberation conceive how to use them hereafter when occasion shall require.

Note.

Master. You say well, goe to now if you will, and try your cunning in the question: But this note take with you by the way, in as much as here is mention made of shillings: turn all your money as you work into shillings, for your more ease in working.

Scholar. If it please you to behold me a little, I will quickly end it: for I have but my first, my second, and my last number to be multiplied together for my Dividend: And my third into my fourth for my Divisor.

li.	*Moneths.*	*Moneths.*	li.	s.
100	—— 12	8 —— 5	—— 8	—— 12
20		20	20	
2000		160	172	
12		5		
4000		800		
2000				
24000				
172				

48000   4128000(
168000   8   00
24000
——4128000

Which 4128000 I divide by 800, and my Quotient is 5160 ſhillings, which in pounds yeildeth 258 my deſire.

Maſter. I will here for this time in whole numbers, end this Rule, and I will inſtruct you in the Rules of fellowſhip. You may at your convenient leaſure for your exerciſe work the ſame by the Rule of Three at twice. And for your aide and incouragement therein, I ſet down here a proffer how to apply it.

	A			B
Moneths.	li.	s.	*pounds.*	li.
5	8	12		100
12	$412\frac{4}{5}$		$412\frac{4}{5}$	258 li.

The

The Rule of Fellowſhip.

The Rule of Fellowſhip without time.

BUt now will I ſhew you of the Rule of Fellowſhip or Company, which hath ſundry operations according to the divers number of the Company. This Rule is ſometime without difference of time, and ſometimes there is in it difference of time. Firſt I will ſpeak of that without difference of time, of which let this be an example.

A queſtion of company.

Foure Merchants of one Company made a banck of money diverſly: for the firſt laid in 30 pound, *the ſecond* 50 pound, *the third* 60 pound, *and the fourth* 100 pound, *which ſtock they occupy ſo long, till it was increaſed to* 3000 pound. *Now I demand of you what ſhould each receive at the parting of this money.*

Scholar. **I perceive that this Rule is like the other, but yet there is a difference which I perceive not.**

Maſter. **Then will I ſhew it to you: Firſt by** Addition, **you ſhall bring all the particular** ſummes **of the** Merchants **into one** ſumme, **which ſhall be the** firſt ſumme **in your working by the** Golden Rule, **and the whole** ſumme **of the gaines by that** ſtock **ſhall be the** ſecond ſumme. **Now for the** third ſumme **you ſhall ſet the** portion of each man **one after another, and then work by the** Golden Rule, **and the** fourth ſumme **will ſhew you each** mans gaines: **as in** Example.

30
50
60
100
240

The

The parcels of the foure Merchants make in one summe 240 pounds: set that in the first place, the gaines in the second, and the first mans portion of stocke in the third place, thus:

240 Z 3000
30

Now multiply the second by the third, and it will be 90000, which you shall divide by 240, and there will appeare 375 pounds, thus:

And that is the gains for the first man.

240 Z 3000
30 375

Now for the second man set the 50 pound that he brought, in the third place, and work as before: and his part will be 625 pound: as this figure sheweth.

340 Z 3000
50 625

Likewise for the third man, set his money which was 60 pounds, and his part of gains will be 750 pounds, as here appeareth.

240 Z 3000
60 750

And so for the fourth man; if you set his summe which is 100 pound, his gaines will be 1250 pound, as the work will declare.

240 Z 3000
100 1250

Scholar. This I perceive: but is there any way to examine whether I have well done or no?

Master. For the triall hereof, adde together all their foure portions, and if their addition make the whole summe of their gaines, then is the work well done. Note this common Proofe.

Scholar. That will I trie by and by; the foure parcels are these, which added together make

3000,

3000, which is the just summe of money that they gained, whereby I know the work is well done.

375
625
750
1250
3000

Master. Well, now another example will I put to you not of gains, but of loss: for one reason serveth for both.

A question of loss. *If three Merchants in one ship, and of one fellowship, had bought Merchandise, so that the first had laid out* 200 *pound, the second* 300 *pound, the third* 500 *pound, and it chanced by tempest that they did cast over board into the Sea Merchandise of the value of* 100 *pound, how much should each man bear in this loss?*

Scholar. If I shall do in this as you did in the other question, then must I joyne their 3 portions together, 200, 300, 500, which maketh 1000. Then say I, If 1000 lose 100 then shall 200 lose 20, and 300 shall lose 30, and 500 shall lose 50, as by the three figures it doth appear plain.

1000 Z 100
200 Z 20

1000 Z 100
300 Z 30

1000 Z 100
500 Z 50

Master. Well, sith now you have done these, I will propound a question of more importance, which shall make you not onely the abler to understand this Rule, but also it will greatly aid you in the next Rule of fellowship with time, if such need be that your money be of divers Denominations.

For

For this may not be forgotten in all such questions: If the number be of divers kinds, you must by reduction bring it into one kinde, that is to say, to the least value that is named in the question. And likewise shall you doe, if the time be of divers kindes, as some years, some moneths, weeks, and dayes, you shall make all moneths, weeks, or dayes, according as the least name of time in the question is, as for example.

First in diversity of money. Three companions bought 2000 sheep, and paid for them 241 pound 13 shillings 4 pence, of which summe one payd 101 pound 10 shillings. The second 82 pound 17 shillings 10 pence. And the third paid 57 pound 5 shillings 6 pence: How many sheep must each of them have? Answer. *The first shall have 840, The second 686, And the third 474, And that must you work thus:* A question of sheep.

First, considering that your money is of divers Denominations, you shall (by reduction) bring it all into the smallest Denomination which is in it, that is to say, pence; and so will the Totall summe be 58000 pence. Solution.

Now if you turne each mans money into pence also, the first mans summe will be 24360 pence: The second mans money will be 19894 pence; and the third mans money will be 13746 pence.

Now to know how many sheep every man shall have, let the whole summe of money, that is, 58000 pence be set in the first place, and in the second place set the number of sheep, and then orderly in the third place set each mans money, and then multiplying the third and the second summes together, and dividing that that amounteth by the first,

there will appear the number of sheep that each man ought to have: as these 3 figures doe shew.

a

58000 2000
24360 840

b

58000 2000
19894 686

c

58000 2000
13746 474

S. Why doe you set the money in the first place, seeing in the question you say 2000 sheep cost 58000 pence, and not thus, 58000 bought 2000 sheep?

Master. You remember I taught you at the beginning of the Golden Rule, that the first and the third numbers must be of one name, and of like things: and evermore the number that the question is asked of, must be set in the third place.

Now is the question plainly this: If foure men bought 2000 sheep for 58000 pence, how many sheep shall each man have?

But seeing in this question, there ought more respect to be had to the summe of money, then to the summe of the persons; (for in the summe of money is their proportion toward the sheep, and not in the number of persons.)

If 58000 pence bought 2000 sheep, how many did 24360 buy? Again, how many did 19894 pence buy? And how many bought 13746 pence?

Scholar. I perceive it reasonable, and so shall I doe in all questions.

Note. M. Even so. But for easinesse of the work, marke this: Whensoever the first & second numbers

have

haue Cyphers in the firſt places, you may both in the Multiplication and in the Diviſion leaue out thoſe cyphers, ſo that you leaue out like many out of both ſummes, as in this queſtion the firſt number 58000 hath 3 cyphers, and ſo hath the ſecond, that is 2000: therefore caſt away their cyphers, and ſo will the firſt number be 58, and the ſecond 2. ſet them in their places, and worke according to the Rule, and you ſhall perceiue that will be all one, ſauing that this is the ſhorter and eaſier way, as theſe three figures doe ſhew.

a

58 Z 2
24360 Z 840

b

58 Z 2
19894 Z 686

c

58 Z 2
13746 Z 474

And this you ſee is both eaſier, and alſo the more certaine way to know the anſwer to this queſtion.

Scholar. Truth it is as you ſay: But Sir me ſeemeth I might aske a further queſtion here, not onely how many ſheep each man ſhould haue, but alſo what euery ſheep coſt.

Maſter. That queſtion doth not onely belong to this Rule, but may alſo be diſcuſſed by Diviſion, eſpecially if the queſtions number be one onely, as thus: Divide the totall ſumme 58000 pence by 2000 (or 58 by 2) omitting the cyphers, and the Quotient will be 29 pence, that is 2 ſhillings 5 pence. Howbeit, by this Rule you may doe it, and beſt when the number of the queſtion doth exceed 1; as if I

should aske this question, 2000 sheep cost 58000 pence, how much doe 20 cost? Then shall I set my figures as before.

2000 Z 58000
20

And doing after the Rule, there will amount 580 pence, that is, 2 pound 8 shillings 4 pence, the price of one score: but if you will use that easie way that I did teach you now, you may change the first and second numbers thus.

2 Z 58
20

Thus doe you perceive the use of the Rule without time.

Scholar. All this I understand very well: I pray you now instruct mee in the Rule of Fellowship with Time.

The Rule of Fellowship *with Time.*

Master.

The Rule of Fellowship with time.

TO the intent you may as well perceive the same Rule with diversity of time, I propose this example.

Foure Merchants made a common stocke, which at the years end was increased to 35145 pound. *Now to know what shall be each mans portion of gain, you must know each mans stocke, and time of continuance.*

The first man of these foure laid in 669 l. which he

he did take from the stock again at the end of 10 moneths. The second man laid in 810 pound, for 8 moneths. The third man laid in 900 pound, for seven moneths. And the fourth laid in 1040, for 12 moneths. Question of a Bank.

This question shall you examine as you did the other before, saving that whereas in the third place of the figure you did set each mans summe alone, here you shall set the same being multipled by the number of their time: and likewise in the first place of the figure you shall set the number which amounteth of their whole summes so multiplied by their time, and added into one whole summe, as thus. Note. A generall Rule.

The first mans summe is *669* pound, which I multiply by 10 (that was the number of his time) and it maketh *6690*. The second mans summe 810 pound, multiplied by 8 (which was his time) maketh *6480*. The third mans summe *900* pound, multiplied by 7 (for that was his time) yeildeth *6300*. The fourth mans summe was 1040 pound, and his time 12: multiply the one by the other, and it will be 12480.

The foure summes thus multiplied by their time, must be set orderly in the third place of the figure, and in the first place must be set the whole summe of all foure, which is 31950; and the gain must be in the second place, which is 35145. Now to end the question, I say first, If 31950 did get 35145, what did *6690* get? Answer, *7359* pounds, as by this figure appeareth.

a
31950 Z 35145
6690 7359

Likewiſe, the ſecond man had to his part 7128 pound, the third muſt have 6930 pounds, and the fourth man ſhall have for his part 13728 pounds, as theſe figures doe partly declare.

b

31950 Z 35145
6480 Z 7128

c

31950 Z 35145
6300 Z 6930

d

31950 Z 35145
12480 Z 13728

Another proof.

Scholar. This I like very well: but what proof is there of this work?

Maſter. The ſame that I taught you for the other: howbeit, there is uſed both for this work and the other alſo, this manner of proof, to adde all the portions together, and it may agree to the whole ſumme, then ſeemeth your work well done: but this is no ſure proof.

S. Yet will I prove in this example: the 4 parcels are theſe, which if I adde together, there will amount 35145, and that was the whole ſumme, whereby I perceive the work is well done.

7359
7128
6930
13728
35145

Maſter. If it fall out otherwiſe, be ſure it is not well.

Scholar. Then do I underſtand this work alſo very well: But what have I now to learn?

M. There are many other excellent parts behind, of which I will not as now make mention, becauſe that without the knowledge of Fractions they cannot be duely taught, and much leſs underſtood. Therefore will I propoſe to you two or three

queſtions

questions more (that thereby you may better perceive the use of this Rule and all other the like) and so make an end for this Time.

¶ *Three Partners by some ill adventure sustained the loss of* 160 pound,*whereof the first laid into the common stocke* 200 pound, *for ten Moneths. The second laid in* 350 pound, *and the third* 100 pound, *but for how long the two latter, is unknown: But breaking off their Partnership, the first found himself a loser* 80 pound, *the second* 56 pound, *and the third* 24 pound. *The question is, for how long time was the money of the two latter in company.* A question of loss.

For the solution hereof, and of such other like, you must also multiply the first mans 200 pounds, that he put into the stock by his time of continuance, which was ten moneths; and it maketh 2000: wherefore now I affirme, if his money that lost 80 pound multiplied by his time make 2000: what shall his money make that lost 56 pound, and his that lost 24 pound, which two numbers I commit to the triall of the Rule of Three at two workings, thus;

If 80 give 2000, what giveth 56? And again, if 80 give 2000, what giveth 24?

80	Z	2000	80	Z	2000
56		1400	24		600

To conclude, if you now divide 1400, the second mans portion, by 350, which was his stock that he laid into company, you shall find in your quotient 4 moneths, and for so long time did the second man put his money into the common stock.

Lastly, if you divide the third mans new laying in, which was 600 by 100, which was his stock

 that

that he put into the company, the quotient declareth his time of continuance, which was six months. And thus is the question resolved.

Scholar. Sir, I have attentively beheld your working, and the more we travell herein, the more me think I am in love with this excellent Art.

Master. Then what say you to this Question?

A question of Canons. *There is in a Cathedrall Church 20 Canons, and 30 Vicars, those may spend by year 2600 pound, but every Canon must have to is part 5 times so much as every Vicar hath: how much is every mans portion say you?*

Scholar. I pray you make the answer your self also, so shall I perceive best the meanes to answer to such other like.

Master. In this Question, you must doe as in those beforesaid, that have diversitie of time, for here is diversity of portions. Therefore shall you multiply the number of the persons by their difference of portions: (as you did in the other by time:) Then must you multiply the 20, (which is the number of Canons) by 5, (for that is the number of their portion) so will it be 100. Then 30, (that is the number of Vicars) by 1, (that is the number of their portion) and it will be 30: put these two summes together, and they make 130. Then say thus; If 130 spend 2600 pounds, what may 100 spend? The Rule sheweth 2000 pounds.

Again for Vicars if 130 spend 2600 pound, what may 30 spend? Answer 600 pound, as these figures shew.

130 Z 600
100 Z 2000

130 Z 2600
30 Z 600

But

But if every Canon should have so often times 4 pound as the Vicar should have 3 pound, then should I multiply 20 by 4, (that were 80) and 30 by 3, (that were 90) and then both were 170. Then should the figures be set as followeth.

	li.	s.	d.		li.	s.	d.
170 Z 2600				170 Z 2600			
80 Z 1223,		10,	7	90 Z 1376,		9	5

But this sort is too hard for you, by reason of the fractions, therefore I will let it rest to that place.

And by this rule you see what the 20 Canons may spend; which summe if you divide by 20, you shall see each Canons proportion: and so of the Vicars, if you divide their summes by 30, the quotient will declare every Vicars portion.

The second Dialogue.

The accounting by Counters.

Master.

Now that you have learned Arithmetick *with the Pen, you shall see the same Art in* Counters: *which feat doth not onely serve for them that cannot write and read, but also for them that can do both; but have not at some time their pen or tables ready with them.*

This sort is in two formes commonly, The one by lines, and the other without lines. In that

that

that hath lines, the lines doe ſtand for the order of places: and in that that hath no lines, there muſt be ſet in their ſtead ſo many Counters as ſhall néed for each line one; and they ſhall ſupply the ſtead of lines.

—100000——
—10000——
*—1000——
—100——
—10——
—1——

Scholar. By examples I ſhould better perceive your meaning.

Maſter. For example of the lines; loe here you ſée ſix lines, which ſtand for ſix places, ſo that the nethermoſt ſtands for the firſt place, and the next above is for the ſecond, and ſo upward till you come to the higheſt, which is the ſixth line, and ſtandeth for the ſixth place.

Numeration by Counters.

Now what is the value of every place or line, you may perceive by the figures which I have ſet on them, which is according as you learned before in Numeration of figures by the Pen: for the firſt place is the place of unites or ones, and every counter ſet in that line, betokeneth but one: and the ſecond line is the place of 10, for every counter there ſtandeth for 10: the third line the place of hundreds, the fourth of thouſands, and ſo forth.

Scholar. Sir, I doe perceive that the ſame order is here of lines, as was in the other figures by places, ſo that you ſhall not néed longer to ſtand about Numeration, except there be any other difference.

Maſter.

Master. If you do understand it, then how will you set 1543?

Scholar. Thus as I suppose.

Master. You have set the places truly, but your figures be not meet for this use: for the meetest figures in this behalf, is* the figure of a counter round, as you see here, where I have expressed that same summe.

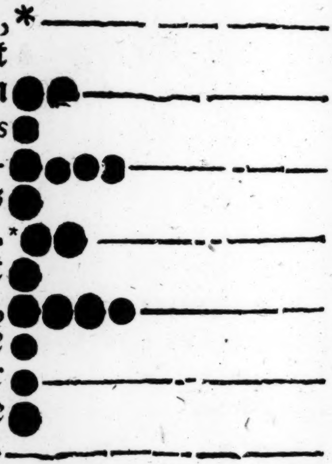
Scholar. So that you have not one figure for 2, nor 3, nor 4, and so forth, but as many digits as you have, so many counters you set in the lowest line, and for every 10 you set one in the second line, and so of other. But I know not by what reason you set that one Counter for 500 between two lines.

Master. You shall remember this, that whensoever you need to set down 5, 50, or 500, or 5000, or set forth any number whose numerator is 5, you shall set one counter for it in the next place above the line that it hath his denomination of: As in this example of that 500, because the numerator is 5, it must be set in a void space, and because the denomination is a hundred, I know that the place is the void place next above hundreds, that is, to say, above the third line.

And further you shall marke, that in all working by this sort, if you shall set down any

summe

ſumme betwéen 4 and 10, for the firſt part of that number you ſhall ſet down 5, & then ſo many counters more as there reſt numbers above 5. And this is true both of digits & articles. And for example, I will ſet down this ſumme 297965, which ſumme if you marke well, you néed none other examples for to learne the numeration of this forme.



But this ſhall you marke, that as you did in other kindes of Arithmetick, ſet a prick in the places of thouſands, in this work you ſhall ſet a Starre, as you ſée before.

Scholar. Then I perceive Numeration: But, I pray you, how ſhall I do in this Art, to adde two ſummes or more together?

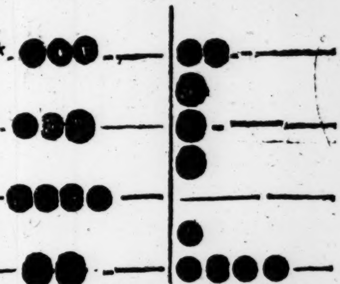
Addition.

Maſter.

THe eaſieſt way in this is to adde but two ſummes at once together: Howbeit you may adde more, as I will tell you anon.

Therefore when you will adde two ſummes, you ſhall firſt ſet downe one

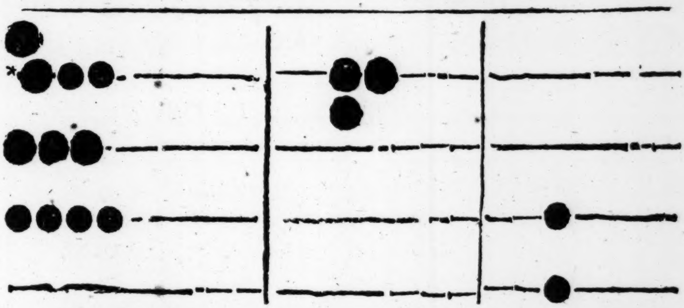
one of them, it forceth not which, and then by it draw a line croſs the other lines. And afterward ſet down the other ſum, ſo that the line may be between them: as if you would adde 2659 to 8342, you ſet your ſums as you ſee here.



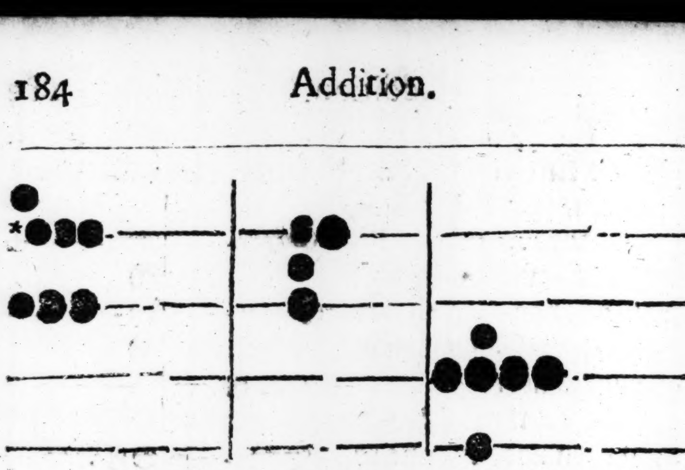
Addition of two ſummes.

And then if you liſt, you may then adde the one to the other in the ſame place: or elſe you may adde them both together in a new place: which way, becauſe it is moſt plaine, I will ſhew you firſt.

Therefore will I beginne at the Unites, which in the firſt ſumme is but 2, and in the ſecond ſumme 9, that maketh 11: Thoſe do I take up, and for them I ſet 11 in the new room, thus:



Then doe I take up all the Articles under a hundred, which in the firſt ſumme are 40, and in the ſecond ſumme 50, that maketh 90: or you may ſay better, that in the firſt ſumme there are foure Articles of 10, and the ſecond ſumme 5, which maketh 9, but then take heed that you ſet them in their right lines, ſee here.



Where I have taken away 40 from the first summe, and 50 from the second, and in their stead I have set 90 in the third room, which I have set plainly, that you might well perceive: howbeit seeing that 90 with the 10 that was in the third room, already, doth make 100, I might better for those 6 Counters, set 1 in the third line, thus:

For it is all in one summe, as you may see, but it is best never to set five counters in any line, for that may be done with one counter in a higher place.

Scholar. I judge that good reason, for many are unneedfull where one will serve.

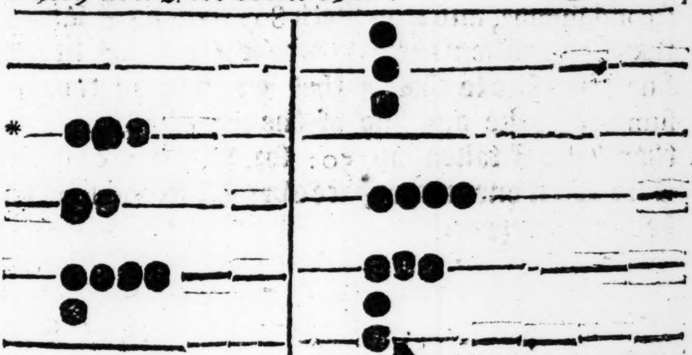
Master. Well, then will I adde forth of hundreds: I find 3 in the first summe, and 6 in the second, which maketh 6000, them doe I take up, and set in the third room, where is 100 already, to which I put 900, and it will be 1000: therefore I set one counter in the fourth line for them all, as you see here.

Then

Then adde I the thousands together, which in the first summe are 8000, and in the second 2000 that maketh 10000, them doe I take up for those two places, and for them I set one counter in the fifth line, and then it appeareth as you see to be 11001, for so many doth amount of the Addition of 8342 to 2659. To adde summes together.

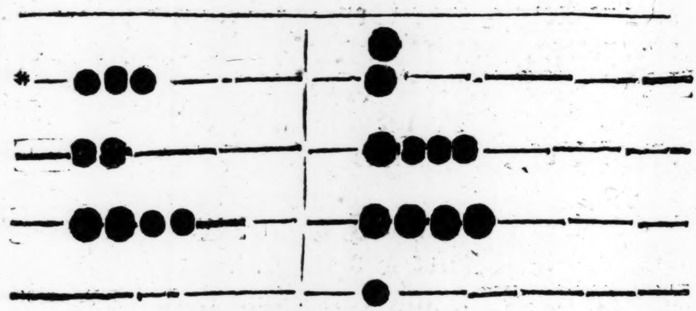
Scholar. Sir, this I do perceive: but how shall I set one summe to another, not changing them to a third place?

Master. Marke well how I doe it. I will adde together 65436 and 3245, which first I set down thus:

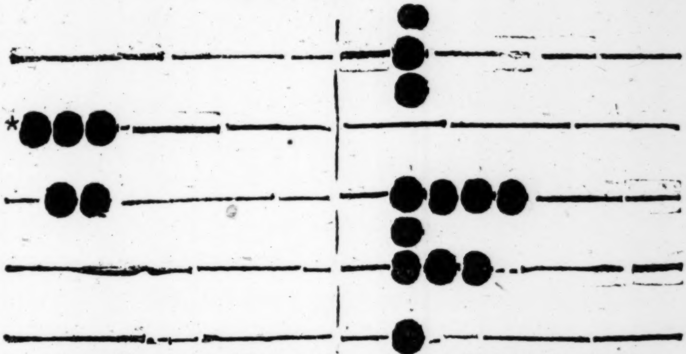


Then

Then doe I begin with the ſmalleſt Denomination, which is 1 in the ſecond ſumme, and ſet it in his place: then doe I find 5 in the firſt ſumme, and 5 in the ſecond, which put together, ſaving the two Counters, cannot be ſet in a void place of 5, but for them both I muſt ſet one in the ſecond line, which is the place of 10, therefore I take up the five of the firſt ſumme and the 5 of the ſecond, and for them I ſet one in the ſecond line, as you ſee here.

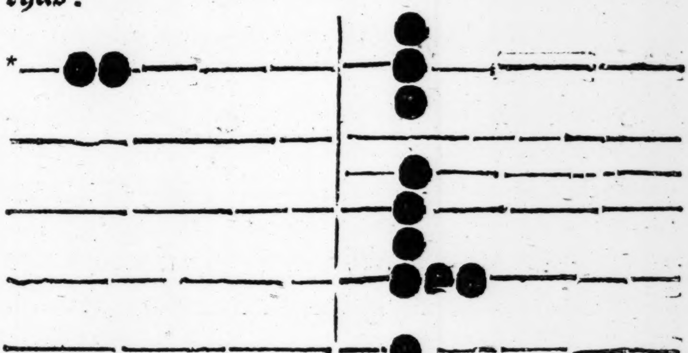


Then do I likewiſe take the 4 Counters of the firſt ſumme and ſecond line (which maketh 40) and adde them to the 4 counters of the ſame line in the ſecond ſumme, and it maketh 80: but as I ſaid, I may not conveniently ſet above 4 counters in one line, therefore to thoſe 4 that I took up in the firſt ſumme, I take one alſo of the ſecond ſumme, and then have I taken up 50: for which 5 counters I ſet down one in the ſpace over the ſecond line, as here doth appear.



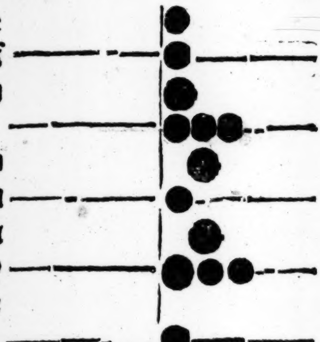
And then is there 80, as well with thoſe 4 counters, as if you had ſet down the other 4 alſo.

Now do I take the 200 in the firſt ſumme, and adde them to the 400 in the ſecond ſumme, and it maketh 600, therefore I take up the two counters in the firſt ſumme, and three of them in the ſecond ſumme, and for them 5, I ſet 1 in the ſpace above, thus:



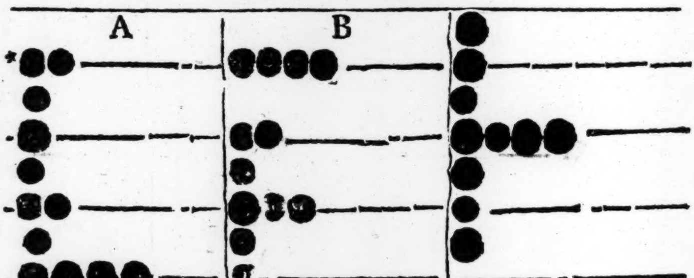
Then take I the 3000 in the firſt ſumme unto which there are none in the ſecond ſumme agreeing, therefore I doe onely remove thoſe three counters from the firſt ſumme into the ſecond, as here doth appear.

And you ſée the whole ſumme that amounteth of that Addition of 65436 with 3245, to bée 68681.



And if you have marked thoſe two examples well, you néed no further inſtruction in Addition of two onely ſums: but if you have more then two ſummes to adde, you may adde them thus:

Firſt adde two of them, and then adde the third and fourth, or more, if there be ſo many: As if I would adde 2679, with 4286, and 1391. Firſt I adde the two firſt ſummes thus:



And then I adde the third thereto thus:

And

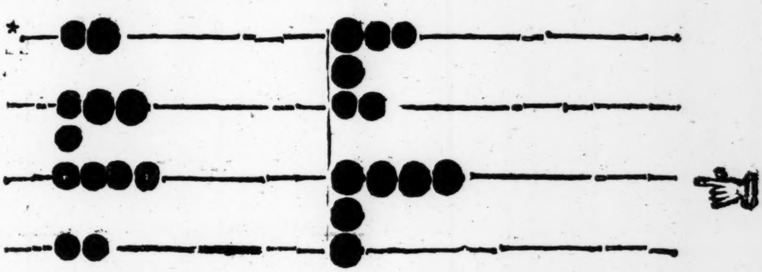
And so of more if you have them.

Scholar. Now I think it best that you pass forth to Subtraction, except there be any way to examine this manner of Addition, then I think that were good to be known next.

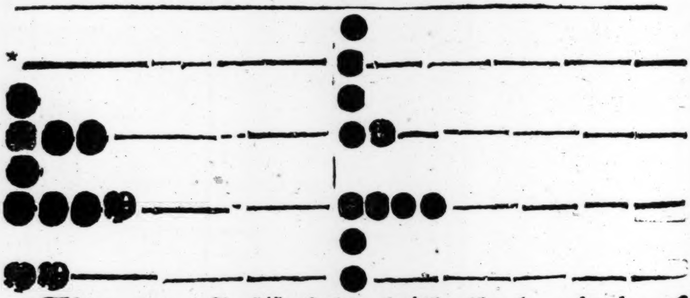
Master. There is the same proof here that is in the other Addition by the Pen, I meane Subtraction; for that onely is a sure way: but considering that Subtraction must be first known, I will first teach you the Art of Subtraction, and that by this Example.

Subtraction.

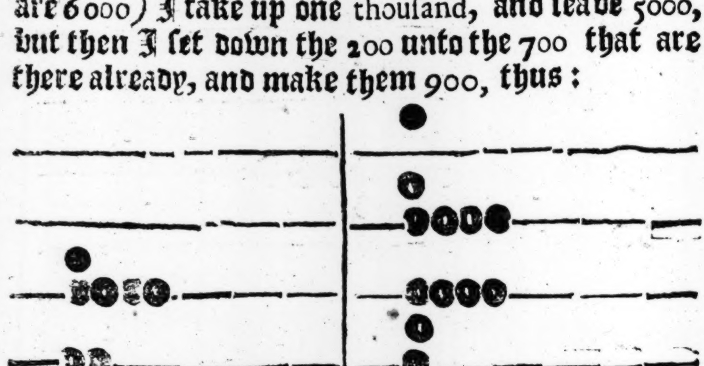
I Would subtract 2892 out of 8746. These summes must I set down as I did in Addition: but here it is best to set the lesser number first, thus:



Then shall I begin to subtract the greatest numbers first, (contrary to the use of the Pen) that is the thousand in this example: therefore I find among the thousands 2, for which I withdraw so many from the second summe, (where are 8) and so remaineth there 6, as this example sheweth.



Then doe I likewise with the hundreds, of which in the first summe I find 8, and in the second summe but 7, out of which I cannot take 2, therefore this must I doe: I must look how much my sum differeth from 10, which I find here to be 2, then must I abate for my summe of 800, one thousand, and set down the excess of hundreds, that is to say 2, for so much as 1000 is more then I should take up: therefore from the first summe I take that 800, and from the second summe (which are 6000) I take up one thousand, and leave 5000, but then I set down the 200 unto the 700 that are there already, and make them 900, thus:



Then come I to the Articles of tennes, where in the first summe I finde 90, and in the second summe but only 40. Now considering that 90 cannot be abated

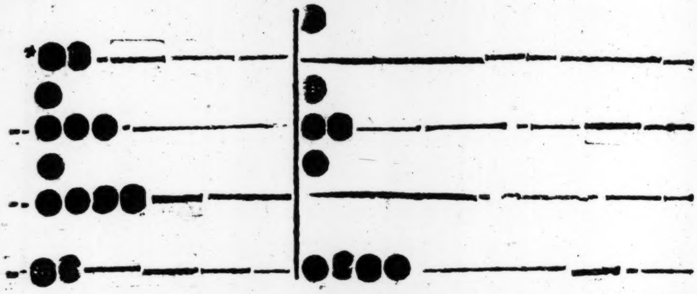
abated from 40, I looke how much that 90 doth differ from the next ſumme above it, that is, 100; or elſe (which is all to one effect) I looke how much 9 doth differ from 10, and I find it to be 1: then in the ſtead of that 90, I doe take from the ſecond ſumme 100: but conſidering that is 10 too much, I ſet down 1 in the next line beneath for it, as you ſee here.

Saving that here I have ſet 1 Counter in the ſpace inſtead of 5 in the next line. And thus I have ſubtracted all ſave 2, which I muſt abate from 6 in the ſecond ſumme, and there will remain 4 thus.

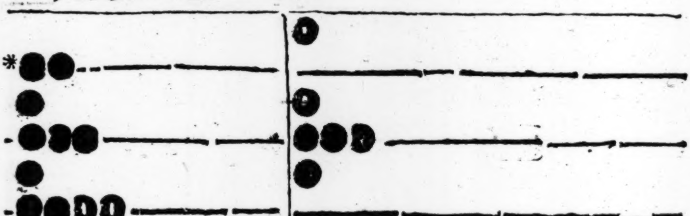
So that if I ſubtract 2892, from 8746 the remainer will be 5854.

And that this is truely wrought you may prove by Addition: for if you adde to this remainer the ſame ſumme that you did ſubtract, then will the former ſumme 8746 amount again.

Scholar. That will I prove, and firſt I ſet the ſumme that was ſubtracted, which was 2892, and then the remainer 5854 thus: A proof of Subtraction.



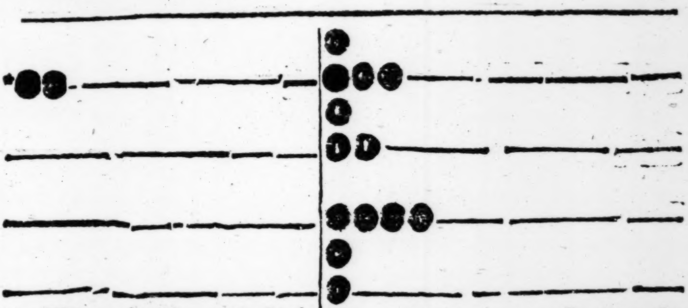
Then doe I adde the first 2 to 4, which maketh 6: so take I up 5 of those Counters, and in their stead I set 1 in the space: and one in the lowest line, as here appeareth.



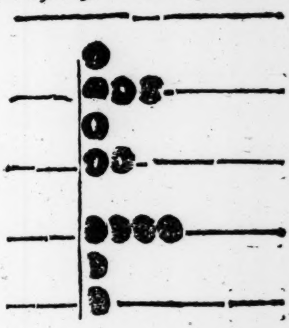
Then doe I adde the 90 next above to the 50, and it maketh 140. Therefore I take up those 6 Counters, and for them I set 1 to the hundreds in the third line, and 4 in the second line thus:

Then

Then doe I come to the hundreds, of which I find 8 in the firſt ſumme, and 8 in the ſecond, that maketh 1600, therefore I take up thoſe 8 counters, and in their ſtead I ſet 1 in the fourth line, and 1 in the ſpace next beneath, and in the third line, as you may ſee here.



Then is there left in the firſt ſumme but only 2000, and in the ſecond 5000, which is 7000, which I ſhall take up from thence, and ſet in the ſame line in the ſecond ſumme to the one that is there already: and there will the whole ſumme appeare as you may well ſee to be 8746, which was the firſt groſs ſum: and therefore I do perceive that I had well ſubtracted before.



And thus may you ſee how Subtraction may be tried by Addition.

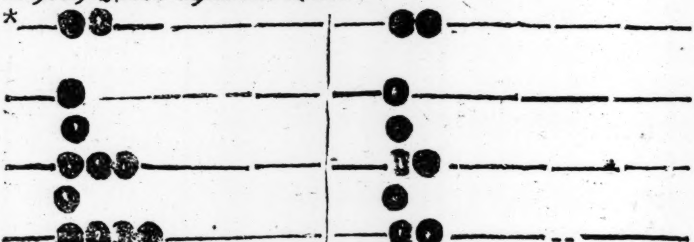
Scholar. I perceive the ſame order here with Counters, that I learned before in figures.

Maſter. Then let me ſee how you can trie Addition by Subtraction.

Scholar. Firſt I will ſet forth this example of Addition, where I haue added 2189 to 4988. And the whole ſumme appeareth to be 7177.

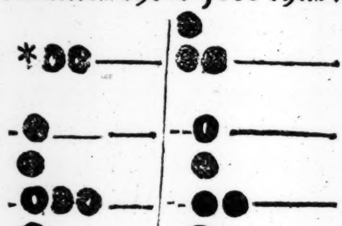
Proof of Addition by Subtraction.

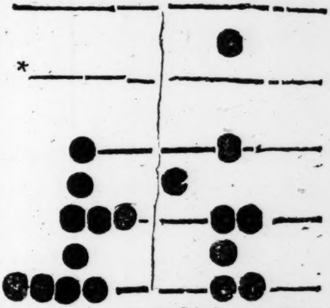
Now to try whether that ſumme be well added or no, I will ſubtract one of the firſt two ſummes from the third. And if I haue well done, the remainer will be like that other ſumme: as for example, I will ſubtract the firſt ſumme from the third, which I ſet thus in order.



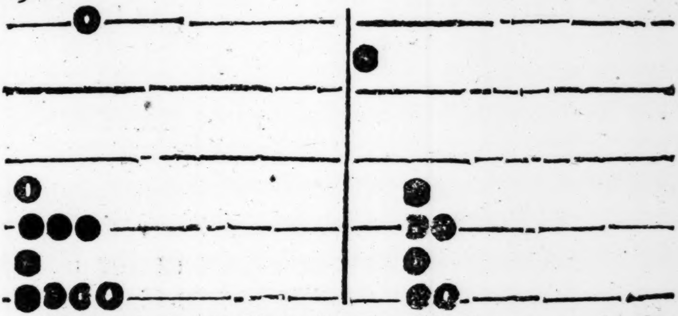
Then do I ſubtract 2000 of the firſt ſumme, from the ſecond ſumme, & then remaines there 5000 thus:

Then in the third line I ſubtract the 100 of the firſt from the ſecond ſumme, where is onely 100 alſo: and then in the third line, reſteth nothing, as you may ſee in this example following.

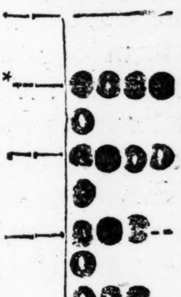




Then in the ſecond line with his ſpace over him I find 80, which I ſhould ſubtract from the other ſumme: then ſeeing there are but only 70, I muſt take it out of ſome higher ſumme, which is here onely 5000: therefore I take up 5000: and ſeeing that is too much by 4920, I ſet down ſo many in the ſecond roome, which with the 70 being there already do make 4990, and then the ſummes do ſtand thus.



Yet remaineth therein the firſt ſumme 9, to be abated from the ſecond ſum, where in that place of unites doth appear only 7: then muſt I abate a higher ſum, that is to ſay 10, but ſeeing that 10 is more then 9 (which I ſhould abate) by 1, therefore ſhall I take up one Counter from the ſecond, and ſet down the ſame in the firſt line, or lowermoſt line, as you ſee here.

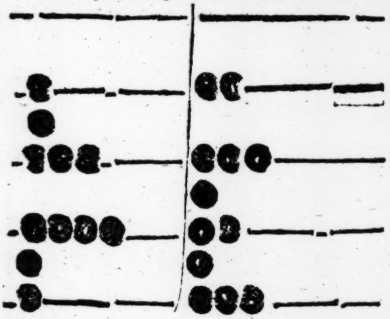


And

And ſo have I ended this work, and the ſumme appeareth to be the ſame which was the ſecond ſumme of mine Addition, and therefore I perceive I have well done.

Another way of Addition.

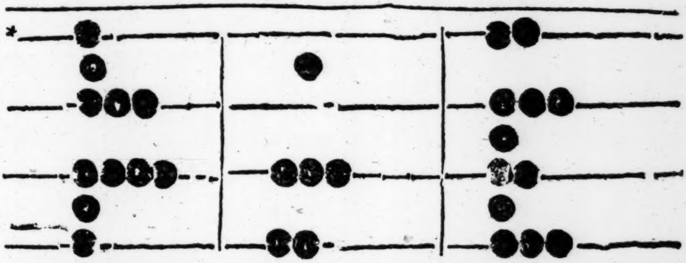
Maſter. To ſtand longer about this, it is but folly: except that this you may alſo underſtand, that many doe beginne to ſubtract with Counters, not at the higheſt ſumme, as I have taught you, but at the nethermoſt, as they doe uſe to adde; and when the ſumme to be abated in any line appeareth greater then the other, then doe they borrow out of the next higher roome, as for example.



I ſhould abate 1846 from 2378, they ſet the ſummes thus:

Firſt they take 6 which is the lower line, and his ſpace from 8 in the ſame roomes in the ſecond ſumme, and yet there remaineth two Counters in the loweſt line. Then in the ſecond line muſt 4 be ſubtracted from 7, and ſo remaineth there 3. Then 800 in the third line, and his ſpace, from 300 of the ſecond ſumme cannot be, therefore doe they abate it from a higher roome, that is, from 1000, and becauſe 1000 is too much by 200, therefore muſt I ſet down 200 in the third line, after I have taken up 1000 from the fourth line. Then is there yet 1000 in the fourth line of the firſt ſumme, which if I withdraw from the ſecond ſumme, then doe

doe all Figures stand in order thus: 532.



So that (as you sée) it differeth not greatly whether you begin Subtraction at the higher lines, or at the lower.

Howbeit, as some men like that one way best, so some like the other: therefore you now knowing both may use which you list.

Multiplication.

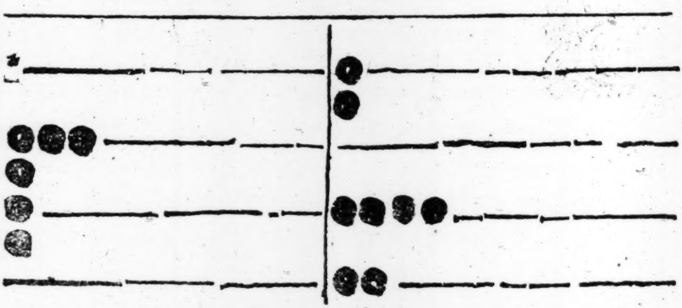
But now touching Multiplication: you shall set your numbers *into two* roomes, *(as you did in those other kinds) but so that the multiplier be set in the first* roome, *then shall you begin with the highest numbers of the second* roome, *and multiply them first after this sort.*

Take the overmost line in your first working as it were the lowest line, seting on it some moveable mark (as you list) and looke how many Counters bée in him, take them up, and for them set downe the whole multiplier so many times as you took up Counters: reckoning (I say) that line for the Unites. And when you have done with the highest number, then

then come to the next line beneath, and doe ſo even with it, and ſo with the next, till you have done all. And if there bee any number in a ſpace, then for it ſhall you take the multiplier five times, and then muſt you reckon that line for the Unites, which is next beneath that ſpace. Or elſe after a ſhorter way, ye ſhall take onely half the multiplier, but then ſhall you take the line next above the ſpace for the line of Unites. But in each working, if by chance your multiplier be an odde number, ſo that you cannot take the half of it juſtly, then muſt you take the greater half, and ſet down that, as if that it were the juſt half: and further, you ſhall ſet one Counter in the ſpace between that line, which you reckon for the line of unites, or elſe only remove forward the ſame that is to be multiplied.

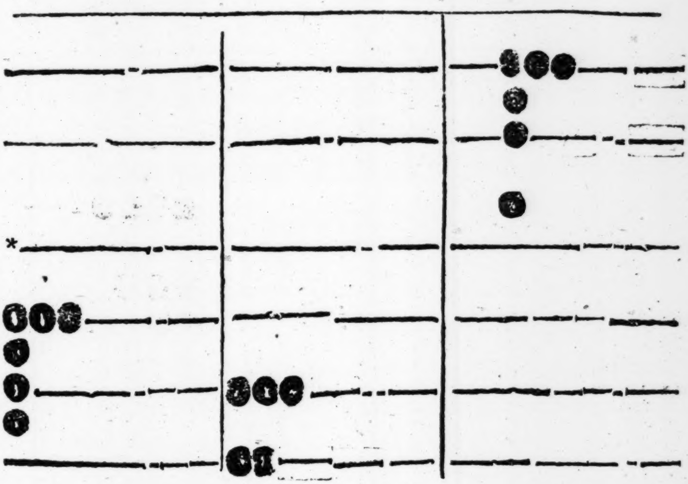
Scholar. If you ſet forth an example hereof, I think I ſhall perceive you.

Maſter. Take this example: I would multiply 1542 by 365. therefore I ſet my numbers thus.



Then firſt I begin at the 1000 in the higheſt roome, as if it were the firſt place, and I take it up ſetting down for it ſo often (that is once) the Multi-

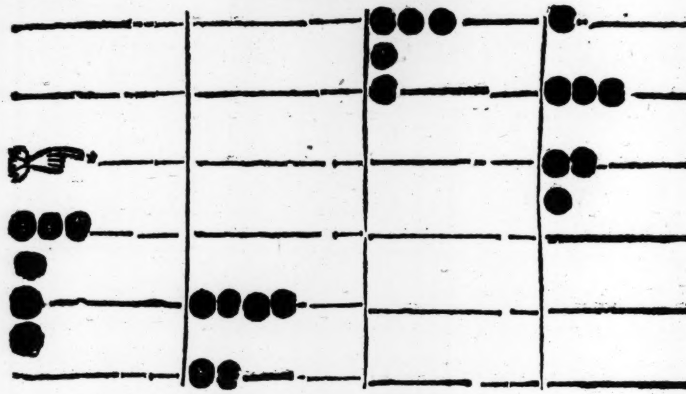
Multiplier, which is 365, thus as you ſée here: where, for the one Counter taken up from the fourth line I have ſet down other ſixe which make the ſumme of the Multiplier, reckoning the fourth line, as if it were the firſt, which thing I have marked by the Starre ſet at the beginning.



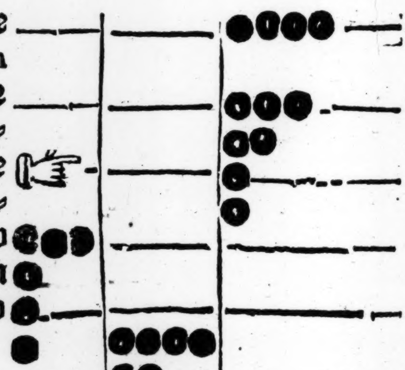
Scholar. I perceive well, for indéed this ſumme that you ſet down, is 365000: for ſo much doth amount of 1000, multiplied by 365.

Maſter. Well then goe forth, in the next ſpace I find one Counter, which I remove forward, but take it not up, but (as in ſuch a caſe I muſt) ſet down the greater half of my Multiplier (ſéeing it is an odde number) which is 182, and here do I ſtill let that fourth place ſtand as if it were the firſt, as in theſe examples you ſhall ſée.

Where

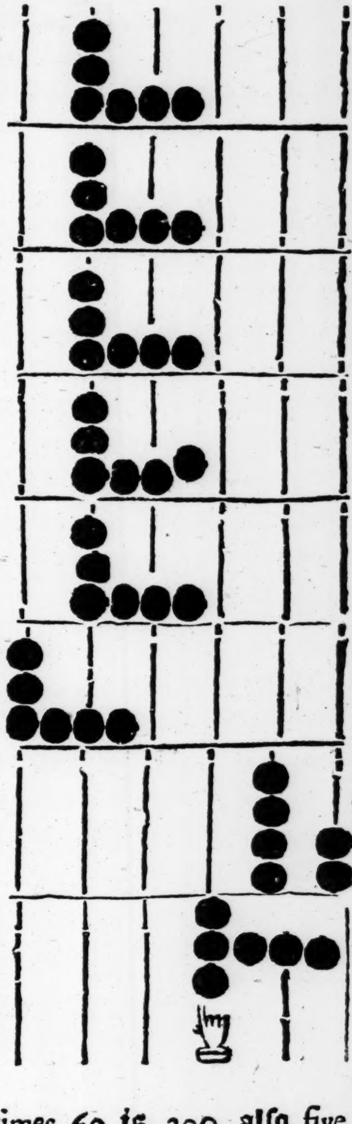


Where I have set the Multiplication with other, but for the ease of your understanding, I have set a little line between them. Now should they both in one summe stand thus.



Now,

Howbeit another forme to multiply ſuch Counters in ſpace is this: firſt to remove the finger to the next line beneath the ſpace, and then to take up the Counter, and to ſet downe the Multiplier five times, as here you ſee. Another forme of Multiplication.



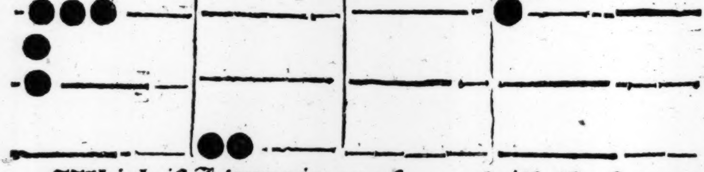
Which ſummes, if you doe adde together, into one ſumme, you ſhall perceive that it will be the ſame that appeareth of the other working before, ſo that both ſorts are to one intent: but as the other is ſhorter, ſo this is plainer to reaſon, for ſuch as have had ſmall exerciſe in this Art.

Notwithſtanding you may adde them in your minde before you ſet them down: as in this example you might have ſaid five times 300 is 1500, and five times 60 is 300, alſo five times five is 25. which all put together doe make 1825.

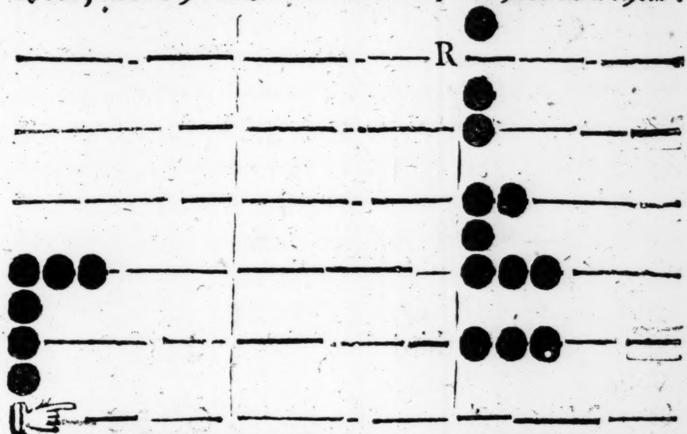
1825, which you may at one time set down if you list.

But now to go forth, I must remove the hand to the next Counters, which are in the second line, and there must I take up those foure Counters, setting down for them my multiplier foure times severally, or else I may gather the whole summe in my mind first; and then set it down: as to say, 4 times 300 is 1200: 4 times 60 are 240: and 4 times 5 make 20, that is in all 1460: that shall I set down also, as here you see.

Which if I joyne in one summe with the former numbers, it will appear thus.



Then to end this Multiplication, I remove the finger to the loweſt line where are only 2, them doe I take up, and in their ſtead doe I ſet down twice 365, that is 730, for which I ſet one in the ſpace above the 3 line for 500, & 2 more in the 3 line with that one that is there already & the reſt in their order, and ſo have I well ended the whole ſum thus:

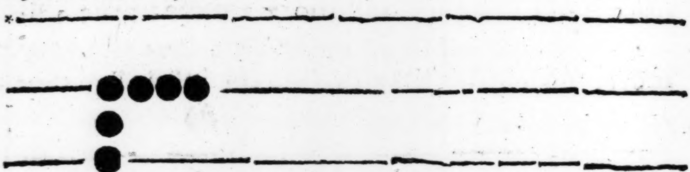


Whereby you ſee, that 1542 (which is the number of years ſince Chriſt his Incarnation) being multiplied by 365 (which is the number of the daies in 1 year) doth amount to 562830, which declareth the number of daies ſince Chriſts incarnation unto the end of 1542 years, beſides 385 daies, and 12 hours for leap years.

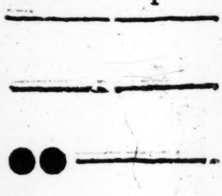
Scholar. Now will I prove by another example, as this: 40 Labourers (after 6 pence the day for each man) have wrought 28 daies. I would know what their wages doth amount unto. Example of wages.

In this caſe muſt I work doubly; firſt I muſt multiply the number of the Labourers, by the wages of a man for one day, ſo will the charge of every day amount.

Then secondly, shall I multiply the charge of one day by the whole number of dayes, and so will the whole summe appear: first therefore I shall set the summes thus.

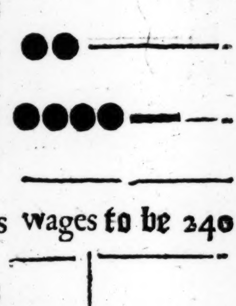


Where in the first place is the Multiplier (that is one daies wages for one man) and in the second space is set the number of the workmen to be multiplied.



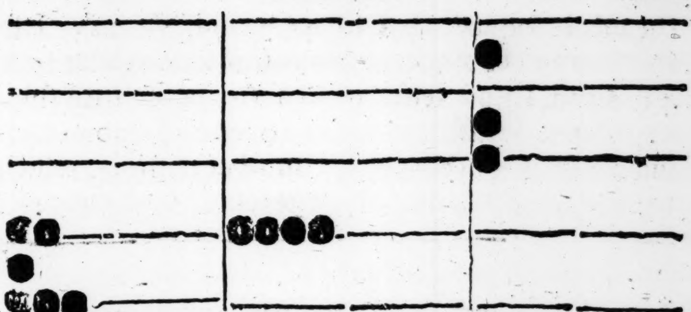
Then say: If 6 times foure (reckoning that second line as the line of unites) maketh 24, for which summe I should set two counters in the third line, and 4 in the second, therefore do I set two in the third line, and let the 4 stand still in the second line thus.

So appeareth the whole dayes wages to be 240 pence, that is 20 shillings.

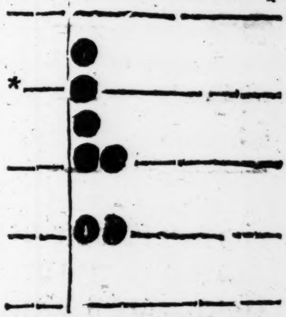


Then do I multiply again the same summe by the number of dayes; and first I set the numbers thus: then because there are counters in divers lines I shall begin with the highest, and take them up setting for them the Multiplier so many times as I took up counters, that is twice, then will the summe stand thus.

Then



Then come I to the ſecond line, and take up thoſe 4 counters, ſetting for them the Multiplier 4 times, ſo will the whole ſumme appear thus:



So is the whole wages of 40 workmen for 28 dayes after 6 pence each day for a man, 6720 pence, that is, 560 ſhillings, or 28 pound.

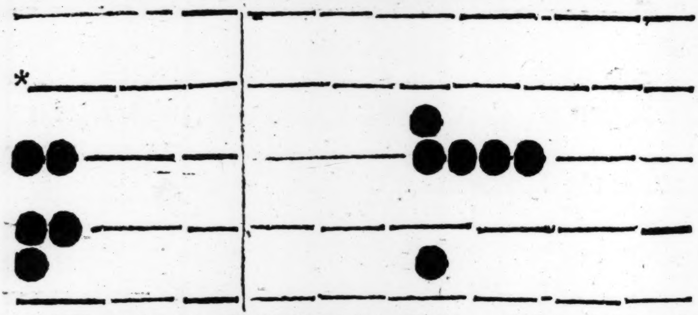
Maſter. Now if you would prove Multiplication, the ſureſt way is by diviſion: therefore will I overpaſs it till I have taught you the Art of diviſion, which you ſhall work thus.

Diviſion.

FIrſt, ſet down the Diviſor, for feare of forgetting, and then ſet that number that ſhall be divided at the right ſide, ſo farre from the Diviſor, that the quotient may be ſet between them: as for example.

If 225 ſheep coſt 45 pound, what did every ſheep coſt? To know this, I would divide the — An example of ſheep.

the whole summe, that is 45 pound, by 225, but that cannot be: therefore must I first reduce that 45 pound, into a lesser denomination, as into shillings, then I multiply 44 by 20, and it is 900; that summe shall I divide by the number of sheep, which is 225, these two numbers therefore I set thus:

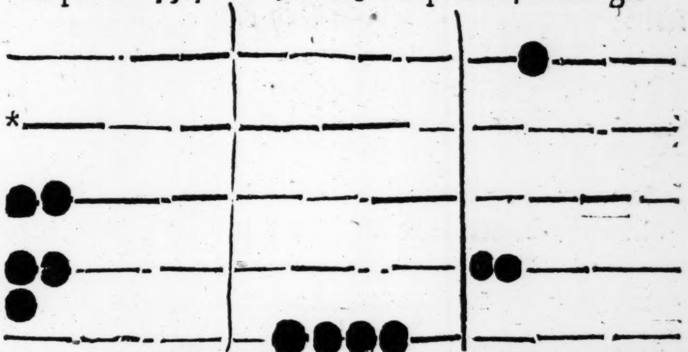


Then begin I at the highest line of the dividend, and seek how oft I may have the divisor therein, and that I may do four times: then say I, foure times 2 are 8, which if I take from 9, there resteth but 1, thus:

And because I found the divisor 4 times in the dividend, I have set as you see, 4 in the middle room, which is the place of the quotient: but now must

muſt I take the reſt of the diviſor as often out of the remainder, therefore come I to the ſecond line of the diviſor, ſaying two times 4 make 8, take 8 from 10, and there remaineth 2, thus:

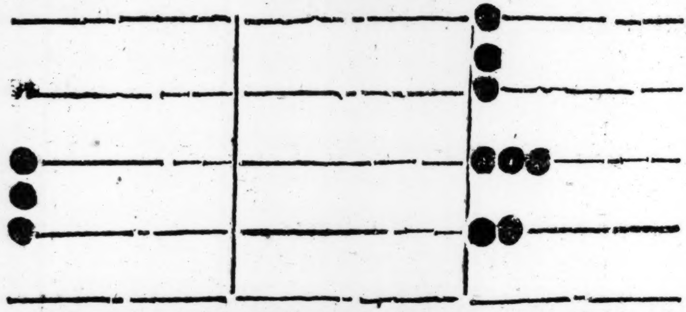
Then come I to the loweſt number, which is 5, and multiply it 4 times, ſo is it 20, that take I from 20 and there remaineth nothing, ſo that I ſee my quotient to be 4. which are in value ſhillings, for ſo was the dividend: and thereby I know that if 225 ſheep coſt 45 pound, every ſheep coſt 4 ſhillings.



Scholar. This can I do as you ſhall perceive by the example. If 160 Souldiers do ſpend every moneth 68 pound, what ſpendeth each man?

Example of ſouldiers wages.

Firſt, becauſe I cannot divide the 68 by 160, therefore I will turn the pounds into pence by Multiplication, ſo ſhall there be 16320 pence: now muſt I divide the ſumme by the number of Souldiers, therefore I ſet them in order thus;



Then begin I at the higheſt place of the dividend, ſeeking my diviſion there, which I find once, therefore I ſet 1 in the nether line.

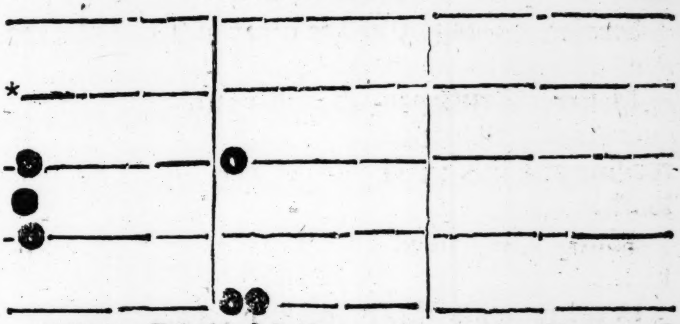
Maſter. Not in the nether line of the whole ſumme, but in the nether line of that work which is the third line.

Scholar, So ſtandeth it with reaſon.

Maſter. Then thus do they ſtand.

Then ſeek I again the reſt, how often I may find my diviſor: and I ſee that in 300 I might find 100 three times: but then the 60 will not be ſo often found in 20, therefore I take 2 for my quotient: then take I 100 twice from 300, and there reſteth 100, out of which with the 20, that maketh 120, I may take 60 alſo twice, and then ſtand the numbers thus:

Where



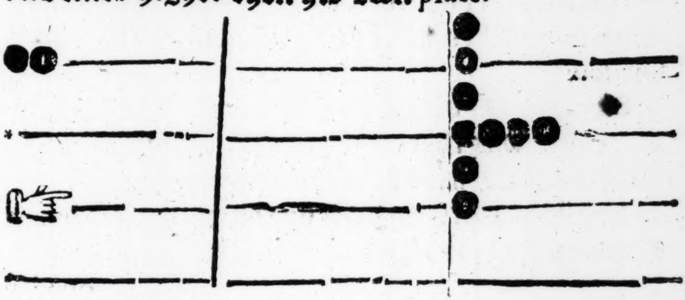
Where I have set the quotient 2 in the lowest line: so is every Souldiers portion 102 pence, that is 8 shillings 6 pence.

Master. But yet because you may justly perceive the reason of division, it shall be good that you set your divisor still against those numbers from which you do take it, as by this example I will declare.

If the purchase of 200 acres of ground did cost 290 pound, what did one acre cost;

First, will I turne the pounds into pence, so will there be 69600 pence. Then in setting down these numbers, I shall do thus: An example of purchase.

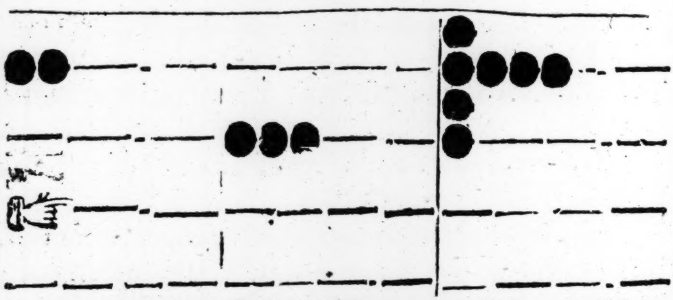
First set the dividend on the right hand as it ought, and then the divisor on the left hand against those Numbers from which I intend to take him first, as here you see, where I have set the divisor two lines higher then his own place.



Scholar. This is like the order of diviſion by the Pen.

Maſter. Truth you ſay, and now I muſt ſet the quotient of this work in the third line, for that is the line of unites in reſpect of the diviſor in this work.

Then I ſee how often the diviſor may be found in the dividend, and that I find 3 times, then ſet I 3 in the third line for the quotient, and take away that 60000 from the dividend: and further I ſet the diviſor one line lower, as you ſee here.



And then ſeek I how often the diviſor will be taken from the number againſt it, which will be four times and 1 remaining.

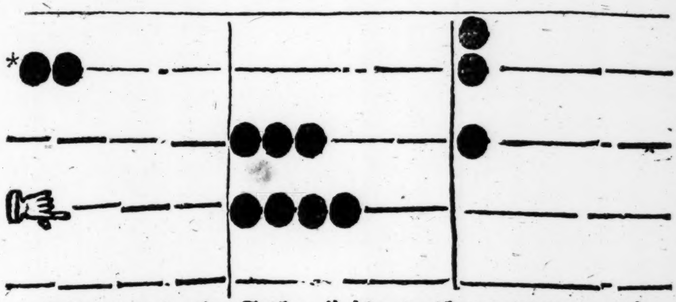
Scholar. But what if it chance that when the diviſor is ſo removed, it cannot be once taken out of the dividend againſt it?

Maſter. Then muſt the diviſor be ſet in another line lower.

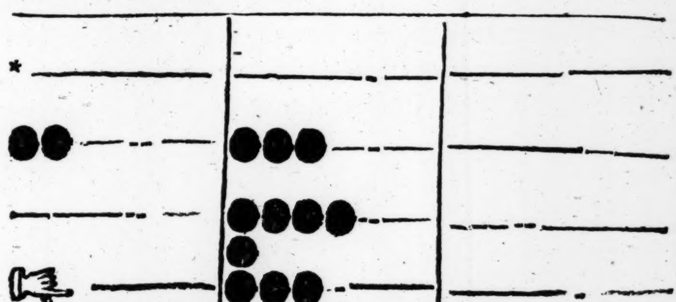
Scholar. So was it in diviſion by the pen, and therefore was there a cypher ſet in the quotient: but how ſhall that be noted here?

Marſter. Here needeth no token, for the lines do repreſent the places, onely look that you ſet your quotient

quotient in that place which standeth for unites in respect of the divisor. But now to return to the example. I find the divisor four times in the dividend, and 1 remaining: for 4 times 2 make 8, which I take from 9, and there resteth 1, as this figure following sheweth: and in the middle space for the quotient, I set 4 in the second line, which is in this work the place of unites.



Then remove I the divisor to the next lower line and seek how often I may have it in the dividend which I may do here 8 times just, and nothing remain, as in this form.



Where you may see that the whole quotient is 348 pence, that is 29 shillings, whereby I know that so much cost the purchase of one Acre.

Scholar.

Scholar. Now resteth the proofs of Multiplication, and also division.

Master. Their best proofs are each one by the other; for Multiplication is proved by division, and division by Multiplication, as in the work by the pen you learned.

Scholar. If that be all, you shall not need to repeat again that which was sufficiently taught already: and except you will teach me any other feat, here may you make an end of this Art, I suppose.

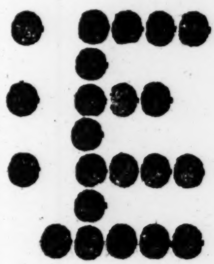
The reason of all the former rules.

Master. So will I doe as touching whole number, and as for broken number I will not trouble your wit with it, till you have practised this so well, that you be full perfect, so that you need not to doubt in any point that I have taught you, and then may I boldly instruct you in the Art of Fractions or broken numbers: wherein I will also shew you the reasons of all that you have now learned. But yet before I make an end, I will shew you the order of common casting, wherein are both pence, shillings and pounds, proceeding by no grounded reason, but onely by a received forme, and that diversly, of divers men: for the Merchants use one forme, and Auditors another.

Merchants uſe.

Merchants Accompt.

BUt firſt for Merchants form, marke this example here, in which I have expreſſed this ſumme, 198 pounds 19 ſhillings 11 pence. *So that you may ſee that the loweſt line ſerveth for pence, the next above for ſhillings, the third for pounds, and the fourth for ſcores of pounds.*



And further you may ſée that the ſpace betwéen pence and ſhillings, may receive but one counter, (as all other ſpaces likewiſe do) and that one ſtandeth in that place for 6 pence.

Likewiſe betwéen the ſhillings and the pounds one counter ſtands for 10 ſhillings.

And betwéen the pounds and 20 pounds, one counter ſtandeth for 10 pounds.

But beſide theſe, you may ſée at the left ſide of ſhillings, that one number ſtandeth alone and betokeneth 5 ſhillings.

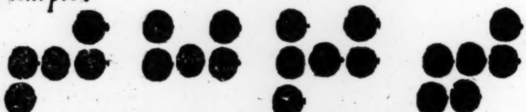
So againſt the pounds, that one counter ſtandeth for 5 pound. And againſt the 20 pounds, the one counter ſtandeth for five ſcore pounds, that is 100 pounds: ſo that every ſide counter is five times ſo much as one of them againſt which he ſtandeth.

Auditors

Auditors Accompt.

Auditors accoumpt.

N*Ow for the Accompt of Auditors, take this example.*



Where I have expreſſed the ſame ſumme 198 pound 19 ſhillings 11 pence.

But here you ſee the pence ſtand towards the right hand, and the other increaſing orderly towards the left hand.

Again you may ſee, that Auditors will make two lines (yea and more) for pence, ſhillings, and all other values, if their ſummes extend thereto. Alſo you ſee that they ſet one Counter at the right end of each row, which ſo ſet there ſtandeth for five of that room, and on the left corner of the row, it ſtandeth for 10 of the ſame row.

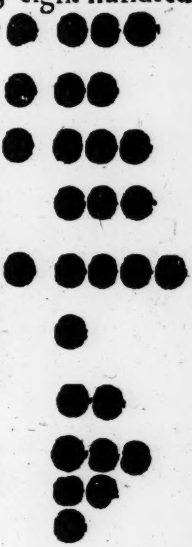
But now if you would adde, or ſubtract after any of both theſe ſorts, if you mark the order of the other feat which I taught you, you may eaſily do the ſame here without much teaching: for in Addition you muſt firſt ſet down one ſumme, and to the ſame ſet the other orderly, and in like manner, if you have many; but in Subtraction, you muſt ſet down firſt the greateſt ſumme, and from it muſt you abate the other, every Denomination from his due place.

Scholar. I do not doubt but with a little practiſe I ſhall attain theſe both: but how ſhall I multiply and divide after theſe forms?

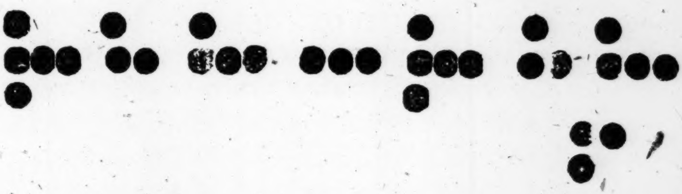
M. You cannot duely do any of both by theſe ſorts: therefore in ſuch caſe you muſt reſort to your other Arts.

Scholar.

Scholar. They that uſe ſuch Accounts that it exceed 200 in the ſumme, they ſet not 5 at the left hand of the ſcores of pounds, but they ſet all the hundreds in another farther row, and 500 at the left hand thereof, and the thouſands they ſet in a farther row yet, and at the left ſide thereof they ſet the 5000, and in the ſpace over they ſet the 10000, and in a higher row 20000, all which I have expreſſed in this example, which is 97869 pounds 12 ſhillings 9 pence ob. q. Ninety ſeven thouſand, eight hundred threeſcore and nine pounds, twelve ſhillings and nine pence half peny farthing, for I had not told you before, where neither how you ſhould ſet down farthings, which (as you ſee here) muſt be ſet in a voide place ſideling beneath the pence, for a farthing one counter, ob. two counters, for ob. farthing 3 counters, and more there cannot bee: for 4 farthings make 1 peny, which muſt be ſet in his due place.



And if you deſire the ſame ſumme after Auditors manner, lo here it is.



But in this thing you shall take this for sufficient, and the rest you shall observe as you may see by the working of each sort, for the divers wits of men have invented divers and sundry wayes, almost innumerable.

THE SECOND PART OF *ARITHMETICK*, touching Fractions, briefly set forth.

Scholar.

Arithmeticall fractions.

ALbeit I perceive your manifold businesses doth so occupie, or rather oppress you, that you cannot as yet compleatly end the Treatise of Fractions Arithmeticall, which you have prepared, wherein not onely sundry works of Geometry, Musick, and Astronomy be largely set forth, but also divers conclusions and naturall works touching mixtures of Metals, and compositions of Medicines, with other strange examples. Yet in the meane season, I cannot stay my most earnest desire, but importunely crave of you some brief preparation toward the use of Fractions, whereby at the least I may be able perfectly to understand the common works of them, and the vulgar use of those rules, which without them cannot well be wrought.

Master. If my leasure were as great as my will

is good, you should not need to use any importunate craving, for the attaining of that thing, whereby I may be perswaded that I shall any way profit the Common wealth, or help the honest studies of any good Members in the same: wherefore while mine attendance will permit me to walk and talk, I am well willing to help you as I may.

Therefore, first to begin with the explication of this name fraction, what take you it to be?

Scholar. Marry sir, I think a fraction (as I have heard it often named) to be a broken number, that is to say, to be no whole number but part of a number.

Master. A fraction indeed is a broken number, and so consequently the part of another number, but that must be understood of such another number as cannot be divided into any other parts then fractions: for although I may take the third part of 60, or the 4 part of it, and so of other parts diversly, yet those parts be not properly, nor ought to be called fractions, because they may be expressed by whole numbers, for the 3 part of it is 20, the 4 part is 15, the 12 part is 5, and so forth of other parts, all which be whole numbers. What a Fraction is

Wherefore properly a fraction expresseth the parts or part onely of a unite, that is to say, that the number which is the whole or entire summe of any fraction, may not be greater then one: and therefore it followeth, that no one fraction alone can be so great, that it shall make 1, as by example I will declare, as soon as I have taught you to know the form how a fraction is expressed or represented in writing. What a Fraction is Properly.

Nume-

Numeration.

The expressing of fractions.

But first to begin with expressing of a fraction, which is the Numeration of it: you must understand that a fraction is represented by two numbers set one over the other, and a line drawn between them, as thus, $\frac{1}{3}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{10}{17}$, which four fractions you must pronounce thus, $\frac{1}{3}$ one third part, $\frac{3}{4}$ three quarters, $\frac{2}{5}$ two fifth parts, $\frac{10}{17}$ ten seventeen parts.

Scholar. I understand this form of their expression and pronounciation, but their meaning or valuation seemeth more obscure. Yet I think that by the two first Fractions, I understand the valuation of the two latter Fractions, and consequently of other.

Master. Value them then, that I may perceive your taking of them.

Scholar. $\frac{2}{5}$ betokneth two fift parts, that is to say, if one be divided into 5 parts, that Fraction doth expresse two of those 5 parts: $\frac{10}{17}$ doth signifie, that if one be divided into 17 parts, I must take ten of them. And this I gather of the two first examples: for $\frac{1}{3}$, that is, one third part, doth easily declare, that if one thing be divided into three parts, I must take out one of them: so $\frac{3}{4}$, that is three quarters, doth declare that one being divided into foure quarters I must take (for this Fraction) three of those quarters.

If there be no more difficulty in this Numeration, then I pray you goe forward to their Addition and Subtraction, and so to the other kindes of workes. For I understand that the same kinds

of

of works be in fractions, that bee in whole numbers.

Master. There are the same kinds of works in both, albeit the order of them is divers, as I will anon declare: but yet more in Numeration before we leave it. You must understand, that those two numbers which expresse a fraction, have severall names, the overmost which is above the line, is called the Numerator, and the other beneath the line is called the Denominator. Numerator. Denominator.

Scholar. And what is the reason of their divers names? For (in mine opinion) both bee Numerators, seeing both doe expresse the numeration of the fraction.

Master. You are deceived; for one onely (which is the overmost) doth expresse the Numeration, and the Donominator doth declare the number of parts, into which the unite is divided, as in this example; when I say: divide a pound weight of gold between foure men, so that the first man shall have $\frac{2}{15}$, the second $\frac{3}{15}$, the third $\frac{4}{5}$, and the fourth $\frac{6}{15}$.

Now doe you perceive that by the Denominator (which is one in all foure fractions) it is intended that the pound weight should bee divided into so many parts, I meane 15, and by the 4 severall Numerators, is limited the divers portion that each man should have, that is, that when the whole is parted into 15, the first man shall have two of those 15 parts: the second man three of them: the third man foure: and the fourth man six. And so may you see the severall offices (as it were) of those two numbers, I meane of the Numerator and the Denominator.

And hereby you perceive that a man can have no more parts of any thing then it was divided

into, neither yet aptly so many: so that it were unaptly said, You shall have $\frac{15}{15}$ that is fifteen parts of any thing, seeing it were better said, you shall have the whole thing.

Scholar. So doth it appeare reasonably, for the labour is vain to divide any thing, and then to apply the Division to no use. And much lesse reasonable were it to say $\frac{16}{15}$: for if the whole be divided into 15 parts onely, it is not possible to take 16 of them, that is to say, more then all together.

Master. This is true touching the proper and apt use of the name of a Fraction; yet improperly (and after a vulgar acceptation for easinesse in worke) both those formes bee called Fractions, because they be written like Fractions although they bee none indeed: for $\frac{15}{15}$ and generally in such other, where the numerator and Denominator be equall, are not Fractions, but the whole thing with all his parts: And so $\frac{14}{12}$ is not to be called a Fraction, but a mixt number, of a whole number and a Fraction, for it is as 1 $\frac{2}{12}$ that is, one whole and $\frac{2}{12}$ parts; as shall be declared in Reduction. Therefore they do abuse the names that call them Fractions, where the Numerator is either equal or greater then the Denominator.

An improper fraction or a mixt number.

Scholar. But is there any needfull cause, why they should so abuse the name?

Master. There is cause why they shall sometimes for easinesse in work write some numbers after that sort like Fractions: but they needed not to call them Fractions, but (as they be) whole numbers, or mixt numbers, (that is, whole numbers with Fractions) expressed like Fractions, or as improper Fractions.

Now

Now muſt you underſtand, that as no Fraction properly can be greater then one, ſo in ſmalneſſe under one the nature of Fractions doth extend infinitely, as the nature of whole numbers is to increaſe above one infinitely, ſo that not onely one may be divided into infinite Fractions or parts, but alſo every Fraction may be divided into infinite Fractions or parts, which commonly be called Fractions of Fractions : and they be expreſſed diverſly as for example, of $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, that is, three quarters of 2 third parts of one halfe part. Whereby is ſignified, that if one be divided into 2 halfes, and the one halfe into three parts, and two of thoſe three parts be divided joyntly into foure quarters, this fraction of fractions doth repreſent three of thoſe quarters. Fractions of fractions.

Scholar. I pray you let me prove by an example in common money, whether I doe rightly underſtand you or no. One Crowne which I take for an unite, doth containe 60 pence ; therefore the halfe of it is 30 pence, $\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{3}{4}$ is fifteen pence; ſo then 15 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a Crown : and ſo is 3 pence, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$. of a ſhillings.

Maſter. You perceive this well enough : yet this note I give you by the way, that the forme of expreſſing the fractions is voluntary, and hath no other reaſon then the will of the Diviſor, which forme many follow : for ſome expreſſe them thus, $\frac{3}{4}\,\frac{2}{3}\,\frac{1}{2}$ without any figure of diſtinction betweene them, which forme alſo many follow. Some other doe make lines betweene every fraction, and adde words of diſtinction, after this ſort, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ which forme is beſt.

Some other expresse them thus in slope form, to distinct them from fractions of whole numbers, for if they were in one right line thus, $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$ then ought it to bee pronounced, three quarters and two third parts, & a halfe, which maketh almost two whole unites, lacking but one twelfth part. And so is it nothing agreeable with the other fraction of fractions: wherefore it is a great oversight in certain learned men, which do expresse them so confusedly with such severall fractions, that a man cannot know the one from the other.

$\frac{3}{4}$
$\frac{2}{3}$
$\frac{1}{2}$

Therefore some men (as Stifelius*) do expresse without a line, numbers of proportion, being applied to* Addition *or* Subtraction, *because they must be taken as two, where the line in fractions maketh them to be taken for one: for of the* Numerator *and* Denominator *is made one number.*

Three severall varieties.

Scholar. Then I perceive there be three severall varieties in fractions: First, when one onely fraction is set for one number, as $\frac{4}{5}$, that is, foure fifth parts. The second is, when there be set two or more severall fractions of one number; as $\frac{4}{9}$, $\frac{2}{5}$, that is, foure ninth parts, and two fifth parts. The third sort is fractions of fractions as $\frac{4}{9}$, of $\frac{2}{5}$, that is, foure ninth parts of two fifth parts.

Master. You have said well, if you understand well your own words.

Scolar. If it shall please, I will by an example in the parts of an old English Angel, expresse my meaning.

Master. Let me hear you.

Scholar.

Scholar. The old Engliſh Angel did contain 7 ſhillings 6 pence, that is 90 pence: Now $\frac{4}{5}$ of it is 72 pence: And of the ſame 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$, that is foure ninth parts, and two fifth parts, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, which both make 76: but if I take $\frac{4}{9}$ of $\frac{2}{5}$, that is, foure ninth parts of two fifth parts, ſeeing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yeeld but 16 for $\frac{1}{9}$ of 36 is but 4, and that taken foure times maketh 16.

Maſter. This is plainly expreſſed, and truly, and hereby (I doubt not) but you do perceive, that as great a difference, as is betweene 16 and 76, ſo much difference is betweene thoſe two fractions $\frac{4}{9}$ and $\frac{2}{5}$: and $\frac{4}{9}$ of $\frac{2}{5}$.

And now that ye underſtand theſe varieties, I will proceed to the reſt of the works: Firſt admoniſhing you, that there is another order to be followed in Fractions, then there was in whole numbers: for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Diviſion, and Reduction: but in Fractions, (to follow the ſame aptneſſe in proceeding from the eaſieſt workes to the harder) we muſt uſe this order of workes, Numeration, Reduction, Addition, Subtraction, Multiplication and Diviſion.

The order of works in fractions.

Scholar. That Addition and Subtraction ſhould go together, and Diviſion to follow Multiplication, naturall order doth perſwade: but why Reduction ſhould be firſt in order here, next to Numeration, and Addition and Subtraction, in the middle, I deſire to underſtand the reaſon.

Maſter. As in the Art of whole numbers, Order would reaſonably begin with the eaſieſt, and ſo

goe forward by degrees to the hardest: even reason teacheth in Fraction the like order. And consider that Addition or Subtraction of Fractions, can very seldome bee wrought without Reduction: & contrariwise, Reduction may be wrought without this forme of Addition or Subtraction: therefore was it orderly required, that Reduction should goe before Addition and Subtraction, and this reason serveth for the placing of Reduction before the other.

Scholar. Then, if Reduction be the easiest, I pray you declare the forme of it, first by rule, and then by example.

Master. Your request is good.

Reduction of Fractions.

Of Reduction of Fractions, there are five varieties.

THerefore will I now declare the diversities of Reduction of fractions, which commonly hath five varieties, or formes.

1 *First, when there bee sundry Fractions of one intire unite, they must be reduced to one Denomination, and also into one Fraction.*

2 *Secondly, when there be propounded fractions of fractions, they must be reduced likewise into one fraction: for otherwise they cannot bee brought into one Denomination.*

3 *Thirdly, when an improper fraction is propounded, that is to say, a fraction in forme, which indeed is greater*

greater then an unite : it must be reduced into apt forme, expressing the unite or unites of it, and the proper fraction distinctly. And sometimes also it shall be needfull to convert such a mixt number of unites with fractions, into the forme of a fraction, that is, into an improper fraction: which two formes I esteem but as one, because they work one kind of number.

Fourthly, there happneth sometimes fractions to be written in great numbers, which might be written in lesser 4 *numbers : therefore is there a meane to reduce such great numbers into their smallest termes.*

Fifthly, when any fraction betokeneth the parts of a 5 *whole thing, which hath by common partition certain parts, but none of like* Denomination *with that fraction, then may you reduce the said* fractions *into another, whose* Denomination *shall expresse the common parts of that whole thing.*

Scholar. **This distinction in** Doctrine **delighteth mee much, but more with hope then present fruit : for as yet I doe scarcely understand the varieties, & much lesse the practise & use of their works.**

Master. Reduction **is an orderly alteration of** Numbers **out of one forme into another, which is never done orderly but for some needfull use, as in every of the said five severall formes, I will distinctly declare.**

First therefore, when two, or more severall fractions of any unite be propounded : as for example $\frac{3}{6}$ and $\frac{4}{6}$, because it is hard to tell what proportion of the intire number those two fractions do expresse, therefore was Reduction devised to be a mean whereby these severall fractions might be brought into one Denomination and fraction.

The first forme of Reduction

And in these fractions, this is the Art for bringing them to one Denomination.

How to reduce fractions of divers denominations into one denomination.

Multiply first the Denominators *together, and the totall thereof you shall set twice down under two severall lines for two new* Denominators, *or rather for one common* Denominator. *Then multiply the* Numerator *of the first fraction, by the* Denominator *of the second, and set the totall thereof for the* Numerator *over the first line. Likewise multiply the* Numeratitor *of the second fraction by the Denominator of the first, and set that totall over the second line for the Numerator of that fraction: and so are these two first Fractions of severall Denominations, brought to one Denomination.*

Scholar. If I understand you, as I think I doe, my example shall declare the same. The Fractions, which you propounded were these $\frac{3}{16}$ and $\frac{4}{6}$, whose Denominators (being 15 and 6) I multiply together, and there amounteth 96, which I set under two lines, thus: $\frac{}{96}$ $\frac{}{96}$

Then I multiply the Numerator of the first Fraction by the Denominator of the second, saying, 3 into 6 maketh 18, that I set over the first line for a new Numerator, and it will be thus, $\frac{18}{96}$

Likewise I multiply the Numerator of the second Fraction, by the Denominator of the first, saying 4 times 16 maketh 64, that I set for the second Numerator, and the Fraction will appear thus $\frac{64}{96}$.

So that both Fractions brought to one Denomination, must stand thus, $\frac{18}{96}$ and $\frac{64}{96}$.

Master. You have done well.

Scholar. I beseech you let me examine it after my

my accustomed forme, by common parts of coyne or other measure.

Master. Go to.

Scholar. I have a péece of Gold which is accounted worth 8 shillings, and containeth 96 pence, whereof $\frac{1}{16}$, that is, the sixteenth part, is 6 pence, and $\frac{3}{16}$ is 18 pence, that is $\frac{18}{96}$. Againe $\frac{1}{6}$ of the same péece of gold is 16 pence, so that $\frac{4}{6}$ parts maketh 64 pence, that is $\frac{64}{96}$. And so I find the summes to agrée with the other before.

Master. *So have you now the Art to bring two such fractions into one* Denomination *: And if there be more then two, then must you multiply all the Denominators together, and set the totall thereof so many times down as there be fractions; and then to get for each one a new* Numerator, *multiply the Numerator of the* first *by the Denominator of the* second, *and the totall thereof multiply by the Denominator of the* third, *and so forth, if there be more. Likewise multiply the Numerator of the* second, *by the Denominator of the* first, *and the totall thereof by the Denominator of the* third. *And in the same sort multiply the Numerator of the* third, *into the Denominator of the* first, *and the totall thereof into the Denominator of the* second, *and so forth if there were moe. So these three fractions* $\frac{2}{5}$ $\frac{3}{4}$ $\frac{2}{3}$ *doe make by Reduction these other three fractions of Denomination.* $\frac{24}{60}$ $\frac{45}{60}$ $\frac{40}{60}$. *All which you may bring into one fraction by adding the Nnmerators together, and putting the totall for the totall Numerator, reserving still that same common Denominator. And those three fractions make one improper fraction, thus:* $\frac{19}{60}$.

Note the Reduction of three fractions, or more, into one.

Scholar. All this I perceive, and also that this

last

last Fraction is more then an unite, and therefore you did call it an improper Fraction.

Master. There be certain other formes of working in this Reduction, which I will briefly touch also, to give you an occasion to exercise your wit therein.

The first variety of Reduction

The first variety is this: When you have made and written down your common Denomination *(as I have taught before) then to get a* numerator *for the first, do thus: Divide the* common Denominator *by the* denominator *of the first fraction, and the quotient multiplied by the* numerator *of the same, yeeldeth a new* numerator *for the first new fraction. So likewise do with the second and the third, and with all the residue, if there be more.*

Scholar. That will I prove in your last example of these three fractions, $\frac{2}{5}\,\frac{3}{4}\,\frac{2}{3}$, When the Denominators be multiplied they make 60, for 5 into 4 maketh 20 and 20, by 3 yeeldeth 60, that I set down three times thus: $\frac{}{60}\,\frac{}{60}\,\frac{}{60}$ then to have a numerator, for the first, I must divide 60 by 5 (the Denominator of the first) and the quotient is 12, which I must multiply by 2 (the numerator first) and that maketh 24, and so have I for the first Fraction, $\frac{24}{60}$.

Likewise for the second Fraction: I divide 60 by 4, and there cometh 15, which I multiply by 3, and so have I 45, for the second Fraction $\frac{45}{60}$. Then for the third in like sort will come $\frac{45}{60}$.

The second variety.

Master. *Another way is this: If it happen so, that the lesser* denominator, *can by any* multiplication *make the greater, then note the multiplier, and by it multiply the numerator over that lesser denominator, and for the lesser* Denominator *put the greater, as thus in these*

these two fractions $\frac{3}{12}$ *and* $\frac{2}{3}$ *three being the lesser Denominator multiplied by* 4, *will make* 12, *which is the greater* Denominator : *therefore by the same* 4 *I do multiply* 2 *which is the* numerator *over* 3, *and that maketh* 8 *: under which I do put* 12, *being the greater* Denominator, *which is also made by multiplication of* 4 *into* 3, *and so have I these two fractions* $\frac{3}{12}$ $\frac{8}{12}$, *thus shortly reduced, without altering the one* Fraction.

Scholar. This I understand.

Master. *Then mark this third way : if the* Denominators *doe not happen so, that one by* Multiplication *may make the other, then look whether they both may be parts of any other one* Number, *as in* $\frac{5}{12}$, *and* $\frac{7}{18}$, *although the lesser taken but twice, be too much to make* 18, *yet they both may be parts unto* 36, *therefore look how many times twlve is in* 36, *and that quotient being multiplied by the* Numerator *over* 12, *the totall shall be put in stead of the* Numerator *over* 12, *and for* 5 *put* 15, *thus*, $\frac{15}{36}$. *So likewise look how often is* 18 *in* 36, *because it is twice, therefore by* 2 multiply 7, *which is over* 18, *and it will be* 14 *: set that for the* Numerator, *and in stead of* 18 *put* 36 ; *and then your* Fractions *reduced stand thus* $\frac{15}{36}$ $\frac{14}{36}$, *in stead of* $\frac{5}{12}$, *and* $\frac{7}{18}$.

The third variety.

And if you will prove whether you have wrought well or no, that may be proved by Reduction of them again to their former Denominations, which Art shall be taught in the fourth kind of Reduction, where greater termes of Fractions be reduced into smaller in number, but no smaller in proportion. And, if in such Reduction the same termes or numbers come again that were before, then is the worke good, else not.

Proofe.

Scholar.

Scholar. Sir I heare your words, but I doe not understand many of them: which if it please you declare.

Master. With a good will, when convenient place serveth, but that must be in the said fourth kind of Reduction, which teacheth how to reduce fractions of fractions into one fraction, and so to one Denomination.

The second form of Reduction of fractions of fractions into one fraction and Denomination.

When Fractions of Fractions be propounded, you shall multiply the numerators of each into other, and set the totall for the new Numerator, and then multiply all the Denominators likewise, and take their totall for the new Denominator, and so are they speedily reduced,

Scholar. If that be all, then I understand it already, as by this example I will declare. These be the fractions $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$ which I would reduce to one denomination and proper simple fraction.

Therefore begin I with the Numerators, and multiply them together, saying, 3 by 2, maketh 6: and 6 by 6, maketh 36, which multiplyed by 7, yeeldeth 252: that I set over a line for the Numerator thus: $\frac{252}{}$

Then I multiply the denominator, 4 by 3, maketh 12, and that by 7 bringeth 84, which multiplied by 9, yeeldeth 756, the new denominator. And so the whole reduced fraction is this, which is too hard a fraction for me to understand yet. $\frac{252}{756}$

Master. You think so, and no marvell, but anon you shall learn to judge it easily, for this fraction is no more indeed then $\frac{1}{3}$, although it be in greater termes, & therefore more stranger, & more obscure.

And this sufficeth for this Reduction, save that I

I wil ſhew you by a figure of meaſure the juſt rate and reaſon of this kind of fractions, and alſo the due underſtanding of their Reduction.

The entire meaſure parted into 9.

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	$\frac{7}{9}$	
1	2	3	4	5	6	$\frac{6}{7}$		
1	2	3	4	$\frac{2}{3}$				
1	2	3	$\frac{3}{4}$.					

Here you ſee the longeſt meaſure, (which ſtandeth for the whole and entire quantity) firſt parted into 9 diviſions, whereof 7 are ſevered by the ſecond meaſure: and thereof againe are parted out 6, and that 6 being diſtinct into three parts, two of them are parted by the fourth meaſure, of which fourth meaſure being divided into foure parts, the loweſt meaſure doth contain $\frac{3}{4}$, ſo that the ſame $\frac{3}{4}$ muſt be named not $\frac{3}{4}$ of the whole meaſure, but indeed is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

Scholar. This example is ſo ſenſible, that I cannot chuſe but ſee it. And furthermore ſee alſo, that the ſame fraction is equall to $\frac{3}{9}$ of the entire meaſure, as the lines which run up and down doe expreſly ſet forth. Alſo I ſee here that $\frac{2}{3}$ of $\frac{6}{7}$ is equall to $\frac{4}{7}$, and further yet that $\frac{2}{7}$ of $\frac{7}{9}$ is equall to $\frac{6}{9}$ or $\frac{2}{3}$.

Maſter. I am glad that you ſee it ſo well, not doubting but you will gather greater light of knowledge hereby.

But now it is time that wee come to the third forme of Reduction, which teacheth of improper Fractions, *that*

that is to say, mixt numbers of unites and fractions although they appear like fractions, as the $\frac{26}{5}$, which doth conclude 5 unites wholly, and $\frac{1}{5}$ over. Wherefore first you shall know them, by that the Numerator is greater then the Denominator.

The third forme of Reduction of improper fractions.

Scholar. Indeed Sir, that appeareth reasonable, that if the Numerator do expresse more parts to be taken of any unite then the Denominator doth signifie that unite to be divided into, it must needs follow, that such a Fraction importeth more then the whole, that is to say, the whole with certain parts over: but what Reduction is there in it?

Two severall waies in this Reduction.

Master. There be two severall kindes of Reduction, concerning such fractions. Sometimes it shall bee needfull to convert these fractions into unites, and the proper fraction, that will remaine. And sometimes, contrariwise, it shall be meet to reduce mixt numbers, that is, unites written with fractions, into the forme of one simple fraction, and so be there two waies.

Scholar. What is the mean of the first way to turn improper Fractions into unites with their proper Fractions?

The fifth way.

Reduction of improper fractions into unites, with their proper fractions.

Master. *That is thus; Your* numerator *being greater then the* denominator, *must be divided by the same* denomination, *and the* quotient *thereof expresseth the* unites; *the remainer shall be put for the* numerator *of the fraction that resteth, and the* denominator *must be the same that was before.*

Scholar. For example, I take $\frac{17}{5}$, and dividing 17 by 5, the quotient will be 3, and there will remain 2.

Master. That you must write thus, $3\frac{2}{5}$, where (you see) I have written 3 without any line, as entire

entire numbers ought to bee written, and the 2 that remained, I have set over the former denominator, with a line; as a proper fraction. And this number doth signifie now three unites, & $\frac{2}{5}$ of one.

Scholar. Then if I would by unites here understand Crownes, so it were 3 Crownes, and $\frac{2}{5}$, that is 2 s.

Master. Even so: and therefore $\frac{17}{5}$ did signifie the same: But this happneth sometimes that when the Reduction is so wrought, there remaineth nothing: And then it is not a mixt number, but a simple intire number, represented like a Fraction.

Scholar. As $\frac{15}{5}$ will make 3 just: and $\frac{18}{3}$ will make even 6. This I will remember. But now, what is the second form of reduction that you spake of for these sorts of fractions. The second way.

Master. *Whensoever you have any of these two sorts of* numbers, *that is to say, whole* numbers *without* fractions, *or whole* numbers *with* fractions, *and you would turn them into the form of* a fraction, *you must multiply the whole* number *by that* denominator *which you will have to remain still, and to the totall thereof adde the* numerator, *which you have already, and all that you shall set for the new* numerator, *keeping still the former* denominator: *As if you have* 6 $\frac{3}{4}$ *which you would convert into an improper* fraction, *you must multiply* 6 *by* 4, *whereof cometh* 24, *and thereto adde the* numerator, *which is* 3, *and so have you* 27 *for the* numerator, *and* 4 *still for the* denominator.

Reduction of whole numbers either alone, or joyned with fractions into improper fractions.

Scholar. Then is $\frac{27}{4}$ equall to 6 $\frac{3}{4}$.

Master. Even just, and so backward (as appeareth by the former Reduction) 6 $\frac{3}{4}$ maketh $\frac{27}{4}$. Note.

And

And thus one of their Reductions may be the proof of the other worke.

Scholar. This I perceive: But now if you would turn whole numbers without fractions into any fractions, I see not how that may be done, because there is no Denominator to make the multiplication by.

Master. That is well marked: but this you know, that no man intendeth to turn any whole number into a fraction, but he hath in his minde that Denominator by which the multiplication must be made: for the proofe whereof I set down 7, which is a whole number. And if you will have this nmuber converted into any certain fraction, will mee to doe it.

Scholar. I pray you reduce 7 into a Fraction.

Master. Then you care not what the Fraction be, so it be some Fraction,

Scholar. No, I passe not for the sort of the Fraction.

Master. Then how can you think that you require me to doe any thing certain, when you leave me to doe as I list? And seeing you stand at that stay, whether think you that I must first intend in minde what fraction I will make of it before I can doe it indeed?

Scholar. Else you should doe ignorantly.

Master. Then will I limit my self (seeing you will not) to turn it into quarters. And therefore I multiply 7 by 4 (which is the denomination by quarters) and there amounteth 28 to be set for the Numerator, and the 4 must be set for the denominator, and the fraction will be thus $\frac{28}{4}$.

Scholar.

Scholar. Indeed I perceive this to be reasonable, for without much triall I understand that $\frac{28}{4}$ of any thing doth make 7. And so then if I would turne 8 into 5 parts, it will make $\frac{40}{5}$ which is all one with 8: for 8 Crownes turned into 5 parts (that is into shillings) will make 40 shillings, that is $\frac{40}{5}$ of a Crowne.

Master. *Seeing you understand now these three kinds of* Reduction, *I will declare unto you the fourth kind, that is, when* fractions *be written in greater termes then they need, how they may be brought to lesser termes.* The fourth forme of Reduction

Scholar. To write any thing in greater termes then needeth, seemeth to be a fault, and so this Rule seemeth to amend that fault.

Master. It were a fault to doe any thing without need, which after must bee redressed: but in this case it is not so, neither did I say absolutely (as you doe) that it needeth not to expresse those Fractions in so great termes, but that the Fractions doe not need, I meane for their value, to be understood: but yet it may be needfull for the ease of these works whereto they be applyed; as for example, In the first kind of Reduction this was your own example, $\frac{3}{16}$ and $\frac{4}{6}$ which when you would reduce, you were fain to turne them first into one denomination, and so appeared they thus, $\frac{18}{96}$ and $\frac{64}{96}$, where the Fractions (for their own understanding) needed not to be turned out of smaller termes into greater, but yet the easinesse of working needed it.

Scholar. Sir, I understand now, not onely the difference of this need (for the Fractions might better be understood as Fractions severall,

each in his value, when they were in lesser termes, although they could not so well be reduced) but also I understand what you mean by greater termes, and lesser termes, whereof before I was in doubt: for I see you call the Numerator and Denominator, the termes of the fraction.

Termes of Fractions.

Master. *I am glad you understand it so well: now then when you would value any* fractions, *because they may best be done when the termes are smallest, you shall reduce them to the smallest that you can, which thing you may doe thus: Divide the greatest of any such two termes by the lesser, and if any thing remain, by that remainer divide the last* Divisor *: and if any thing remain now, by that divide the* first divisor *(which was before the remainer of the last* division *) and so continue still, till nothing do remain in the* division *: and then mark your last* divisor, *for it is the* number *that will easily reduce your* fractions, *if you divide both the* numerator *and the* denominator *by the same* number, *and put for the* numerator *the* quotient *of his* division, *and for the* denominator *also his* quotient, *that riseth by his* division.

Scholar. I take for example $\frac{18}{96}$, and because 96 is the greatest number, I divide it by 18, and the quotient is 5, and there resteth 6, what shall I doe with this quotient?

Master. Nothing in this work, but now seeing there remaineth somewhat, by that remainer must you divide the last divisor.

Scholar. If I shall divide 18, (which was the last divisor) by 6, that was the remainer, so is the Quotient 3, and nothing resteth.

Master. As for the Quotient, I omit him yet: but

but because there doth remain nothing, therefore is 6 (which was your last divisor) that number by which you may reduce the fraction propounded.

Scholar. Then as you taught me, I must divide the Numerator 18 by 6, and the quotient is 3, which I must put for the numerator over a line thus: And then by the said 6 must I divide also the denominator 96, and the Quotient will be 16, which I must take for the denominator, and so is the Fraction $\frac{3}{16}$. And so me thinketh this rule doth prove the work of the first Reduction.

$$\frac{3}{16}$$

Master. That is true, if the first Reduction were made of fractions into their least termes, and else not, without some help, as the second number in that place will declare.

Scholar. The second number was $\frac{4}{6}$, which was turned into $\frac{64}{96}$ by that Rule. Now if I shall by this Rule reduce it againe into the least termes, I must divide 96 by 64, and there remaineth 32, wherefore I must take that 32 for the divisor, to reduce the said Fractions. Then doe you divide 64 by 32, and the Quotient is 2, which I set for my numerator. Againe, I divide 96 by 32, and the Quotient will be 3, and so I have but $\frac{2}{3}$.

Master. Muse not at the matter, for you have done well enough: but you think you have not the fraction that you looked for, that is, $\frac{4}{6}$, yet have you one equall to it, as by the parts of a shilling you may prove.

Scholar. Truth it is, for each of them will bring forth 8 pence, so that $\frac{8}{12}$ and $\frac{4}{6}$ and $\frac{2}{3}$ be all three equall. And now I perceive that because $\frac{4}{6}$

was not written in the least termes that it might be, therefore this Reduction brought forth not it, but that other which is written in the least termes. Now understand I this Rule well. But is there any other way to work this Reduction?

Another way to work this Reduction.

Master. Yes: but first note this, that if you find no such Divisor, to reduce the fraction till you come to 1, because one doth make no division, therefore that fraction is already in his least termes, as by $\frac{71}{100}$ you may prove, and so $\frac{85}{98}$, and many other like.

Note that to mediate any number is to divide by two.

But now for your better aid to find the due proportion in less termes, with more ease for a young learner, you shall mediate or take the halfe of the Numerator, and also of the Denominator as long as you may upon a line, alwaies parting them with a right down dash of your pen as you work, which may easily be done, if the numbers be even, as 2. 4. 6. 8. or 10, but if they be odde (though it be but one of them) then must you abbreviate them by 3. 5. 7. or 9. &c.

And because examples doe most instruct, I have here set down the manner of two or three, whose last number at the end of the line sheweth the least terme of valuation of that Fraction.

As for example: I would reduce $\frac{288}{576}$ into his least terme or value, whereupon I set forth $\frac{288}{576}$ with a long line drawn from it, thus,

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

And because both the Numerator and the Denominator end in even numbers, I see this may be abbreviatec by 2, or 4 or 6, &c. Therefore on the

the other side of the right down dash toward the right hand, I first take the halfe of the Numerator, saying, the halfe of 2 is 1, the halfe of 8 is 4 : and again, the halfe of 8 is 4 : which 144 is now a new Numerator, and therefore I part it with a right down dash as before.

Then doe I also take the halfe of 576, in saying, the halfe of 5 is 2, and the halfe of 17 is 8, and the halfe of 16 is 8, and so have I 288 for a new Denominator.

Then beginning again; saying the halfe of 144 is 72, and the halfe of 288 is 144: thus continuing the mediation of division by 2, untill you come to the last worke, as appeareth here in the example, where the same is reduced to $\frac{1}{2}$ which is equall to $\frac{288}{576}$.

So the second example $\frac{28}{112}$ first abbreviated by 2, and again by 2, and last by 7, is reduced to $\frac{1}{4}$ which is equall to $\frac{28}{112}$.

28	14	7	1
112	56	28	4

Again, $\frac{1465}{4395}$ abbreviated first by 5, then by 293.

1465	293	1
4395	879	3

Scholar. Sir, I thank you much, this is very easie and good for a young learner.

Master. So it is, but yet notwithstanding, if you can without that division by memory, espy the greatest number that may divide exactly both termes of your Fractions proposed, then need you not to use that division, as in this Fraction $\frac{60}{96}$,

I see that 12 is the greatest number that can divide them both: and therefore without any work, by memorie onely, I turn that into $\frac{5}{8}$; but this ability in knowledge is got by exercise.

Yet one other way of easie Reduction in this kind there is; when your fraction hath any cyphers in the first places of both termes, then may you by casting away the Cyphers, make a briefe Reduction as thus $\frac{300}{400}$ Here take away the Cyphers, and it will be $\frac{3}{4}$, which is the same in value with $\frac{300}{400}$.

Scholar. And so if I have $\frac{400}{650}$, it will be $\frac{4}{65}$.

Master. you are deceived, for you take away more cyphers from the Numerator then you do from the Denominator, which you may not doe.

Scholar. I confess my fault which came of too much hast, I was gladder of the Rule then wise in using it: but now I understand it I trust.

Master. Then may I goe in hand with the fifth or last kind of Reduction, which teacheth how to turn any fraction proposed into any other Denomination that you list, or into any part of common coynes, weights, or measures, or such like.

The fifth kind of Reduction

For declaration whereof, first you shall marke whether your fraction *be a simple* fraction, *either else a* fraction *of sundry parts, I mean of more termes then two. And if your* fraction *be a* fraction *of* fractions, *or otherwise compound, you must reduce it to one simple* fraction: *And then mark well the* denomination *of that other* fraction, *into which you would turn this: for by that* denominator *you must multiply the* numerator *of your first* fraction, *and the totall* Product *thereof shall you divide by the* denominator *of your first* fraction, *& that* quotient *shall be the* numerator *of the* deno-

To reduce fractions to a denomination appointed.

denominator *proposed :* as for example, *I have this* fraction $\frac{3}{5}$, *which I would turn into ten parts: therefore I multiply this* 10 *by* 3, *that is the* numerator *of my* fraction, *& there ariseth* 30, *which I divide by* 5, *and the* quotient *is* 6, *which must be the* numerator *to* 10, *and so* $\frac{3}{5}$ *will be* $\frac{6}{10}$.

Scohlar. This is easie enough to do.

Master. Then shall you see another example of the same fraction that is not so easie: as if I would turne $\frac{3}{5}$ into 8 parts, prove you that work.

Scholar. I must multiply 8 by 3, and there amounteth 24, which I divide by 5, and the quotient is 4, then is the new fraction $\frac{4}{8}$.

Master. And see you nothing doubtfull in this work?

Scholar. I see that when 24 was divided by 5, there remained 4, which I did not pass of, because ye spake nothing of any remainer, but onely of the quotient.

Master. By likelihood you remember what I said to you in Division of whole numbers, that you should not pass of the remainer there, but onely note it as a summe that could not be divided without knowledge of Fractions. Wherefore now mark this, that in all divisions of whole numbers, when there is any remainer, you shall set it over a line as a Numerator, & set the divisor for the denominator, and that Fraction doth make the Division compleat, & is part of the quotient: As if I would divide 48 by 5, the quotient will be $9\frac{3}{5}$: so in your former work when 24 was divided by 5, the quotient should be $4\frac{4}{5}$, & so the new Fraction should be thus $\frac{4}{8}$ & $\frac{4}{5}$ of $\frac{1}{8}$, that is, $\frac{4}{8}$ of the entire number, & $\frac{4}{5}$ of $\frac{1}{8}$

part of any thing, which you may prove by example of some Coyne.

Scholar. Then I take a Crowne, whose $\frac{3}{5}$ is 3 s. Now I would prove whether the 3 s. be $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$ I shall have a cumbrous work to doe.

Master. Indeed for whole pence, your example is a little troublesome : yet turning the crowne into halfe pence, it is easie enough.

Scholar. that will I try.

¶ First I see that $\frac{3}{5}$ of a Crowne is 3 shillings which is 36 pence, or 72 halfe pence. Now if I can find that this Fraction $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$ be equall unto 3 shillings, then am I fully answered.

Because I cannot take $\frac{4}{8}$ of a Crowne, I turne the Crowne into halfe pence, as you willed me, which makes 120, which I divide by 8, my Quotient is 15, which taken foure times, make 60 ob. Now resteth me to have $\frac{4}{5}$ of the $\frac{1}{8}$ part of a Crowne, whereof $\frac{1}{8}$ part is 15 ob. the 15 being parted into 5 parts, the Quotient is 3, which taken foure times maketh 12 ob. which with my 60 before amounteth to 72, which are then equall to $\frac{3}{5}$, my desire.

Master. I commend you for your diligence, you might have wrought it thus: either $\frac{4}{8}$ being abbreviated as before I taught, is $\frac{1}{2}$. Now halfe a Crowne is 2 shillings 6 pence. Now $\frac{4}{5}$ of $\frac{1}{8}$ is a Fraction of Fractions, which if you doe reduce into one entire Fraction, as before you have learned, in saying, 5 times 8 is 40, for a new Denominator, and once 4 is 4, for a new numerator: it maketh $\frac{4}{40}$, and abbreviated also make $\frac{1}{10}$. now the tenth part of a Crown is 6 pence, which put to 2 shillings 6 pence make also 3 shillings your desire.

But

But now one example more for this Rule, and then we shall end it. If I have $\frac{7}{15}$ of a Soveraign (accounting the Soveraign 20 shillings) how many shillings is that $\frac{7}{15}$?

Scholar. I must multiply 7 by 20 and that maketh 140, which I shall divide by 15, and the quotient will be $9\frac{5}{15}$, or in lesser termes $\frac{1}{3}$.

Master. That is 9 shillings, and one third part of a shilling, that is 4 pence, as by the same Rule you may prove. And this for this time shall suffice for Reduction. And now I will proceed to Addition.

Addition.

Addition of fractions of one denomination.

W*Hensoever you have any* Fractions *to be added, you must consider whether they be of one* denomination, *or not, and if they be of one* denomination, *then adde the* Numerators *together, and set that that amounteth for the* numerator *over the common* denominator, *and so have you done: The reason is, because that such differ little in* Addition *or* Subtraction *from the work of vulgar* denominations, *where the* denominators *be of the* number, *as* 3 pence *and* 5 pence *make* 8 pence, *where the* denomination *is not altered. But if the* fractions *be not of one* denomination, *or any of them be mixt of whole* numbers *and* fractions, *then must you first reduce them to one* denomination, *and after adde them. And if they be many, then adde first two of them, and so the* summe *that doth amount of the* Addition, *and the third, and then the 4 &c. if you have so many.*

Scholar.

Scholar. This seemeth easie enough, now that I have already learned to reduce, without which I could never have wrought this: And therefore now I see good reason why you did place Reduction before Addition.

Master. It is well considered, but yet refuse not to expresse your understanding of it by an example.

Scholar. Then would I adde first $\frac{7}{18}$ with $\frac{5}{18}$ and because the denominators are like (and so needeth no Reduction) I adde 7 to 5, which maketh 12, and then is my summe $\frac{12}{18}$, that is in smaller numbers, being abbreviated $\frac{2}{3}$.

To adde fractions of divers Denominations.

And if you have many numbers to be added, as here $\frac{3}{8}$ $\frac{4}{5}$ $\frac{9}{10}$, first I must reduce them (because they have divers denominators) into one denomination, and then they will be thus, $\frac{150}{400}$ $\frac{320}{400}$ $\frac{360}{400}$, or in lesser termes. $\frac{15}{40}$ $\frac{32}{40}$ $\frac{36}{40}$, which by Addition do make $\frac{83}{40}$, that is $2\frac{3}{40}$.

Master. Now may we go to Subtraction.

Subtraction of Fractions.

Subtraction of Fractions.

Subtraction *hath the same precepts that* Addition *had, for if the* denominators *bee like, then must you subtract the one* numerator *from the other, and the rest is to be set over the common* denominator, *and so your* subtraction *is euded: but and if you have many* fractions *to be subtracted out of many, then must you reduce them to one* denomination, *and into two severall* fractions, *that is, all*

all that must be subtracted into one fraction. *and the residue into another* fraction, *and then work as I said before.*

Scholar For the first example I take $\frac{15}{12}$ to be subtracted out of $\frac{17}{12}$, and the rest will be $\frac{2}{12}$ or $\frac{1}{6}$.

For another example I take $\frac{3}{4}$ to be subtracted out of $\frac{7}{8}$, which I must reduce, and it will be thus $\frac{14}{32}$ and $\frac{28}{32}$.

Then do I subtract 24 out of 28, and there resteth 4, which I set over the common denominator for a Remainer, thus, $\frac{4}{32}$: that is $\frac{1}{8}$.

Now for the third example, I take $\frac{3}{4}$ and $\frac{5}{6}$ to bee subtracted from $\frac{7}{8}$ and $\frac{9}{10}$: and because their denominators be divers, I do reduce them into one denomination thus $\frac{38}{24}$ and $\frac{142}{80}$ $\frac{3408}{1920}$ and $\frac{3040}{1920}$.

Then do I adde the two first, and they make $\frac{3040}{1920}$. Also I adde the two last, and they yeild $\frac{3408}{1920}$. Then do I subtract 3040 out of 3408, and there resteth 368, so is the remainer, $\frac{368}{1920}$, that is in smaller termes $\frac{23}{120}$: And thus have I done with Subtraction, except you have any more to teach me.

Master. Prove one example or more out of Fractions of divers Denominations.

Scholar. I take two Fractions $\frac{7}{8}$ to bee subtracted from $\frac{9}{24}$ which being reduced, will stand thus $\frac{168}{192}$ and $\frac{72}{192}$: Now would I subtract 168 out of 72, but I cannot.

$$\begin{array}{ccc} 168 & & 72 \\ \frac{7}{8} & & \frac{9}{24} \\ & 192 & \end{array}$$

Master. Then may you perceive that you mistook the Fractions: for you can never subtract the greater out of the lesser, although you may adde, multiply or divide the greater with the lesser.

And

The greatest of two fractions.

And albeit that $\frac{7}{8}$ hath both his termes lesser then $\frac{9}{24}$ yet is $\frac{9}{24}$ the lesser Fraction: for generally if you multiply the Numerator and the Denominators of two Fractions crossewaies, that fraction is the greatest of whose Numerator commeth the greatest summe, as in this example, 7 multiplied by 24 maketh 168, and 9 being multiplied by 8 yeildeth but 72, therefore is the first fraction 7 the greatest of these two, so can you not subtract it out of a lesser Fraction.

But if you should subtract a Fraction out of a whole number, what should you doe?

Scholar. Marry I would reduce the whole number into a Fraction of the same Denomination that my Fraction is, and then work by Subtraction.

Master So may you doe, but it is much easier, if your Fraction be a proper Fraction, that is to say, less then an unite, to take an unite from the whole number, and then turn it into an improper Fraction, and so work your Subtraction. As if I would subtract $3\frac{2}{5}$ from 4, I may take 1 from 4, and turne it into $\frac{5}{5}$, from which I abate $3\frac{2}{5}$, there will remain, $\frac{3}{5}$. And if the first be an improper Fraction, then may I take so many units from the whole number, that they may make an improper Fraction, greater then that first, and then work by Subtraction. As if there be proposed $\frac{10}{3}$ to bee subtracted from 6, because $\frac{10}{3}$ is more then 3, & not so much as 4, I must take 4 from 6, and turne them into thirds thus, $\frac{12}{3}$ then abate $\frac{10}{3}$ from $\frac{12}{3}$, there resteth $\frac{2}{3}$; so the whole remainer is $2\frac{2}{3}$. Or else you may at your pleasure take $3\frac{1}{3}$, which is $\frac{10}{3}$: from 6 whole: then set 1 under 6, as thus $\frac{6}{1}$: And then

then to reduce those two Fractions into one Denomination, as here appeareth $\frac{10}{3}$ from $\frac{6}{1}$: Then $\frac{10}{3}$ from $\frac{18}{3}$ resteth $\frac{8}{3}$, which maketh $2\frac{2}{3}$ your desire. And thus will I make an end of the worke of subtraction of Fractions, and proceed to Multiplication.

$$\begin{array}{ccc} & 8 & \\ \frac{10}{3} & \times & \frac{18}{3} \\ & 3 & \end{array}$$

Multiplication of Fractions.

Herefore when any two fractions *be proposed to be multiplied together, the* numerator *of the one must be multiplied by the* numerator *of the other : and the summe that amounteth thereof must be set for a new* numerator : *likewise the* Denominator *of the one must be multiplied by the* Denominator *of the other , and that that amounteth shall be set for the* Denominator, *and this new third* fraction *expresseth teh* Product *of the* Multiplication *of the two first* Fractions *proposed, whereof take this* Example, $\frac{3}{5}$ *multiplied by* $\frac{5}{12}$ *doth make* $\frac{15}{60}$. Multiplication of Fractions.

$$\begin{array}{ccc} & 15 & \\ \frac{3}{5} & & \frac{5}{12} \\ & 60 & \end{array}$$

Scholar. I perceive then that 3 being the Numerator of the first Fraction is multiplied by 5 being the Numerator of the second Fraction, whereof amounteth 15, the Numerator of the third Fraction, And so likewise 5 being the Denominator of the first Fraction, is multiplied by 12 the Denominator of the second Fraction, whereof amounteth 60 the new denominator, so

that I perceive how the work is done, I doe not perceive how $\frac{15}{60}$ is greater then $\frac{3}{5}$, for if I shall use my former manner of examination by the parts of some coine, I see that $\frac{3}{5}$ of a Crowne is 36 pence, and $\frac{5}{12}$ of a Crowne is 25 pence, whereof the one multiplied by the other, doth make 900 pence, which is 15 Crownes, but by your multiplication there amonnteth $\frac{15}{60}$, which is but 15 pence, and that is much less then any other of both the first Fractions.

Master. That difference is betweene multiplication in whole numbers, and Multiplication in broken numbers, that in whole numbers, the summe that amounteth is greater then both the other whereof it came: but in fractions it is contrariwise: for the summe that amounteth is lesser then any of the other two fractions whereof it is produced.

S. I desire much to understand the reason thereof.

Master. Although I purposed to reserve the reasons of works Arithmeticall for the perfect Booke of Arithmetick, yet I will shew you this, because of the strangenesse of the work.

You see in whole numbers, that of two numbers, being multiplied together, is made the third number, which third number doth beare the same proportion to the number multiplied, that the multiplier doth beare to an unite. And so in Fractions the third number which amounteth of Multiplication, beareth the same proportion to each of the two first Fractions, that the other of those two fractions doth bear to an unite.

Scholar. Sir, I understand your words thus: when 40 is multiplyed by 12, there doth amount

480

480, which 480 doth containe 40 so many times in it, as 12 doth containe Unites, that is to say, twelve times. And so it appeareth that 480 doth contain twelve so many times also as 40 doth contain unites, that is 40 times. But now I see not how the third number in this example of Fractions can containe any of the two former (as it happened in whole numbers) seeing it is lesser then either of them.

Master. No marvell if you connot see that thing which is not possible to be seen of any man, how the third number in multiplication of Fractions should be greater then any of the two former Fractions: but yet this may you see (which I said) that the third number in fractions so multiplyed, doth beare the same proportion to any of the two former Fractions that the other of those two Fractions doth beare to an unite, as in your example, $\frac{3}{5}$ being multiplyed by $\frac{5}{12}$ doth make $\frac{15}{60}$ Now I say that $\frac{15}{60}$ doth beare the same proportion to $\frac{3}{5}$ that $\frac{5}{12}$ doth bear to a unite, as you may in your own forme of examination by Coine, try it: for in an old angell (which in times past was currant for 7 shillings 6 pence) are 180 halfe pence which I set for the intire unite, whose parts (according to the Fractions aforesaid) are these, for $\frac{15}{60}$ set 45 halfe pence, for $\frac{3}{5}$ take 108 halfe pence, and for $\frac{5}{12}$ put 75 halfe pence. Now doth 45 bear the same proportion to 108, that 75 doth bear to 180, for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180.

But these reasons may be better reserved till another time, when the knowledge of proportions in due order shall be taught: yet in the mean season

I

I will shew you how it commeth to pass, that in Fractions the third summe must needs be lesser then any of the other two.

Consider this, that when a Fraction is proposed, as in the former example $\frac{3}{5}$ if it be multiplied by more then 1, it will make more then one entire number. As if I multiply $\frac{3}{5}$ by 5, that is to say, if I take it 5 times, it will make three entire unites. Example: in a Crown $\frac{3}{5}$ of it maketh 3 shillings, which if I take five times, it will amount to 15 shillings that is, three entire Crowns; so if I take the same $\frac{3}{5}$ but twice, it will yeeld 6 shillings, that is, one entire Crown, and $\frac{1}{5}$, now if I take it but once, it cannot be more then it was before, that is 5 shillings, And if I take it less then once, it cannot bee so much as it was before. Then seeing that a Fraction is less then one, if I multiply a Fraction by another Fraction, it followeth that I doe take the first Fraction less then once, and therefore the summe that amounteth must needs be less then the first Fraction.

Scholar. Sir, I thank you much for this reason. And I trust I doe perceive the thing, as by example of this same Fraction $\frac{3}{5}$ I will express. If I take $\frac{3}{7}$ of a Crown once, that is to say, if I multiply $\frac{3}{5}$ by 1, it will be as it was before, but 3 shillings: so if I doe multiply it by $\frac{1}{2}$, that is, if I take but halfe one time, then will it be but halfe so much: likewise if I multiply it by $\frac{1}{3}$ that is, if I take but the third part of one, it will yeild but 12 pence, that is, the third part of the first Fraction.

And

And so to make an end: if I take but the twelfth part of one, that is, if I doe multiply it by $\frac{1}{12}$ it will yeild but the twelfth part of the first Fraction, which is but 3 pence. And it followeth, that if $\frac{1}{12}$ make 3 pence, then $\frac{5}{12}$ must needs make five times so much, that is, 15 pence, which was the summe that hath given the occasion of all this doubt.

Master. Then I perceive you have sufficient understanding in this sort of Multiplication for this time, wherefore I will proceed to the rest.

In Multiplication *it happeneth sometime, that there be whole numbers to be multiplyed with Fractions, and may be in two sorts: for either the whole number is severall from the fraction, and is the* multiplier, *or else the whole number is joyned with one, or both of the fractions, and so maketh a mixt number thereof. If it be in the first sort, then needeth there no* Reduction, *but onely multiply the* numerator *of the fraction by that whole number, and the totall thereof set for the new* numerator.

To multiply a whole number into a fraction.

Scholar. I understand you thus. If I have $\frac{6}{23}$ to be multiplyed by 16 then must I multiply that 16 with 6, which is the Numerator, whereof commeth 96, and that must I set for the new Numerator: keeping still 23 for the Denominator, and so the Fraction will be $\frac{96}{23}$ that is $4\frac{4}{23}$.

Master. And in this sort of work you may abridge the labour thus. If it happen the denominator to be such a number as may evenly be divide by the said whole number proposed, then divide it thereby, and set the Quotient of that division for the former denominator, but reserve still the numerator, and so is the multiplication ended.

Scholar. Then ſain this example $\frac{7}{20}$ to be multiplied by 5, and becauſe 5 will juſtly divide 20, therefore I take the Quotient of that diviſion, which is 4, and ſet it in ſtead of 20, and ſo the Fraction will be $\frac{7}{4}$ that is $1\frac{3}{4}$.

Maſter. Which is all one with $\frac{35}{20}$ that would have followed of the other ſort of work.

Scholar. I perceive it well.

How to multiply mixt numbers.

Maſter. *Now then for the other ſort, where the number is mixt, take this way: firſt to reduce the ſaid whole number and Fraction into one improper Fraction (as I ſhewed you in* Reduction*) and then multiply them together, as if they were proper Fractions.*

Scholar. $13\frac{3}{5}$ being ſet to be multiplyed by $\frac{5}{8}$ firſt I muſt reduce the mixt number, as in this example appeareth, by multiplying 13 by 5, and that maketh 65, whereto I muſt adde the numerator 3, and ſo the Fraction will be $\frac{68}{5}$ which two Fractions now I ſhall multiply after the accuſtomed forme, and it will be $\frac{340}{40}$, or $\frac{34}{4}$.

$13\frac{3}{5}$	340
68 by	5
5 40	8

Maſter. You have done well: and ſo may you ſee, that although moſt part of the formes of multiplication may be wrought without Reduction, yet ſome cannot, as namely mixed numbers.

Duplation.

And yet one note more I will tell you of Multiplication *before we leave it: That is, whenſoever you would multiply any Fraction by 2, which commonly is called* duplation, *you may doe it not only by doubling the* numerator, *but alſo by parting the* denominator *into half, if it be eaven.*

Scholar. Then if I would double $\frac{5}{12}$ I may chuſe

chuſe whether I will make it $\frac{10}{12}$ or elſe $\frac{5}{6}$. And indeed I ſee that is all one, but the dividing of the Denominator ſeemeth the better way to make ſmaller termes of the Fraction, and ſo they ſhall need the leſs Reduction.

Maſter. It is ſo: and now I ſhall not need to tell you that Multiplication is proved by Diviſion, and Diviſion likewiſe by multiplication: but the like work that I ſhewed you in multiplication, will I ſhew you in diviſion.

Diviſion of Fractions.

Whenſoever two Fractions *bee propoſed, that one ſhould be divided by the other, I muſt ſet down firſt the* Fraction *that ſhall be divided (which is called the* dividend) *and then after it the other which is the* diviſor: *Then ſhall I multiply the* numerator *of the* dividend *by the* denominator *of the* diviſor, *and that which amounteth I muſt put for a new* numerator. *Again I ſhall multiply the* denominator *of the* dividend *by the* numerator *of the* diviſor, *and the* number *that amounteth thereof I muſt put for the new* denominator. *And this third* fraction *is the* Quotient *of the ſaid* diviſion. Diviſion of Fractions.

Scholar. This ſeemeth eaſie in forme, as by example thus: If I would divide $\frac{5}{8}$ by $\frac{4}{6}$, firſt I multiply 5, (being the numerator of the dividend) by 6, which is the denominator of the diviſor,

and thereof riſeth 30: then I multiply 8 (being the denominator of the dividend) by 2 being the numerator in the diviſor: and ſo riſeth 16, the which I muſt make a third Fraction thus $\frac{30}{16}$.

	30	
5		2
8		9
	16	

Maſter. Me ſeemeth you are quicker in underſtanding now, then you were when I taught you the Art of whole numbers, but that is no marvell: for the more knowledge that a man getteth the readier ſhall he find his wit, and be quicker in underſtanding: but yet of two things I will admoniſh you, which you, might have obſerved here for the eaſe of worke and lightneſs of underſtanding the nature of the Quotient.

Whenſoever you divide one Fraction by another, either they be both equall together, or elſe the one is greater then the other: if they be equall, their Quotient ſhall be ſuch that the Numerator and the Denominator of it ſhall be equall alſo. And if the two firſt Fractions be unequall their Quotient ſhall declared the ſame by the inequality of the Numerator and Denominator, as in theſe examples following ſhall appear.

Firſt, if equall Fractions $\frac{4}{9}$ and $\frac{12}{27}$ be equall together, and if the one be divided by the other, the Quotient will be $\frac{108}{108}$, as you may perceive by that Rule aforeſaid.

Now in the unequall Fractions, as $\frac{4}{9}$ and $\frac{3}{10}$ the Quotient will be $\frac{40}{27}$, where the Numerator is greater then the Denominator.

Scholar. I ſee it is ſo: but I ſee not the reaſon why it ſhould be ſo.

Maſter

Master. The reason is this; when any Fraction is divided by another, the Quotient declareth what proportion the dividend beareth to the Divisor. So $\frac{1}{2}$ divided by $\frac{1}{4}$, maketh 2, which must be sounded, not 2, but twice, declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$. Note how to know the proportion between two numbers.

And note this, that the Numerator in the Quotien representeth the Dividend, and the Denominator representeth the Divisor. And this is alwayes true, whether the greater Fraction be divided by the lesser, or the lesser by the greater. But this proportion will not be exactly known, till you have learned the Art of proportions: notwithstanding somewhat of it I have declared in the Rule of Reduction. But now for the easie remembreance of the Quotient in Division, as soon as you have set down your two Fractions the one against the other, then make a straight line for the Quotient: and as soon as you have multiplied the Numerator of the Dividend, by the Denominator of the Divisor, set the Number that amounteth over the said line, and then multiply the other two Numbers, and set their totall under the same line.

Scholar. I perceive you would not have me trust to memory till I were better expert, lest oftentimes I happen by misse-remembrance to be abused. This Example I take for that declaration.

If I would divide $\frac{2}{3}$ by $\frac{3}{4}$ I must set the numbers one against the other (as here doth appear) and then make another line for the Quotient in some good

$$\frac{2}{3} \text{ by } \frac{3}{4}$$

 distance,

distance, where I may set the numbers of the Quotient, as soon as any of them is multiplied. So then as soon as I have multiplied 2 by 4 which maketh 8, I shall set that 8 over that line, thus: And then multiply 3 by 3, which yeildeth 9: and that 9 must be set under the same line, and then will the whole Quotient appear thus $\frac{8}{9}$: whereby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$ as 8 is to 9, but how may I perceive that?

$$\frac{8}{9}$$

Master. Although you might better perceive it by the Rule of Reduction, yet this example may be declared in common coines, as in a common shilling of 12 pence, of which $\frac{2}{3}$ maketh 8 pence, and $\frac{3}{4}$ doth make 9 pence, and so you may easily see that their proportions doe agree. And if you had taken this example before when you took the example of $\frac{5}{8}$ and $\frac{2}{6}$, your Quotient would appear (as this doth) more easie to understand; whereas that Quotient being $\frac{30}{16}$, is not an easie proportion for you to perceive, being yet little acquainted with proportions.

Scholar. If there be whole Numbers to bee divided by a Fraction, how shall I performe it?

To divide a whole number by a fraction.

Master. *When any whole number shall be divided by a fraction, you must multiply the said whole number with the Denominator of the Fraction, and set the totall thereof for the new Numerator, and for the Denominator set the Numerator of the fraction.*

Scholar. Then 20 divided by $\frac{3}{4}$ will make $\frac{80}{3}$, as here appeareth $\frac{80}{3}$.

$$\frac{20}{1} \text{ by } \frac{3}{4} \qquad \frac{80}{3}$$

To divide the fraction by the whole number.

Master. *Even so: but if you would divide the Fraction by the whole number, then multiply the Denominator by the same whole number, and set the totall*

for

for the Denominator, without changing the Numerator.

Scholar. Then to divide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$, as here appeareth $\frac{20}{23}$ by $\frac{4}{1}$ in this Example $\frac{20}{92}$.

$$\frac{20}{23}\ \text{by}\ \frac{4}{1} = \frac{20}{92}$$

Master. You say well. And by the same Example you give me cause to remember another briefe way to doe the same: for if you had divided the said Numerator by 4, and set the Quotient for the Numerator, keeping still the old Denominator, it would have been not only as well done, but also in a Fraction of lesser termes. Another brief way.

Scholar. I guess it to be even so, by a like work that you taught me in multiplication: And for proof thereof $\frac{20}{23}$ being the Dividend, and 4 the Divisor, I divide the Numerator 20 by 4, and the Quotient is 5, which I set for 20 over 23, thus $\frac{5}{23}$: And I see that it is all one with $\frac{20}{92}$, as by dividing or abbreviating both these termes by 4, and so reducing them to their least Denomination, I may easily prove: as appeareth by this example $\frac{20}{92}$ $\frac{5}{23}$.

Master. You conceive it well. And if there be mixt numbers, (either one or both) you must first reduce that mixt number into an improper Fraction, and then work as you have learned.

Scholar. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

Master. Then take this note yet for Division: if the denominators be like, then divide the numerators, as it were in whole numbers, and the Quotient, whether it be Fraction, whole number or mixt, is a

distance, where I may set the numbers of the Quotient, as soon as any of them is multiplied. So then as soon as I have multiplied 2 by 4 which maketh 8, I shall set that 8 over that line, thus: And then multiply 3 by 3, which yeildeth 9: and that 9 must be set under the same line, and then will the whole Quotient appear thus $\frac{8}{9}$: whereby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$ as 8 is to 9, but how may I perceive that?

$$\frac{8}{9}$$

Master. Although you might better perceive it by the Rule of Reduction, yet this example may be declared in common coines, as in a common shilling of 12 pence, of which $\frac{2}{3}$ maketh 8 pence, and $\frac{3}{4}$ doth make 9 pence, and so you may easily see that their proportions doe agree. And if you had taken this example before when you took the example of $\frac{5}{8}$ and $\frac{2}{6}$, your Quotient would appear (as this doth) more easie to understand; whereas that Quotient being $\frac{30}{16}$, is not an easie proportion for you to perceive, being yet little acquainted with pooportions.

Scholar. If there be whole Numbers to bee divided by a Fraction, how shall I performe it?

To divide a whole number by a fraction. Master. *When any whole number shall be divided by a fraction, you must multiply the said whole number with the Denominator of the Fraction, and set the totall thereof for the new Numerator, and for the Denominator set the Numerator of the fraction.*

Scholar. Then 20 divided by $\frac{3}{4}$ will make $\frac{80}{3}$, as here appeareth $\frac{80}{3}$.

$$\frac{20}{1} \text{ by } \frac{3}{4} \quad \frac{80}{3}$$

To divide the fraction by the whole number. Master. *Even so: but if you would divide the Fraction by the whole number, then multiply the Denominator by the same whole number, and set the totall*

for

for the Denominator, without changing the Numerator.

Scholar. Then to divide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$, as here appeareth $\frac{20}{23}$ by $\frac{4}{1}$ in this Example $\frac{20}{92}$.

$$\frac{20 \text{ by } 4}{23 \quad 1} \quad 92$$

Master. You say well. And by the same Example you give me cause to remember another briefe way to doe the same: for if you had divided the said Numerator by 4, and set the Quotient for the Numerator, keeping still the old Denominator, it would have been not only as well done, but also in a Fraction of lesser termes. Another brief way.

Scholar. I guess it to be even so, by a like work that you taught me in multiplication: And for proof thereof $\frac{20}{23}$ being the Dividend, and 4 the Divisor, I divide the Numerator 20 by 4, and the Quotient is 5, which I set for 20 over 23, thus $\frac{5}{23}$: And I see that it is all one with $\frac{20}{92}$, as by dividing or abbreviating both these termes by 4, and so reducing them to their least Denomination, I may easily prove: as appeareth by this example $\frac{20}{92}$ $\frac{5}{23}$.

Master. You conceive it well. And if there be mixt numbers, (either one or both) you must first reduce that mixt number into an improper Fraction, and then work as you have learned.

Scholar. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

Master. Then take this note yet for Division: if the denominators be like, then divide the numerators, as it were in whole numbers, and the Quotient, whether it be Fraction, whole number or mixt, is a

good Quotient for that Diviſion. And generally, if one of the numerators may juſtly divide the other by that Quotient, multiply the Denominator of the leſſer numerator, and ſet it that doth amount in the roome of the ſame denominator, and then for a numerator to it, ſet the denominator of the other Fraction.

Scholar. Then if I would divide $\frac{3}{4}$ by $\frac{12}{17}$ I ſee that 3 will divide 12, and the Quotient will be 4, by which I muſt multiply the other 4, that is the denominator under 3, and then it is 16, which is ſet for the denominator 4, and over it in ſtead of 3 I muſt ſet 17 the other denominator, and ſo it is thus $\frac{17}{16}$.

Maſter. And ſo is $\frac{7}{16}$ in ſtead of $\frac{51}{48}$, which would have riſen by the common work, as here appeareth.

	51	
3		12
4	by	17
	48	

And now for Mediation (which is to divide by 2) marke this, if the Numerator *be an even number, ſet the half of it in his place without the* diviſor, *and ſo have you done: and if the* numerator *be not even, then double the* denominator.

Scholar. That is, if I would mediate $\frac{16}{11}$ I may make the Quotient $\frac{8}{11}$, and if I would mediate $\frac{7}{11}$, I muſt make it $\frac{7}{22}$.

Maſter. And thus will I make an end of the works of common fractions for this time, not doubting but you can apply them both to the Rule of progreſſion, and alſo to the Golden Rule, without any other teaching then you have learned before, which might ſeem tedious to repeat, in regard you have ſufficient knowledge in Reduction, Addition,

Addition, Subtraction, Multiplication, and Division: And therefore will I goe in hand with the Rule of Proportion, or Golden Rule, which now will appear easie enough.

The Golden Rule direct in Fractions.

Master.

THerefore as touching the Golden Rule for the placing of the three numbers proposed in the question whereby to find the fourth and for the form of their work, with other like notes I referre you to that which you have already learned.

The rule of proportion in Fractions.

But this easie form of working by fractions shall you note, that if your three numbers be fractions, for an apt work and certain, multiply the numerator of the first number in the question, by the denominator of the second: And all that again multiply by the denominator of the third number, and the totall thereof shall you keep for to bee the divisor. Then multiply the Denominator of the first number by the numerator of the second, and the whole thereof by the numerator of the third, and the totall thereof shall be your dividend.

Note this for a generall Rule.

Now divide this dividend by the divisor which you found out before, and that number shall be the fourth number of the question which you seek for, as in this example.

If $\frac{3}{4}$ of a yard of Velvet cost $\frac{2}{3}$ of a soveragin, esteemed at 20 shillings, what shall $\frac{5}{6}$ cost?

A question of Velvet.

Scholar. If it please you to let me make the answer, I would first place these three numbers as I learned in whole numbers, thus: $\frac{3}{4}\ \mathrm{Z}\ \frac{2}{3}$ $\frac{5}{6}$

And

And then according to your new rule, I must multiply 3, being numerator in the first number, by 3, the denominator of the second: and thereto commeth 9, which I multiply again by 6, the denominator of the third number, and so have I 54, which I keep for the divisor. Then multiply I 4 the denominator of the first, by 2 the numerator of the second, and there ariseth 8, which again I multiply by 5 the numerator of the third, and it maketh 40. Then must I divide 40 by 54, and it will be $\frac{40}{54}$ that is $\frac{20}{27}$ in lesser termes, and then the figure will stand thus:

$\frac{3}{4}$ Z $\frac{2}{3}$
$\frac{5}{6}$ $\frac{20}{27}$

But what that is in money I cannot tell, except I shall work it by Reduction, as you taught me.

Master. It forceth not now, you may reduce it when you list, but it were disorderly done here to mingle divers works together, where we do not seek the value of the thing in common money, but in apt number, which yee have well done: and therefore will I yet shew you another like way of easiness in work, how you may change your three Fractions into three whole numbers, by which you shall work, as if the question wise proposed in whole numbes. The first number you shall find as I taught you: now to find the divisor of the second number, take the numerator for the second fraction: and for the third number, take that that ariseth of the multiplication of the denominator of the first, by the numerator of the third, and then work your question.

A question of Silver.

Scholar. *For example hereof, I put this question. If $\frac{11}{12}$ of 1 pound weight of silver, be worth $\frac{1}{4}$ of a Soveraigne, what is, $\frac{1}{2}$ of 1 pound weight worth?*

For

For the anſwer, firſt I place the Fractions in order thus:

$\frac{11}{12}$ Z $\frac{12}{4}$
$\frac{1}{2}$

Then to turne theſe Fractions into whole numbers, I multiply 11, which is the numerator of the firſt by 4 (the denominator of the ſecond) and there commeth 44, which I multiply by 2 the denominator of the third, and ſo amounteth 88, which I ſet for the Diviſor in the firſt place. Then in the ſecond place I ſet 12, which is the numerator of the ſecond fraction, and in the third place I ſet the ſumme that amounteth of 12, being the denominator of the firſt number, multiplied by one, being numerator in the third number, and ſo the figure will ſtand as here you ſee.

88 Z 12
12

Then to work it forth, I multiply 12 by 12, and there amounteth 144, which I divide by 88, and the quotient will be 1 $\frac{56}{88}$, or in leſſer termes, 1 $\frac{7}{11}$ and then the figures well ſtand thus:

$\frac{11}{12}$ Z $\frac{12}{4}$
$\frac{1}{3}$ 1 $\frac{7}{11}$

Maſter. Theſe two formes now you underſtand well enough, and as for any other at this time I will not repeat, only this ſhall you mark for the proof of this Rule, whether your work be well wrought or no. Multiply the firſt number by the fourth, and note what amounteth; then multiply the ſecond by the third, and mark what amounteth alſo. Now if thoſe two numbers ſo amounting be equall, then is your work well done, elſe you have erred. And this ſhall ſuffice for the former Rule.

The proof of the golden rule.

The

The Backer Rule, or Reverſe Rule in Fractions.

The back-er Rule in Fractions.

BUt in the Backer Rule, this ſhall you note for your eaſe of work, that you multiply the Numerator of the firſt by the Numerator of the ſecond, and the whole thereof by the Denominator of the third, and that amounteth thereof, ſhall be the dividend. Then multiply the Denominator of the firſt, by the denominator of the ſecond, and that whole by the Numerator of the third, and that that ariſeth thereof, ſhall be the Diviſor. Example of this.

Note this alſo for a generall Rule.

A queſtion of Loan.

I did lend my friend $\frac{3}{4}$ *of a* Porteguiſe, *ſeven Moneths upon promiſe that he ſhould do as much for me againe; and when I ſhould borrow of him he could lend me but* $\frac{5}{12}$ *of a* Porteguiſe: *now I demand how long time I muſt keep his money in juſt recompence of my loane, accounting* 13 *Months in the year?*

Scholar. **The firſt number muſt be the firſt money borrowed, that is** $\frac{3}{4}$ **of the** Porteguiſe: **the ſecond number the** 7 **moneths, that is** $\frac{7}{13}$ **of a** year: **and the third number the money that was lent in recompence, that is** $\frac{5}{12}$ **of a** Porteguiſe: **then I ſet the numbers thus:**

$$\frac{3}{4}\ \mathrm{Z}\ \frac{7}{13}$$
$$\frac{5}{12}$$

Then (as you taught me) **I multiply** 3 (being Numerator in the firſt number) **by** 7, **the** Numerator of the ſecond number, **and it maketh** 21, **which I multiply by** 12 **the** Denominator of the third, **and ſo have I** 252 **for the** dividend: **then I multiply** 4, **the** Denominator of the firſt, **by** 13 **the** Denomina-

tor

tor of the 2d, & it yeildeth 52, which I multiply again by 5, the Numerator of the third, & it will make 260, that is the divisor. Then must I divide 252 by 260 so it will be in the small Fraction $\frac{63}{65}$ of a year.

Master. And thus do you see some ease in working, better then to multiply and divide tediously so many Fractions.

Another question yet I will propose, to the intent you may see thereby the reason of the Statute of Assise of Bread and Ale, which in all statute Books, in Latine, French, and English, is much corrupted for want of knowledge in this Art; for the right understanding whereof I propose this question.

Statute of Assise of bread and ale.

When the price of a quarter of wheat is 2 shillings, *the farthing white loafe shall weigh* 68 shillings; *then I demand what shall such a loafe weigh, when a quarter of Wheat is sold for* 3 shillings.

Question of bread.

Scholar. This question must be wrought as it is proposed in whole numbers: and not in Fractions.

Master. You seem to say reasonably, howbeit in the Statute of Assise, the rate is made by the proportion of parts in a pound weight Troy, else could it not be a Statute of any long continuance, seeing the shillings doe change often as all other moneys doe: but this Statute being well understood, is a continuall Rule for ever, as I will anon declare by a new Table of Assise, converting the shillings into ounces, and parts of ounces.

Wherefore here by a shilling you must understand $\frac{1}{20}$ of a pound weight, and so by a penny $\frac{1}{20}$ of an ounce: wherefore although you might work this question proposed by whole numbers well enough, for that time when the Statute was made,

yet

yet to apply it to your time, and make it ſerve for all times generally, it is beſt to work it by fracti-ons, ſetting for 2 ſhillings $\frac{2}{20}$, and for 68 ſhillings $\frac{68}{20}$, and ſo for three ſhillings $\frac{3}{20}$, and then will the figure of the queſtion ſtand thus

$$\frac{2}{20}\ \mathbf{Z}\ \frac{68}{20}$$
$$\frac{3}{20}$$

In which queſtion, becauſe all the Denominators be like, you ſhall work only with the Numerators.

Scholar. Then ſhall I multiply 68 by 2, where-of commeth 136, which if I divide by 3, the quoti-ent will be $45\frac{1}{3}$: but how ſhall I make a fraction of that, to ſtand with the other?

Maſter. Have you ſo ſoon forgotten what was taught you ſo lately? this is his forme.

$$\frac{45\frac{1}{3}}{20}$$

Scholar. I remember it now, and then it ſigni-fieth 45 twenty parts, and the third deale of one twenty part.

Note what a ſhilling is.

Maſter. So is it that maketh in ſhillings 54 ſhil-lings 4 pence, whereby you may note one great er-rour in the Statute Books; which have conſtantly 48 ſhillings in that Aſſiſe. And by this Rule, if you examine the Statute, you ſhall find many ſummes falſe. Wherefore for the true underſtanding of that Statute, and ſuch like, as I have made mention of it, and ſomewhat recognized it, ſo doe I wiſh that all Gentlemen and other ſtudents of the Lawes would not neglect this Art of Arithmetick, as un-needfull to their ſtudies. Wherefore to encourage them thereto, and to gratifie both them and all other in generall I will exhibite a Table of that part of the Statutes in two Columnes and in a third Co-lumne, I will adde the correction of thoſe errours which have crept into it.

Here followeth the Table.

The price of a quarter of Wheat.		The weight of farthing white loaf, by the Statute Books.			The Correction by just Assiise.		
s.	d.	l.	s.	d.	l.	s.	d.
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	2	8	0	3	8	0
2	6	2	14	$4\frac{1}{5}$	2	14	$4\frac{4}{5}$
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	$10\frac{2}{7}$
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	$2\frac{2}{7}$
5	0	1	8	$2\frac{1}{3}$	1	7	$2\frac{2}{5}$
5	6	1	4	$8\frac{1}{4}$	1	4	$8\frac{8}{11}$
6	0	1	2	8	1	2	8
6	6	0	16	11	1	0	$11\frac{1}{13}$
7	0	0	19	1	0	19	$5\frac{1}{7}$
7	6	0	18	$1\frac{1}{2}$	0	18	$1\frac{3}{5}$
8	0	0	7	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	$0\frac{1}{4}$	0	15	$1\frac{1}{3}$
9	6	0	14	$0\frac{3}{4}$	0	14	$3\frac{15}{19}$
10	0	0	13	$7\frac{1}{2}$	0	13	$7\frac{2}{5}$
10	6	0	12	$11\frac{1}{2}$	0	12	$11\frac{3}{7}$
11	0	0	12	$4\frac{1}{4}$	0	12	$4\frac{4}{11}$
11	6	0	11	10	0	11	$9\frac{21}{23}$
12	0	0	11	4	0	11	4

In

In the common Books there is no further rate of Assise made, then unto 12 s. the quarter of wheat, but in an ancient copy of 200 years old (which I have) there is added the rate of Assise unto 20 s. the quarter, but yet was that Assise also either wrong cast at the first penning, or else corrupt since that time, for lack of just knowledge in the Rule of Proportion, *which I will adde here also to gratifie such as be desirous to understand truth exactly.*

The price of a quarter of Wheat.		The weight of a farthing white loaf, by the Statute Books.			The Correction by just Assiise.		
s.	d.	l.	s.	d.	l.	s.	d.
12	6	0	11	0	0	10	$10\frac{14}{25}$
13	0	0	11	$0\frac{2}{7}$	0	10	$5\frac{7}{13}$
13	6	0	10	$1\frac{1}{2}$	0	10	$0\frac{8}{9}$
14	0	0	9	7	0	9	$8\frac{4}{7}$
14	6	0	9	$2\frac{1}{2}$	0	9	$4\frac{16}{29}$
15	0	0	9	$1\frac{1}{2}$	0	9	$0\frac{4}{5}$
15	6	0	9	$1\frac{1}{4}$	0	8	$9\frac{9}{31}$
16	0	0	9	0	0	8	6
16	6	0	8	6	0	8	$2\frac{10}{11}$
17	0	0	8	3	0	8	0
17	6	0	7	10	0	7	$9\frac{9}{35}$
18	0	0	7	6	0	7	$6\frac{2}{3}$
18	6	0	7	3	0	7	$4\frac{8}{37}$
19	0	0	7	2	0	7	$1\frac{17}{19}$
19	6	0	5	10	0	6	$11\frac{9}{13}$
20	0	0	5	6	0	6	$9\frac{3}{5}$

These

These two Tables I have set severall, because no man should think that I would either add or take away from any Law, those parts which might of right seem either superfluous, either diminute: but yet I may not be so curious as to neglect manifest errours, which is not only my part, but every good Subjects duty with sobriety to correct. And for avoiding of offence, I have rather done it in this private Book, then in any Book of the Statutes it self, trusting that all men will take it in good part.

Scholar. I would wish so, but I dare not so hope, sith never good man that would reform errour, could reform the venemous tongues of envious detractors, which because they either cannot or list not to doe any good themselves, doe delight to bark at the doings of others, but I beseech you to stay nothing for their perverse behaviour.

Master. I consider many things that some may object, whereunto I am not unprovided of just answers, but I will not seem so hasty to make the answers before I heare their Objections, but as I trust that men are of a better nature, and more gratefull now then some have been in times past. As I have done in the Statute of Assise of bread in rate of shillings, so will I set forth the like Table in pounds and ounces, and the parts thereof, that it may be easily applyed to all times: But I mean not by this to alter any word of the Statute, (being so good an Ordinance, and of so great continuance) but onely to make it as a kind of exposition and declaration of the said Statute, trusting that thereby the Statute may be better understood and con-

Concerning the following Tables.

A pound weight.

sequently better put in execution. And here you shall note, that I have accounted the shillings after the rate of 60 shillings to the pound weight, because I esteem it the most apt for our time. Wherefore in the first Columne you find the price of Wheat directly against it; in the second Columne, you may find the weight of a farthing white loafe, in this our time: and if you double the number (as I have done in the third Columne) then have you the weight of the half penny white loafe; and so in the fourth Columne is set the weight of a penny white loafe; It needeth not to tell, that the sight doth testifie how that every Columne is parted into three smaller pillers, whereof the first Columne hath these three titles; pounds, ounces and penny weights. And as in the first Columne 12 pence make a shilling, and 20 shillings make a pound, so in the other three Columnes 20 pence weight maketh an ounce, and 12 ounces make a pound.

GEntle Reader, touching the understanding of the *Table* following, wherein according to our time, *Master Record* alloweth 60 *pence* to the *ounce*, and 3 *pound* or 60 *shillings* to the *pound*, and thereupon after the rate of 60 *shillings* to the *pound Troy*, doth he frame or produce this his *Table*, beginning at 3 *shillings the quarter* till it come to 40 *shillings 6 pence the quarter*. And this is his proportion (for that he hath not set down any one *Example* to continue the work) hath been hard for many to conceive or comprehended, and therefore the only chief cause why I have written this *digression* for the better understanding of him therein.

The first thing therefore that is sought for in this *Table*, as in the other aforesaid, is a *Maxime* grounded upon the *Statute*, which is this. When the *quarter of Wheat* is sold for *two shillings*, then the *farthing white loafe* shall weigh 68 *shillings*, where by a *shilling* is $\frac{1}{10}$ meant of a *pound*, and by a *penny* $\frac{1}{20}$ of an *ounce*. Now therefore

fore for a generall Rule, to find what weight the *farthing white loaf* shall weigh at 3 *shillings the quarter*, till you come to 40 *shillings 6 pence the quarter*, is thus to be wrought. Comming to the *first ground*, and working by the *Backer Rule*, say, If *two shillings the quarter* give, or allow the *farthing white loaf* to weigh 68 *shillings*, what weight ought the *farthing white loaf* to weigh at 3 *shillings the quarter?* Work, and you shall find 45 *shillings* 4 pence, as before in the correction of the *first table* is noted. Then for the *second work*, say by the *Rule of Three Direct*, if 20 *pence* give one *ounce*, what giveth 45 *shillings* 4 *pence?* multiply and divide, and you shall find 544 *ounces*, which 544 *ounces* being multiplied by 3, for 3 *pounds* or 60 *shillings*, yeeldeth 1632 *ounces*, which divided by 20, produceth 81 *ounces*, and $\frac{12}{20}$ or rather $\frac{3}{5}$ of an *ounce*, equall unto 12 *penny weight*, which is *half an ounce*, and 2 *penny weight*, and so maketh in all 6 *pounds*, 9 $\frac{1}{2}$ *ounces* and 2 *penny weight*. Now the next way to continue this *table*, to know the weight of the half *penny white loaf*, is this, multiply 1632 *ounces* by 2, and it bringeth forth 3264 *ounces*, and divided by 20, it yeildeth 163 *ounces* and $\frac{1}{5}$, which is equall to 13 *pounds* 7 *ounces*, and 4 *penny weight*, as M. *Record his table noteth*.

Thirdly, for the weight of the penny white loaf, multiply 1632 *ounces* by 4, and divide by 20, and after by 12 as before, and you shall find 27 *pounds* 2 *ounces*, and 8 *penny weight*, &c. This *Method*, or else by doubling the *farthing white loaf*, for the *weight of the half penny white loaf*, and so doubling the *half penny white loaf*, for the *weight* of the *penny white loaf*, is the order to continue the *table* to the end thereof.

The price of a quarter of Wheat.			The weight of a farthing white loaf.		
li.	s.	d.	Po.	Ounces.	Penny weight.
0	3	0	6	9	12
0	4	6	4	6	8
0	6	0	3	4	16
0	7	6	2	8	12 $\frac{4}{5}$
0	9	0	2	3	4
0	10	6	1	11	6 $\frac{2}{7}$
0	12	0	1	8	8
0	13	6	1	6	2 $\frac{2}{3}$
0	15	0	1	4	6 $\frac{2}{5}$
0	16	6	1	2	16 $\frac{8}{11}$
0	18	0	1	1	12
0	19	6	1	0	11 $\frac{1}{13}$
1	1	0	—	11	13 $\frac{1}{7}$
1	2	6	—	10	17 $\frac{3}{5}$
1	4	0	—	10	4
1	5	6	—	9	12
1	7	0	—	9	1 $\frac{1}{3}$
1	8	6	—	8	11 $\frac{15}{19}$
1	10	0	—	8	3 $\frac{1}{5}$
1	11	6	—	7	15 $\frac{3}{7}$
1	13	0	—	7	8 $\frac{4}{11}$
1	14	6	—	7	1 $\frac{21}{23}$
1	16	0	—	6	16
1	17	6	—	6	10 $\frac{14}{25}$
1	19	0	—	6	5 $\frac{7}{13}$
2	0	6	—	6	0 $\frac{8}{9}$

The

The price of a quarter of Wheat.			The weight of a half penny white loaf.			The weight of a half penny white loaf.		
li.	s.	d.	Po.	ounces.	Penny weight	Po.	ounces.	Penny weight.
0	3	0	13	7	4	27	2	8
0	4	6	9	0	16	18	1	12
0	6	0	6	9	12	13	7	4
0	7	6	5	5	5 $\frac{3}{5}$	10	10	11 $\frac{1}{5}$
0	9	0	4	6	8	9	0	16
0	10	6	3	10	12 $\frac{4}{7}$	7	9	5 $\frac{1}{7}$
0	12	0	3	4	16	6	9	12
0	13	6	3	0	5 $\frac{1}{3}$	6	0	10 $\frac{2}{3}$
0	15	0	2	8	12 $\frac{4}{5}$	4	5	5 $\frac{3}{5}$
0	16	6	2	5	13 $\frac{5}{11}$	4	11	6 $\frac{10}{11}$
0	18	0	2	3	4	4	6	8
0	19	6	2	1	2 $\frac{2}{13}$	4	2	4 $\frac{4}{13}$
1	1	0	1	11	6 $\frac{2}{7}$	3	10	12 $\frac{4}{7}$
1	2	6	1	9	15 $\frac{1}{5}$	3	7	10 $\frac{2}{5}$
1	4	0	1	8	8	3	4	16
1	5	6	1	7	4	3	2	8
1	7	0	1	6	2 $\frac{2}{3}$	3	0	5 $\frac{1}{3}$
1	8	6	1	5	3 $\frac{11}{19}$	2	10	7 $\frac{3}{19}$
1	10	0	1	4	6 $\frac{2}{5}$	2	8	12 $\frac{4}{5}$
1	11	6	1	3	10 $\frac{6}{7}$	2	7	1 $\frac{5}{7}$
1	13	0	1	2	16 $\frac{8}{11}$	2	5	13 $\frac{5}{11}$
1	14	6	1	2	3 $\frac{19}{23}$	2	4	7 $\frac{15}{23}$
1	16	0	1	1	12	2	3	4
1	17	6	1	1	1 $\frac{8}{25}$	2	2	2 $\frac{6}{25}$
1	19	0	1	0	11 $\frac{1}{13}$	2	1	2 $\frac{2}{13}$
2	0	6	1	0	1 $\frac{7}{9}$	2	0	3 $\frac{5}{9}$

 Having

The price of a quarter of Wheat. | The weight of a farthing white loaf.

li.	s.	d.	Po.	Ounces.	Penny weight.
0	3	0	6	9	12
0	4	6	4	6	8
0	6	0	3	4	16
0	7	6	2	8	12 4/5
0	9	0	2	3	4
0	10	6	1	11	6 2/7
0	12	0	1	8	8
0	13	6	1	6	2 2/3
0	15	0	1	4	6 2/5
0	16	6	1	2	16 3/11
0	18	0	1	1	12
0	19	6	1	0	11 1/13
1	1	0	—	11	13 1/7
1	2	6	—	10	17 3/5
1	4	0	—	10	4
1	5	6	—	9	12
1	7	0	—	9	1 1/3
1	8	6	—	8	11 15/19
1	10	0	—	8	3 1/5
1	11	6	—	7	15 3/7
1	13	0	—	7	8 4/11
1	14	6	—	7	1 21/23
1	16	0	—	6	16
1	17	6	—	6	10 14/25
1	19	0	—	6	5 7/13
2	0	6	—	6	0 8/9

The

The price of a quarter of Wheat. li.	s.	d.	The weight of a half penny white loaf. Po.	ounces.	Penny weight	The weight of a half penny white loaf. Po.	ounces.	Penny weight.
0	3	0	13	7	4	27	2	8
0	4	6	9	0	16	18	1	12
0	6	0	6	9	12	13	7	4
0	7	6	5	5	5 3/5	10	10	11 1/5
0	9	0	4	6	8	9	0	16
0	10	6	3	10	12 4/7	7	9	5 1/7
0	12	0	3	4	16	6	9	12
0	13	6	3	0	5 1/3	6	0	10 2/3
0	15	0	2	8	12 4/5	4	5	5 3/5
0	16	6	2	5	13 5/11	4	11	6 10/11
0	18	0	2	3	4	4	6	8
0	19	6	2	1	2 2/13	4	2	4 4/13
1	1	0	1	11	6 2/7	3	10	12 4/7
1	2	6	1	9	15 1/5	3	7	10 2/5
1	4	0	1	8	8	3	4	16
1	5	6	1	7	4	3	2	8
1	7	0	1	6	2 2/3	3	0	5 1/3
1	8	6	1	5	3 11/19	2	10	7 3/19
1	10	0	1	4	6 2/5	2	8	12 4/5
1	11	6	1	3	10 6/7	2	7	1 5/7
1	13	0	1	2	16 8/11	2	5	13 5/11
1	14	6	1	2	3 19/23	2	4	7 15/23
1	16	0	1	1	12	2	3	4
1	17	6	1	1	1 8/25	2	2	2 6/25
1	19	0	1	0	11 1/13	2	1	2 2/13
2	0	6	1	0	1 7/9	2	0	3 5/9

Having

HAving ſpoken before for the underſtanding of the *Table* placed by M. *Record*, a man indued with rare knowledge in *Arithmeticall* and *Geometricall proportions*, touching the *Statute* of *Coynage, and the Standard thereof*, as appeareth in his Epiſtle of this *Book dedicated to King Edward the ſixth*, inſinuating unto his Highneſs that the *Standard of Coyne* is much altered from the 14. year of *King Edward the third* (when this *Statute and Aſſiſe* was confirmed) to the *Standard of this our time.* For it appeareth that in *King Edward the thirds time*, when the *Aſſiſe* of *Bread and Drink* was eſtabliſhed, that a *Sterling penny, round without clipping*, did then weigh 32 *cornes of wheat dried*, and taken out of the middle of the *eare*, and 20 of theſe *pence* made an *ounce*, and 12 *ounces* made a *pound Troy.* And ſo from the *weight of a penny* to 20 *ſhillings ſterling*, which then weighed 12 *ounces*, took *Bread his weight* and proportion: And now finding 60 *pence* is an *ounce.* That onely cauſe (I perceive, for the zeale of a Common-wealth) moved him to ſet down the ſame *Table in this private Book*; meaning not thereby to alter any word of the *Statute* being ſo good an ordinance and of ſo long continuance, but as a kind of expoſition by the way, that thereby the *Statute* may be better underſtood, and ſo conſequently better put in execution. Which *Aſſiſe of his*, is three times greater then the *Statute* now alloweth. Therefore alſo, to gratifie ſuch as are deſirous of knowledge, according to theſe *prices of a quarter of wheat*, I have added to this *Author theſe three other new tables following*, and reduced their *prices* into their juſt proportions of *Sterling money*, and alſo reduced the *money* into known *weight Troy*, according to the Statute. And thereafter according to proportion in my other three *Tables*, have I noted the *juſt weight* that a *Farthing, Half penny, and penny white-loaf*, ought to weigh by the *Statute*.

The

The price of a Quarter of Wheat.			☞ The weight of a Farthing white loaf in Sterling money by Assise.			The weight of a Farthing white loaf in Troy weight by Assise.		
l.	s.	d.	l.	s.	d.	ounces.	Po.	Penny weight.
0	3	0	2	5	4	2	3	4
0	4	6	1	10	2 2/3	1	6	2 2/3
0	6	0	1	2	8	1	1	12
0	7	6	0	18	1 3/5	—	10	17 1/5
0	9	0	0	15	1 1/5	—	9	1 1/3
0	10	6	0	12	15 3/7	—	7	15 3/7
0	12	0	0	11	4	—	6	16
0	13	6	0	10	0 8/9	—	6	0 8/9
0	15	0	0	9	0 4/5	—	5	8 4/5
0	16	6	0	8	2 1/10	—	4	18 10/11
0	18	0	0	7	6 2/3	—	4	10 2/3
0	19	6	0	6	11 9/13	—	4	3 7/13
1	1	0	0	6	9 3/5	—	4	1 1/2
1	2	6	0	6	0 8/13	—	3	12 8/15
1	4	0	0	5	8	—	3	8
1	5	6	0	5	4	—	3	4
1	7	0	0	5	0 4/9	—	3	0 4/9
1	8	6	0	4	9 5/19	—	3	17 5/19
1	10	0	0	4	6 2/5	—	2	14 2/5
1	11	6	0	4	3 17/21	—	2	11 17/21
1	13	0	0	4	2 1/11	—	2	9 1/2
1	14	6	0	3	11 7/23	—	2	7/23
1	16	0	0	3	9 2/3	—	2	5 2/3
1	17	6	0	3	7 13/25	—	2	3 13/25
1	19	0	0	3	5 11/13	—	2	1 11/13
2	0	6	0	3	4 8/17	—	2	0 8/27

The price of a Quarter of Wheat.			The weight of a half penny white loaf in Troy weight by Assise.			The weight of a half penny white loaf in Troy weight by Assise.		
l.	s.	d.	Po.	ounces.	Penny weight.	Po.	ounces.	Penny weight.
0	3	0	4	6	8	9	0	16
0	4	6	3	0	$5\frac{1}{3}$	6	0	$10\frac{2}{3}$
0	6	0	2	3	4	4	6	8
0	7	6	1	9	$15\frac{1}{5}$	3	7	$10\frac{2}{5}$
0	9	0	1	6	$2\frac{2}{3}$	3	0	$5\frac{1}{3}$
0	10	6	1	3	$10\frac{6}{7}$	2	7	$1\frac{5}{7}$
0	12	0	1	1	12	2	3	4
0	13	6	1	0	$1\frac{7}{9}$	2	0	$3\frac{5}{9}$
0	15	0	0	10	$17\frac{3}{5}$	1	9	$15\frac{1}{5}$
0	16	6	0	9	$17\frac{8}{11}$	1	7	$15\frac{7}{9}$
0	18	0	0	9	$1\frac{1}{3}$	1	6	$3\frac{2}{3}$
0	19	6	0	8	$7\frac{1}{13}$	1	4	$14\frac{2}{11}$
1	1	0	0	8	$3\frac{1}{3}$	1	4	$0\frac{2}{5}$
1	2	6	0	7	$5\frac{1}{15}$	1	2	$10\frac{2}{15}$
1	4	0	0	6	16	1	1	12
1	5	6	0	6	8	1	0	16
1	7	0	0	6	$0\frac{8}{9}$	1	0	$1\frac{7}{9}$
1	8	6	0	5	$14\frac{10}{19}$	0	11	$9\frac{1}{19}$
1	10	0	0	5	$8\frac{4}{5}$	0	10	$17\frac{3}{5}$
1	11	6	0	5	$3\frac{14}{21}$	0	10	$7\frac{5}{12}$
1	13	0	0	4	19	0	9	18
1	14	6	0	4	$14\frac{14}{23}$	0	9	$9\frac{5}{23}$
1	16	0	0	4	$11\frac{1}{3}$	0	9	$2\frac{1}{3}$
1	17	6	0	4	$7\frac{8}{25}$	0	8	$14\frac{2}{25}$
1	19	0	0	4	$3\frac{9}{23}$	0	8	$7\frac{5}{28}$
2	0	6	0	4	$0\frac{16}{27}$	0	8	$1\frac{5}{29}$

Scholar. Sir I do thank you most heartily for this, not only in mine own name, and in the name of all students, but also in the name of the whole commons, to whom the restitution of this Assise (I trust) shall bring restitution of the weight in Bread, which long time hath been abused. And if you know any thing more, wherein you would vouchsafe to declare the errours, and set forth the truth, you cannot but obtain great thanks of all good hearted men that love the Commonwealth.

Mast. I have sundry things to declare, but I have reserved them for a private Book by it self, yet notwithstanding because the statute of the rate of measuring of grounds is so common, that it toucheth all men, and yet no more common then needfull, but so much corrupt, that is, too farre out of all good rate; not only in the English Books of statutes, commonly Printed, but also in the Latin Books, and in the French also, (for I have read of each sort, and conferred them diligently) I will give you a Table for the restitution of these errours, as may suffice for this present time. And first I will propose one question to you touching the use of that statute, whereby you may perceive the order how to examine the whole statute, and every parcell thereof, and the question is this.

When the Acre of ground doth contain foure Perches in breadth, then must it contain 40 Perches in length. Then do I demand of you, how much shall the length of an Acre be, when there is in the breadth of it 13 Perches. But before you shall answer to this question, I will declare unto you another Statute, which is the ground of the former Statute. And that Statute is this. A question of measuring of ground.

It

Statute measure.

It is ordained, that thrée Barly-cornes dry and round, ſhall make up the meaſure of an inch: 12 inches ſhall make a foot, and 3 foot a yard, (the common Engliſh books have an Elne) five yards and a half ſhall make a Perch, and 40 Perches in length, and 4 in breadth ſhall make an Acre. This is that Statute, (whereby you may perceive, that the intent of the ſtatute is, that one Acre ſhould containe 160 ſquare Perches. Now let me hear you anſwer to the queſtion.

An Acre.

Scholar. As I perceive by the words of the Statute, a Perch to be the $\frac{1}{160}$ part of an Acre, ſo could I make thoſe numbers all in fractions, and ſo work the queſtion: but ſéeing I may doe it alſo in whole numbers, I take that forme for the moſt eaſe; therefore thus I ſet the queſtion in forme. Then doe I multiply 40 by 4, and it maketh 160, which I divide by 13, and the Quotient is 12 $\frac{4}{13}$.

4 Z 40
13 Z 12

Maſter. Now turne that $\frac{4}{13}$ into the common parts of a Perch, as they be named in the former ſtatute: Howbeit it ſhall be beſt to take one of the leaſt parts in Denomination for avoiding of much labour, as Feete, whereof the Perch containeth 16 $\frac{1}{2}$.

Scholar. Then to return $\frac{4}{13}$ into Feet, I multiply 16 $\frac{1}{2}$ by 4, and it maketh 66, which I muſt divide by 13, and the Quotient is 5 $\frac{1}{13}$.

Maſter. So I find, that if the Acre hold in breadth 13 Perches, it ſhall containe in length 12 Perches 5 Foot, and $\frac{1}{13}$ of a foot, which is not fully an Inch, for the Inche is $\frac{1}{12}$ of a foot. But here all the Statute Bookes in Latine and Engliſh (that I have ſéen) doe note it to be 13 Perches 5 foot, and one

Note this errour.

one Inch, which make above 13 Perches too many in the Acre: so that I would have thought the errour to have crept into the Printed Books, by the great negligence that Printers in our time doe use, save that in written copies of great antiquity, I doe find the same; yet have I one French copy which hath 12 Perches ¼ and one foot, and that misseth very little of the truth.

Scholar. Then I see it is true that I have often heard say, that the truest Copies of the Statutes, be the French copies.

Master. That is often true, but not generally, as I have by conference tried diversity: but in this Statute the French Book is most corrupt: in all other places lightly.

But now to performe my promise, I will set forth the Table for measuring of an Acre of ground, onely by such parts as the Statute doth mention, because at this time I doe of purpose write it for the better understanding of that Statute, and hereafter with other things intend to set forth this same more at large.

In this Table *following, I have not done as in the other* Statute *before compared by restitution with the faults crept into the* Statute, *but only have written that true measure, which the equity of the* Statute *doth pretend. For it were vile to judge of so noble Princes and worthy Counsellors, as have authorised and set forth this* Statute, *that they would make one* acre *in any forme greater then another, but every one to be just and equall with each other, which is the ground also of my work: and hereby may all men perceive how needfull* Arithmetick *is to the* Students of Law. *But now*

now I think beſt to make an end of theſe matters for this preſent time, ſith the Table *hath in it no obſcurity that I ſhould need to declare.*

The breadth of the Acre	The length of the Acre.			
Perches	Perches	Feet	Inches.	Parts of an Inch
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	$\frac{12}{13}$
14	11	7	0	$\frac{6}{7}$
15	10	11	0	0
16	10	0	0	0
17	9	6	9	$\frac{9}{17}$
18	8	14	8	0
19	8	6	11	$\frac{7}{19}$
20	8	0	0	0
21	7	10	2	$\frac{4}{7}$
22	7	4	6	0
23	6	15	9	$\frac{9}{23}$
24	6	11	0	0
25	6	6	7	$\frac{1}{5}$
26	6	2	6	$\frac{6}{13}$
27	5	15	3	$\frac{1}{3}$

The breadth of the Acre.	The length of the Acre.			
Perches	Perches	Feet	Inches	Parts of an inch.
28	5	11	9	$\frac{3}{7}$
29	5	8	6	$\frac{12}{29}$
30	5	5	6	0
31	5	2	7	$\frac{29}{31}$
32	5	0	0	0
33	4	14	0	0
34	4	11	7	$\frac{13}{17}$
35	4	9	5	$\frac{1}{7}$
36	4	7	4	0
37	4	5	4	$\frac{8}{37}$
38	4	3	5	$\frac{13}{19}$
39	4	1	8	$\frac{4}{13}$
40	4	0	0	0
41	3	14	10	$\frac{28}{41}$
42	3	13	4	$\frac{2}{7}$
43	3	11	10	$\frac{32}{43}$
44	3	10	6	0
45	3	9	2	0

Scholar. Indeed Sir, I understand the Table (as I think) by those other which you set forth before. For in the first columne is set the Perches of the breadth of an Acre, and then in the two Columnes following appeareth how many Perches, and how many foot that same Acre must have for his length.

Master.

Master. You take it well: howbeit to speak exactly of breadth and length, and the first Columne doth sometime betoken the breadth, and sometime the length: for properly the longest side of any square doth limit his length, and the shorter side doth betoken the breadth, yet it is no great abuse in such Tables, where a man cannot well change the Title, to let the name remaine, although the proportions of the numbers doe change: for still by the first Columne is expressed the measure of the one side, and by the two other pillers in one Columne, is set forth the measure of the other side. And this shall be sufficient now for the use of the Golden Rule.

The Rule of Fellowship.

NOw will I touch certaine other Rules, which for their severall names may seeme divers Rules, and distinct from this, but indeed they are but branches of it: yet because they have severall workings in appearance, but all pleasant in use, I will give you a tast of each of them. As for the Rule of Fellowship, both single and double, with time and without time, I shall need to say little more then I have already said in teaching the works of whole numbers: yet an example or two will we have to refresh the remembrance of the same, and to declare certain proper uses and applications of it, as this for one.

The Rule of Fellowship without time.

A question of inequall society.

Foure men got a booty or prize in time of warre, the prize is in value of money 8190 *pound, and because the*

the men be not of like degree, therefore their shares may not be equall; but the chiefest person will have of the booty the third part, and the tenth part over; the second will have a quarter, and the tenth part over: the third will have the sixth part: and so there is left for the fourth man a very small portion, but such as his lot (whether he be pleased or worth) he must be content with one 20 part of the prey: Now I demand of you what shall every man have to his share?

Scholar. You must be fain to answer to your own question, else it is not like to be answered at this time.

Master. The form to understand the solution of this question, and all such like, is this: Reduce all the Denominators into one number by Multiplication, except that any of them be parts of some other of them, for all such parts you may overpass and take for them all those numbers, whose parts they be: as in this example the shares be these $\frac{1}{3}\ \frac{1}{10}\ \frac{1}{4}\ \frac{1}{10}\ \frac{1}{6}\ \frac{1}{20}$, if I multiply all the Denominators together, beginning with 3, and so goe on unto 20, it will make 144000: but considering that 3 is a part of 6, I will omit that 3, and likewise 10, which is a part of 20, I may overpass also, and then is there but 3 Denominators to multiply, that is, 4, 6, and 20 which make 480, which summe I take for my work, because all the Denominators will be found in it. Then I take such parts of it as the question importeth, that is, for the first man $\frac{1}{3}$ and $\frac{1}{10}$, the $\frac{1}{3}$ is 160, the $\frac{1}{10}$ is 48, which I put in one summe for the first mans share, and it maketh 208, Then for the second mans share, I take $\frac{1}{4}$, which is 120, and $\frac{1}{10}$ which is 48, and that maketh in the whole

whole 168. Now for the third man which must have $\frac{1}{6}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole summe: so that if the whole prey had been but 480 pound, then were the question answered: but because the summe was of greater value, by this meanes now shall I know the partition of it. I must set my numbers by the order of the Golden Rule, putting in the first place the number of that I found by multiplying the Denominators, and in the second place the summe of the booty, And look what proportion is between the first number and the second, the same preportion shall be between the parts of that first number, and the parts of the second, comparing each to his like. Therefore I must put in the third place, one of the parts or shares, and then work by the former Rule of proportion or Golden Rule. And because I have 4 severall parts of the first number, by which I would find out foure like parts of the second number, therefore must I make foure severall figures.

The reason of this rule.

Scholar. Now I trust I can answer to your question, as by your favor I will prove:

And to trie it, I set the foure figures thus, marked with A, B, C, D, to shew their order.

A

A
480 Z 8190
208

B
480 Z 8190
168

C
480 Z 8190
80

D
480 Z 8190
24

And then in each of them I multiply the ſecond number by the third, and divide their totall by the firſt, and ſo amounteth the fourth ſumme which I ſeek for: for if I doe multiply 8190 by 208, it maketh 1703520, which being divided by 480, maketh in the Quotient 3549 for the firſt mans portion.

And ſo working with the other three figures, I find for the ſecond man 2866 $\frac{1}{2}$, and for the third man 1365, and then for the fourth man 409 $\frac{1}{2}$, and ſo every mans ſhare is ſet forth in the figure here annexed.

A
480 Z 8190
208 3549

B
480 Z 8190
168 2866 $\frac{1}{2}$

C
480 Z 8190
80 1365

D
480 Z 8190
24 940 $\frac{1}{2}$

And thus I think I have done well.

Maſter. If you miſdoubt your working, and liſt to prove it, adde all the ſhares together; and if they make the totall, then ſeemeth it well done.

The proof of Addition.

Scholar. I may ſet them thus: and then by Addition the juſt ſumme doth amount, that is, 8190, and therefore (as you ſay) it ſeemeth to be well wrought.

3549
2866 $\frac{1}{2}$
1365
409 $\frac{1}{2}$
8190

But I beseech you, is there any doubt in this triall, that you use that word seemeth?

Master. You may easily conjecture, that if you did assigne the first mans share to the last, and so change all the rest, and one had anothers share, yet would the Addition appear all one, and therefore is not the proof ergo.

The just Proof.

But if you will make a just proof for the first mans part, take $\frac{1}{5}$ and $\frac{1}{10}$ of the whole summe, and if it agree with the number in the figure, then it is well done. And so do for the second, third, and fourth summes, and this proof faileth not. Now will I propound certaine other questions, which have been set forth by certain learned men, albeit not without some oversight, which questions I protest heartily, I doe not repeate to deprave those good men, whose labours and studies I much praise and greatly delight in: But onely according to my profession, to seek out truth in all things, and to remove all occasions of errour as much as in me lieth: and for that cause I will onely name the questions without hurting the Authors name.

The first question is this.

A question of building

Foure men did build an house, which cost them 3000 *crownes, their shares were such that one man should pay* $\frac{1}{2}$ *of the summe and six crownes over; the second should pay* $\frac{1}{3}$ *and 12 crownes over; the third man must lay out* $\frac{2}{3}$ *abating* 8 *crownes; and the fourth man should pay* $\frac{1}{4}$ *and* 20 *crownes more. Can you answer to this question?*

Scholar. Nay I cannot sir, and that you know best of any man, for I know no more then you have taught me.

Master.

Master. Then I dare say you cannot doe it, neither yet the best learned man that ever did propose it: for the question is impossible. For declaration whereof, I will be bold to use first the representation of the Numbers in their aptest formes, (although I have not yet taught that manner of work) because it may appeare plainly that the question is not possible. For here I have set the parts, and added them, and they make the whole sum, and $\frac{3}{4}$ & 30 more. Now how is it possible to divide truly either gaines, either charges, so that the particulars $1\frac{3}{4}$ shall be more then the totall? An impossible question.

$$\left\{\begin{array}{lcr} \frac{1}{2} & + & 6 \\ \frac{1}{3} & + & 12 \\ \frac{2}{3} & - & 8 \\ \frac{1}{4} & + & 20 \\ \hline 1\frac{3}{4} & + & 30 \end{array}\right.$$

Scholar. It is against the forme of proofe by addition of parts.

Master. You say truth. And (because you shall perceive it the better) I will try it after the vulgar forme, as in this figure you see where the $\frac{1}{2}$ with 6 over is 1506, for the totall as you heard before, is 3000, the $\frac{1}{3}$ and the 12 more is 1012: the $\frac{2}{3}$ would be 2000, but then abating 8 it is but 1992, and then last of all, the $\frac{1}{4}$ is 750, and the 20 more namely 770: which all being added in one summe doe make 5280 where the totall summe should be but 3000, which summe of 3000 if you divide by $\frac{1}{4}$, you shall have $\frac{3}{4}$ of it, that is 2250, and thereto adde 30 more, then will those three sums make 5280: whereby you may see how this forme (as well as the other) doth declare that the particulars in that question would make more then the whole summe by $\frac{3}{4}$ and 30 more, and therefore can that question not be accepted as a

$$\begin{array}{r} 1506 \\ 1012 \\ 1992 \\ 770 \\ \hline 5280 \end{array}$$

$$\begin{array}{r} 3000 \\ 2250 \\ 30 \\ \hline 5280 \end{array}$$

But I beseech you, is there any doubt in this triall, that you use that word seemeth?

Master. You may easily conjecture, that if you did assigne the first mans share to the last, and so change all the rest, and one had anothers share, yet would the Addition appear all one, and therefore is not the proof exact.

The just Proof.

But if you will make a just proof for the first mans part, take $\frac{1}{5}$ and $\frac{1}{10}$ of the whole summe, and if it agree with the number in the figure, then it is well done. And so do for the second, third, and fourth summes, and this proof faileth not. Now will I propound certaine other questions, which have been set forth by certain learned men, albeit not without some oversight, which questions I profest heartily, I doe not repeate to deprave those good men, whose labours and studies I much praise and greatly delight in: But onely according to my profession, to seek out truth in all things, and to remove all occasions of errour as much as in me lieth: and for that cause I will onely name the questions without hurting the Authors mame.

The first question is this.

A question of building

Foure men did build an house, which cost them 3000 crownes, their shares were such that one man should pay $\frac{1}{2}$ of the summe and six crownes over: the second should pay $\frac{1}{3}$ and 12 crownes over: the third man must lay out $\frac{2}{3}$ abating 8 crownes: and the fourth man should pay $\frac{1}{4}$ and 20 crownes more. Can you answer to this question?

Scholar. No, I cannot sir, and that you know best of any man, for I know no more then you have taught me.

Master.

Maſter. Then I dare ſay you cannot doe it, neither yet the beſt learned man that ever did propoſe it: for the queſtion is impoſſible. For declaration whereof, I will be bold to uſe firſt the repreſentation of the Numbers in their apteſt formes, (although I have not yet taught that manner of work) becauſe it may appeare plainly that the queſtion is not poſſible. For here I have ſet the parts, and added them, and they make the whole ſum, and $\frac{3}{4}$ + 30 more. Now how is it poſſible to divide truly either gaines, either charges, ſo that the particulars $1\frac{3}{4}$ ſhall be more then the totall? An impoſſible queſtion.

$\frac{1}{2}$	+	6
$\frac{1}{3}$	+	12
$\frac{2}{3}$	—	8
$\frac{1}{4}$	+	20
$1\frac{3}{4}$	+	30

Scholar. It is againſt the forme of proofe by addition of parts.

Maſter. You ſay truth. And (becauſe you ſhall perceive it the better) I will try it after the vulgar forme, as in this figure you ſee where the $\frac{1}{2}$ with 6 over is 1506, for the totall as you heard before, is 3000, the $\frac{1}{3}$ and the 12 more is 1012: the $\frac{2}{3}$ would be 2000, but then abating 8 it is but 1992, and then laſt of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which all being added in one ſumme doe make 5280 where the totall ſumme ſhould be but 3000, which ſumme of 3000 if you divide by $\frac{3}{4}$, you ſhall have $\frac{3}{4}$ of it, that is 1250, and thereto adde 30 more, then will thoſe three ſums make 5280: whereby you may ſee how this forme (as well as the other) doth declare that the particulars in that queſtion would make more then the whole ſumme by $\frac{3}{4}$ and 30 more, and therefore can that queſtion not be accepted as a

1506
1012
1992
770
5280

2000
2250
5280

possible thing, but yet doe certain learned men propound such questions, and answer to them: Therefore somewhat to say to their excuse (rather of their good meaning, then for their doing) I will anon declare what may be said for their defence: but in the meane season, I will propound the question as it may be wrought by good possibility.

The former question of building now possible.

As if foure men build a house together, and it cost them 3000 Crownes, and then for the partition they agree thus: that as often as the first man doth pay 6 Crownes, so often the second man shall pay 4, the third man 8; and the fourth man 3. Or else thus, that the first man shall pay double so much as the fourth, and the second man shall pay ⅔ of the first mans charge: the third man shall pay double so much as the second: (and these two wayes are to one end) but further for their agreement it is appointed also, that the first shall give 6 Crownes overplus, and the second 12, and the fourth shall give 20, but the third man shall give no overplus, but shall have 8 Crownes abated of his charge.

Now is the question possible to be assoiled, and this is the way to doe it. Marke the proportion of the severall charges, and set out small numbers in that Rate, by which you may reduce the work to the Golden Rule, as here in the first forme the numbers are already named, 6, 4, 8, 3: and in the second forme, although they be but plainly named, yet they may be the same numbers: for 6 is double to 3, and 4 is ⅔ of 6: and again 8 is double to 4, Now adde these together, and they make 21, which 21 must be set for the first number in the Golden Rule: for if it, with the overplus of each mans charge, would make the totall summe of the

charges

charges, then were thoſe ſeverall ſummes the charges of each man, beſides his overplus: but now it is not ſo.

But yet this is true; (ſo excellent are concluſions Arithmeticall) that look what proportion each of their ſeverall ſummes doth bear to 21, the ſame proportion doth the juſt charges of every man (beſides his overplus) bear to the totall of the charges, the overplus being deducted: wherefore this may you note, that before you do apply the totall of his charges to the golden rule, you muſt deduct the overplus, which is 6, 12, and 20, that is in the whole 38: but then 8 muſt be reſtored for the abatement of the third man, and then remaineth to be deducted 30: take 30 therefore out of 3000, and there will reſt 2970, which I muſt ſet in the Golden Rule, for the ſecond ſumme: and for the third ſumme, I muſt put each of the ſmall numbers before mentioned, which although they be not ſeverall charges, yet they repreſent them in proportion. And ſo making for every mans charge a ſeverall queſtion, the figures will be 4, which I marke with foure letters, A, B, C, D, thus.

	A			B	
21	Z	2970	21	Z	2970
6		848 $\frac{4}{7}$	4		565 $\frac{5}{7}$
	C			D	
21	Z	2970	21	Z	2970
8		1131 $\frac{3}{7}$	3		424 $\frac{2}{7}$

Where I have ſet for breifneſs the ſumme of every mans charge in the fourth place, preſuppoſing that you can tell how to try out the fourth ſumme

by so many Examples as ye have had.

Scholar. As I trust that I understand this forme, so I desire much to know what may be said for them that mistook this Question.

Master. You seem so desirous to know this errour, that you have forgotten to examine whether this work be without fault.

Scholar. Me seemeth this work to be well done, because the Addition of the foure severall numbers doth make the totall summe of 2970, which was to be divided into such foure parts.

Master. But then have you forgotten that the first man must pay six crownes more besides his share, and the second man 12 crownes more, the third man 8 crownes less, and the fourth man 20 crownes more; for without these, your first totall of 3000 crownes will not be made.

Scholar. Then must I adde to the first mans summe 6 more, and it will be $854\frac{4}{7}$ and to the seconds summe, I must adde 12, and it will be $577\frac{5}{7}$: from the thirds summe I must abate 8, and then will the summe be $1123\frac{3}{7}$: then adding unto the fourths summe 20, it will be $444\frac{2}{7}$, and these foure summes will make 3000, which is the whole charge, as in this example it may appear, where first I gather the $\frac{14}{7}$, that maketh 2, and so proceed I in the Addition to the end.

$$\begin{array}{r} 854\frac{4}{7} \\ 577\frac{5}{7} \\ 1123\frac{3}{7} \\ 444\frac{2}{7} \\ \hline 3000 \end{array}$$

Master. Now have you well done, and this work in the same summes is brought of other Learned men for the true solution of the question, as it was proposed, which (as I said) was impossible

possible: and now examine by these severall sums, and see whether it doth agree with the summes in the Question proposed.

The first man must pay $\frac{1}{2}$ and 6 over of the totall summe: how think you, is $854\frac{4}{7}$ the half and 6 more of 3000?

Scholar. No that it is not, for it should be 1506: and for the second man 1012: and for the third man 1992: and for the fourth man 770: whereof not one summe agreeth to this work. But I marvell, that so wise men could bee so much over-seen.

Master. It is commonly seen, that when men will receive things from elder writers and will not examine the thing, they seem rather willing to erre with their Ancients for company, then to be bold to examine their works or writings. Which scrupulosity hath ingendred infinite errors in all kinds of knowledge, and in all civill administration, and so in every kind of Art. But these Learned men did not mean any other thing by this question, then to find such numbers as should beare the same proportion together, as those number, in the question proposed, did bear one to another: which thing you shall perceive more plainly by another question of theirs, that is this.

A man lying upon his Death-bed, bequeathed his goods (which were worth 3600 Crownes) in this sort: because his Wife was great with child, and he yet uncertain whether the Child were Male or Female, he made his bequest conditionally, that if the Wife bare a Daughter, then should the Wife have half his goods, and the Daughter $\frac{1}{3}$, *but if she were delivered of a Sonne, then that Sonne should* A question of a Testament.

 have

have ½ of the goods, and his Wife but ⅓. Now it chanced her to bring forth both a Sonne and a Daughter; the question is, How shall they part the goods agreeable to the Testatour his Will?

S. If some cunning Lawyers had this matter in scanning, they would determine this Testament to be quite void, & so the Man to die intestate, because the Testament was made insufficient, sith this condition was not expressed in it, and also it might have chanced that she should have brought forth neither Sonne nor Daughter, as often hath been seen: so is the Will insufficient in that point also.

Master. Such Scanners should seem too cunning, and yet not so cunning as cruell: for the minde of the Testator is to be taken favorably for the aid of the Legataries, when there ariseth such doubt. But let us try this work, not by force of Law, but by proportion Geometicall, seeing the Testator did minde to provide for each sort of them.

Scholar. If the Sonne shall have ½ by force of the Testament, so must the Mother have ⅓. Again, because she hath a Daughter also, therefore ought she to have ½ and the Daughter ⅓, that is both wayes $\frac{1}{2}\frac{1}{3}$, & $\frac{1}{3}\frac{1}{2}$, which commeth to the whole goods, & ⅔ more. Wherefore it seemeth also imposible.

Master. In this matter the minde of the Testator is to be understood that such proportion should be between the portion of the Wife, and the Sonne, as is between ⅓, and ½, that is, the Sonne must have $\frac{3}{6}$, for $\frac{2}{6}$, to his Mother, so shall he have 3 to 2, that is as much as his Mother, and half as much more; and the Mother must have the like rate in comparison to her Daughter.

Then

Then muſt I find out 3 numbers in ſuch proportion, that the firſt may have as much as the ſecond, and half as much more (that is) in proportion ſeſquialtera, and the ſecond to the third in that ſame proportion: ſuch numbers, be 9,6,4.

Scholar. I pray you Sir, how ſhall I find out theſe numbers?

Maſter. That will I gladly tell you.

Whatſoever the proportion be of any three numbers, multiply the Termes of that proportion together, and the number that amounteth ſhall be the middle number of the three: then multiply that middle number by the leſſer term, and divide that totall by the greater, and the leaſt number of the three will amount. So if you multipy that middle number by the greater extreame, and divide the totall by the leſſer extreame, then will the greateſt number of that Progreſſion amount.

To find three numbers in any proportion.

Scholar. Then in this exmple to find the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I muſt divide (as you taught me in Diviſion) $\frac{1}{2}$ by $\frac{1}{3}$, and the Quotient will be $\frac{3}{2}$, that is, 1 $\frac{1}{2}$. whereby I perceive that the proportion in this queſtion is as three to 2. Therefore as you taught me even now, I multiply 3, by 2, and the ſumme is 6, which muſt be the middle number: then I multiply the middle number 6 by 2, which is the leaſt terme, and the ſumme is 12, that I doe divide by 3, being the greater terme, and the Quotient is 4: ſo is 4 the leaſt number of the three. Then I multiply 6 by 3, whereof cometh 18, and that I divide by 2, and ſo have I 9 which is the greateſt number of the three.

To find the proportion between 2 numbers.

Maſter. Another way yet may you find the

third

third number in any Progression, if you have two of them: for if the middle number be one of them which you have, then multiply it by it self (as in this example, 6 by 6 maketh 36) and that totall divide by the other number which you have, and the third number will be the Quotient.

Scholar. Then I divide 36 (which commeth of 6 multiplied by it self) by 4, the Quotient will be 9: and if I divide 36 by 9, the Quotient will be 4. But what if I know the first number, and the third, and would have the middle number?

Master. Multiply the 2 numbers together, and in their totall you must seek the root of that number, and it shall be the middle number: but because as yet you have not learned to extract Roots, therefore use the first forme which I have taught you, till I teach you to extract Roots. And now go forwards with the answer to the same question.

Note.

Scholar. I perceive then, that the Sonne must not have $\frac{1}{2}$ of the goods, neither the Mother $\frac{1}{3}$, nor yet the Daughter $\frac{1}{3}$, but yet must the goods be divided into such proportion, that the Sonne shall have 9 crownes for 6 to his Mother, and the Mother shall have 6 crownes for every 4 to her Daughter. Then I apply it to the Golden Rule in three examples, as followeth.

Where the first number is the Addition of those three numbers 9, 6, 4: and the third is one of them severally: the second is the totall of the goods in that Testament: and then by the work of the Golden Rule, I find out the fourth number

19 Z 3600
9

19 Z 3600
9

19 Z 3600
4

number in every work: that is, for the Sonne 1705 $\frac{5}{19}$, for the Mother 1136 $\frac{16}{19}$, and for the Daughter 757 $\frac{17}{19}$: the which summes added together, doe make the summe of the whole goods as may be seen by this Example.

1705 $\frac{5}{19}$
1136 $\frac{16}{19}$
757 $\frac{17}{19}$
3600

And this (me thinketh) I doe perceive, because in this Case there is a necessary remedy divised against an urgent inconvenience: therefore those learned men thought they might use the like liberty in that other question.

Master. Your guess is good, but they had so good reason for them in the one, as they have in the other: As in another example of theirs, it may better appeare, as in this.

A man left unto his three Sonnes 7851 crownes to be parted in such sort, that the first Sonne should have $\frac{1}{2}$, the second Sonne $\frac{1}{3}$, and the third Sonne $\frac{1}{4}$, which is not possible: for $\frac{1}{2}$ and $\frac{1}{3}$ and $\frac{1}{4}$ doe make $\frac{26}{24}$, or $\frac{13}{12}$, that is, 1 $\frac{1}{12}$, so it is more then the whole, but reduce these Fractions *into one denomination, the least that they will come to, and they will be $\frac{6}{12}$, $\frac{3}{12}$, $\frac{2}{12}$ and so may you part the goods into such proportion as these 3 Numerators beare together, that is, the first to have 6 for every 4 to the second, and the second to have 4 as often as the third hath 3: and so their portions will be for the first, 3623 $\frac{7}{13}$, for the second 2415 $\frac{9}{13}$, and for the third 1811 $\frac{10}{13}$: and these 3 shares added together, will make the totall summe of the whole goods, as you may easily see in this example.* Another question of a Testament.

3623 $\frac{7}{13}$
2415 $\frac{9}{13}$
1811 $\frac{10}{13}$
7851

Another

third number in any Progression, if you have two of them: for if the middle number be one of them which you have, then multiply it by it self (as in this example, 6 by 6 maketh 36) and that totall divide by the other number which you have, and the third number will be the Quotient.

Scholar. Then I divide 36 (which commeth of 6 multiplied by it self) by 4, the Quotient will be 9: and if I divide 36 by 9, the Quotient will be 4. But what if I know the first number, and the third, and would have the middle number?

Master. Multiply the 2 numbers together, and in their totall you must seek the root of that number, and it shall be the middle number: but because as yet you have not learned to extract Roots, therefore use the first forme which I have taught you, till I teach you to extract Roots. And now go forwards with the answer to the same question.

Note.

Scholar. I perceive then, that the Sonne must not have ½ of the goods, neither the Mother ⅓, nor yet the Daughter ⅓, but yet must the goods be divided into such proportion, that the Sonne shall have 9 crownes for 6 to his Mother, and the Mother shall have 6 crownes for every 4 to her Daughter. Then I apply it to the Golden Rule in three examples, as followeth.

Where the first number is the Addition of those three numbers 9, 6, 4: and the third is one of them severally: the second is the totall of the goods in that Testament: and then by the work of the Golden Rule, I find out the fourth number

19 Z 3600
9

19 Z 3600
9

19 Z 3600
4

number in every work: that is, for the Sonne 1705 $\frac{5}{19}$, for the Mother 1136 $\frac{16}{19}$, and for the Daughter 757 $\frac{17}{19}$: the which summes added together, doe make the summe of the whole goods as may be seen by this Example.

1705 $\frac{5}{19}$
1136 $\frac{16}{19}$
757 $\frac{17}{19}$
3600

And this (me thinketh) I doe perceive, because in this Case there is a necessary remedy divised against an urgent inconvenience: therefore those learned men thought they might use the like liberty in that other question.

Master. Your guess is good, but they had so good reason for them in the one, as they have in the other: As in another example of theirs, it may better appeare, as in this.

A man left unto his three Sonnes 7851 *crownes to be parted in such sort, that the first Sonne should have* $\frac{1}{2}$, *the second Sonne* $\frac{1}{3}$, *and the third Sonne* $\frac{1}{4}$, *which is not possible: for* $\frac{1}{2}$ *and* $\frac{1}{3}$ *and* $\frac{1}{4}$ *doe make* $\frac{26}{24}$, *or* $\frac{13}{12}$, *that is,* 1 $\frac{1}{12}$, *so it is more then the whole, but reduce these* Fractions *into one denomination, the least that they will come to, and they will be* $\frac{6}{12}$, $\frac{3}{12}$, $\frac{2}{12}$ *and so may you part the goods into such proportion as these* 3 *Numerators beare together, that is, the first to have* 6 *for every* 4 *to the second, and the second to have* 4 *as often as the third hath* 3: *and so their portions will be for the first,* 3623 $\frac{7}{13}$, *for the second* 2415 $\frac{9}{13}$, *and for the third* 1811 $\frac{10}{13}$: *and these* 3 *shares added together, will make the totall summe of the whole goods, as you may easily see in this example.* Another question of a Testament.

3623 $\frac{7}{13}$
2415 $\frac{9}{13}$
1811 $\frac{10}{13}$
7851

Another

Another question is there proposed thus.

Another like question.

There are 450 crownes to be divided between three men, so that the first man must have $\frac{1}{2}$ and $\frac{1}{4}$, the second man $\frac{1}{3}$ and $\frac{1}{4}$; the third man shall have $\frac{1}{4}$ and $\frac{1}{5}$.

Scholar. I marvell that any man should be so overseen, to propose that question as a thing possible, sith $\frac{1}{2}\frac{1}{3}$, $\frac{1}{3}\frac{1}{4}$, $\frac{1}{4}\frac{1}{5}$, doe make 1 $\frac{12}{13}$, that is almost double the whole summe.

But I perceive it might be thus proposed: that as often as the first man did receive 50 crownes, so often the second man should receive 35 and the third man 27: for $\frac{1}{2}$ and $\frac{1}{3}$ is equall to $\frac{5}{6}$: and so is $\frac{1}{3}$ and $\frac{1}{4}$ equall to $\frac{35}{60}$, and $\frac{1}{4}$ and $\frac{1}{5}$ is $\frac{27}{60}$ and so working the question the three figures will appeare in this forme: whereby the first mans portion is found to be 200 $\frac{50}{56}$: the second mans part is 140 $\frac{35}{56}$ the third mans share 108 $\frac{27}{56}$: which in the whole doth make 450 crownes to be divided between them.

112	Z	450
50		200 $\frac{50}{56}$
112	Z	450
35		140 $\frac{35}{56}$
112	Z	450
27		108 $\frac{27}{56}$

Master. And thus you are (I think) sufficiently instructed in the Rule of Fellowship.

The

The Rule of Alligation.

NOw will I goe in hand with the Rule of Alligation; which hath his name, for that by it there are divers parcells of sundry prices, and sundry quantities alligate, bound or mixed together: whereby also it might be well called the Rule of Mixture; and it hath great use in composition of Medicines, and also in mixtures of Metalls, and some use it hath in mixtures of Wines: but I wish it were less used therein then it is now a dayes. The order of this rule is this. The rule of Mixture.

When any summes are proposed to be mixed, set them in order one over another, and the common number (whereunto you will reduce them) set on the left hand; then marke what summes be lesser then that common number, and which be greater, and with a draught of your penne evermore linke two numbers together, so that one be lesser then the common number, and the other greater (for two greater or two smaller cannot well be linked together) and the reason is this, that one greater and one smaller, may be so mixed, that they will make the meane or common number very well: but two less can never make so many as the common number, being taken orderly: no more can two summes greater then the mean, ever make the mean in due order, as it shall appeare better to you hereafter. And as it is of necessity to linke every smaller (once at the least) with one greater, and every greater with one smaller, so it is at liberty The reason of this Rule.

to

to link them oftner then once, and so may there be to one question many solutions. When you have so linked them, then marke how much each of the lesser numbers is smaller then the mean or common number, and that difference set against the greater numbers, which be linked with those smaller, each with his match still on the right hand, and likewise the excess of the greater numbers above the mean, you shall set before the lesser numbers which be combined with them.

Then shall you (by Addition) bring all these differences into one summe, which shall be the first number in the Golden Rule, and the second number shall be the whole masse that you will have of all those particulars: the third summe shall be each difference by it selfe, and then by them shall be found the fourth number, declaring the just portion of every particular in that mixture: As now by these Examples I will make it plaine.

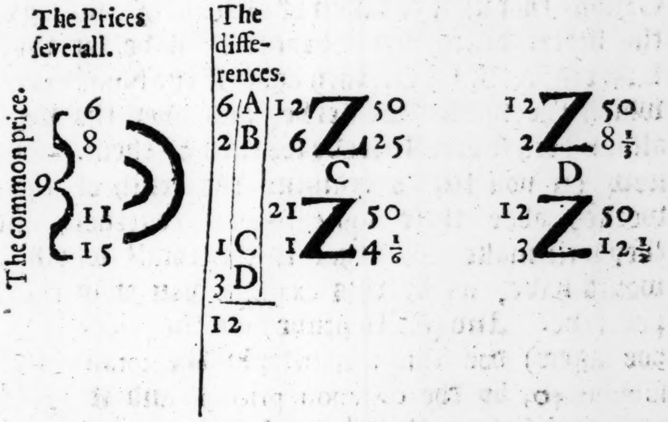
A question of mixing of Wines.

There are foure sorts of Wine, of severall prices, one of 6 pence *a gallon, another of* 8 pence, *the third of* 11 pence, *and the fourth of* 15 pence *the gallon, of all these Wines would I have a mixture made to the summe of fifty gallons, and so the price of each gallon may be* 9 pence. *Now demand I, how much must be taken of every sort of Wine?*

Scholar. If it shall please you to work the first example, that I may mark the applying of it to the Rule: then I trust I shall bee able not onely to doe the like, but also to see the reason in the order of the worke.

Master,

Maſter. Marke then this forme, and the placing of every kind of number in it.



Here (you ſee) I have ſet down the ſeverall prices, which be 6, 8, 11, 15, and have linked together 6 with 15, and 8, with 11. The common price 9, I have ſet on the left ſide, and the difference betwéen it, and every particular price I have ſet on the right hand, not againſt the ſumme (whoſe difference it is) but againſt the ſumme that it is linked withall, ſo the difference of 15 above 9, is 6, which I have ſet, not againſt 15 but 6, that is linked with 15, and the difference betwéen 6 and 9 (that is 3) I have ſet againſt 15. So likewiſe the difference betwéen 8 and 9, is but 1, that I have ſet againſt 11; and the difference of 11 above 9 (which is 2) I have ſet againſt 8. Then adde I all thoſe foure differences, and they make 12, which I ſet for the firſt number in the Golden Rule: the ſecond number I make 50, which is the ſumme

of

of Gallons that I ſhould have, and the third ſumme is every particular difference. Now if you work by the Golden Rule, you ſhall find the number of Gallons that ſhall be taken of each ſort of wine: For the better diſtinction whereof I have ſet theſe Letters, A, B, C, D, both againſt the Numbers for which the works doe ſerve, and over the work alſo, which ſeverally ſerve for each of them. And now (if you liſt to examine the truth of theſe works) adde theſe foure ſummes together, and they will make 50. that is the totall which I would have, as by this example you may eaſily perceive. And (for to prove how the prices doe agree) doe this: Multiply the totall ſumme 50, by the common price 9, and it will make 450: then keep that ſumme by it ſelf, and afterward multiply every ſeverall ſumme of Gallons, by the price belonging to the ſame Gallons, and if that ſumme doe agree with this, which you have kept firſt, then is your work well done. As here 25 is the number of Gallons of 6 pence price, multiply then 25 by ſix, and it maketh 150, which you ſhall ſet down, then multiply $8\frac{1}{3}$ by 8, which is the price for the number of Gallons, and it will make $66\frac{4}{6}$: ſo again $4\frac{1}{6}$ multiplied by 11, doth make $45\frac{5}{6}$. And laſt of all, $12\frac{1}{2}$ multiplied by 15, maketh $187\frac{3}{6}$: and theſe added together doe make 450, as in the example annexed you may ſee, wherefore ſeeing it doth agree with the former ſumme of 50, multiplyed by 9, I may juſtly affirme this work to be good, and well done.

The proof of this rule.

25
$8\frac{1}{3}$
$4\frac{1}{6}$
$12\frac{1}{2}$
50

150
$66\frac{4}{6}$
$45\frac{5}{6}$
$187\frac{3}{6}$
450

And

And now to prove how you can doe the like, I propound the same Question, onely willing you to use some other forme of combining or linking the summes. The variation of this question.

Scholar. That shall I prove with your favour, and therefore I combine 8 with 15, and 6 with 11: and then the form will be as followeth.

9	6	2	A
	8	6	B
	11	3	C
	15	1	D
		12	

A: 12 Z 50; 2 Z $8\frac{2}{6}$

B: 12 Z 50; 6 Z 25

C: 12 Z 50; 3 Z $12\frac{3}{6}$

D: 12 Z 50; 1 Z $4\frac{1}{6}$

Whereby amounteth the same summe in totall of the differences as did before: and yet now the differences be altered as the combination is changed, whereof I understand the reason by your former work. And therefore here appeareth no strange thing, but that now I have $8\frac{2}{6}$ gallons of 6 pence, and 25 gallons of 8 pence: and 12 gallons and $\frac{3}{6}$ of 11 pence, and so consequently 4 gallons and $\frac{1}{6}$ of 15 pence: so that multiplying $8\frac{2}{6}$ by 6, it maketh 50, and then 25 multiplyed by 8, maketh 200: likewise $12\frac{3}{6}$ multiplyed by 11 yield $137\frac{3}{6}$ and $4\frac{1}{6}$ multiplyed by 15, maketh $62\frac{3}{6}$, which 4 summes added into one, will yield in the totall 450, which agreeth with the multiplication of 50 (being the totall summe of Gallons) by 9 the common or mean price.

50
200
$137\frac{3}{6}$
$62\frac{3}{6}$
450

Master. Seeing you conceive this work so

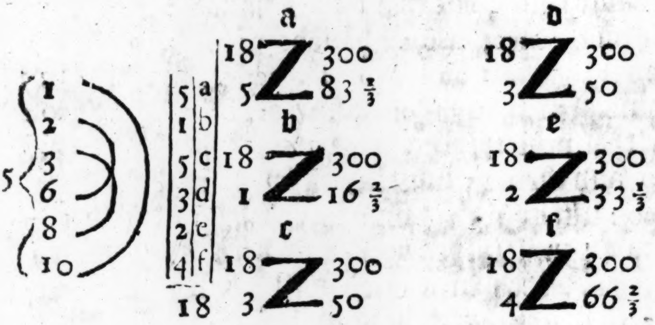
well, I will propound another example unto you of more variety in the Alligations or combinings, as thus.

A queſtion of Spices. *A Merchant being minded to make a bargain for ſpices, in a mixt maſſe (that is to ſay) of Cloves, Nutmegs, Saffron, Pepper, Ginger, and Almonds: the Cloves being at 6 ſhillings, Saffron at 10 ſhillings, Pepper at 3 ſhillings, Ginger at 2 ſhillings, and Almonds at 1 ſhilling.*

Now would he have of each ſort ſome, to the value of 300 pound in the whole, and each pound one with another to bear in price five ſhillings: How much ſhall he have of each ſort?

Scholar. That will I try thus:

Firſt I ſet downe thoſe 6 ſeverall prices, and at the left hand I ſet the common price five ſhillings. Then I link them thus, 1 with 10, 2 with 6, and 3 with 8: as in the example following.



Maſter. I had minded to have combined them in more variety: but I am content to ſee your

own

own work firſt, and then more varieties in combination may follow anon.

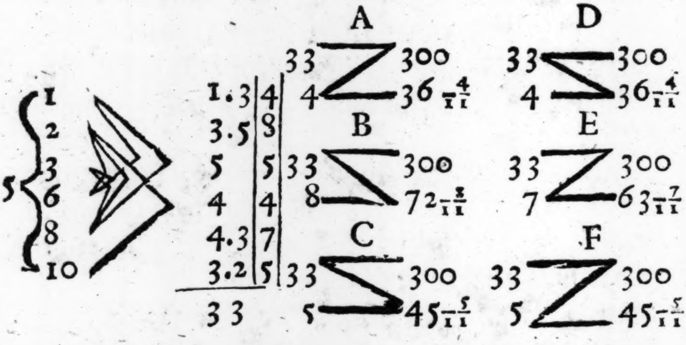
Scholar. Then to continue as I began, I ſeek the difference betweene 1 and 5, (which is 4) and that I ſet againſt 10, then againſt 1 I ſet 5, which is the exceſs of 10 above 5, ſo I gather the difference betweene 2 and 5, which is 3, and that I ſet againſt 6, becauſe it is combined with 2, and likewiſe the difference of 6 above 5 (which is 1) I ſet againſt 2. Then take I the difference of 3 from 5, which is 2, and that I ſet againſt 8: and before that 3, I ſet the difference of 8 above 5, which is 3. Then gather I all theſe differences by Addition, and they make 18, which I ſet for my firſt Number in the Golden Rule, and ſo appeareth by thoſe works, that of Almonds I muſt take 83 $\frac{1}{3}$ pound, of Ginger 16 $\frac{2}{3}$ pound, Pepper 50 pounds, of Cloves 50 pound, of Nutmegs 33 $\frac{1}{3}$ pound, and of Saffron 66 $\frac{2}{3}$ pounds.

Then for triall hereof, I multiply every parcell by his ſeverall price, as 83 $\frac{1}{3}$ which is the ſumme of Almonds, I multiply by 1 which is their price.

Alſo 16 $\frac{2}{3}$ the ſumme of Ginger, I multiply by 2, which is the price of it: and ſo each other in his kind, as this Table annexed doth repreſent, and then adding them all together I find the totall to be 1500, which alſo will amount by the multiplication of the groſs maſs of 300, by the common price 5, wherefore it appeareth well wrought.

83 $\frac{1}{3}$
33 $\frac{1}{3}$
150
300
266 $\frac{2}{3}$
666 $\frac{2}{3}$
1500

Maſter. Now I will make the alligation to prove your cunning ſomewhat better: but becauſe you ſhall not think your ſelf preſſed too much, I will alſo note the differences, as by this Example you may ſee, where I have alligated 1 with 6



and 8, and therefore have I ſet againſt 1 both their differences, that is 1 and 3: Likewiſe, becauſe 2 is combined with 8 and 10, I ſet before him their differences, 3 and 5. Againſt 3 I have ſet onely 5, which is the difference of 10, with whom 3 is combined onely. Likewiſe 6 is onely alligate to 1, and therefore is the difference of 1 from 5 which is 4, onely ſet againſt it: 8 is linked with 1 and 2, and therefore hath ſet 4 againſt him both their differences, 4 and 3: and 10 is joyned with 2 and 3, therefore hath he their differences, 3 and 2. And becauſe of eaſe for you, in another columne I have ſet the differences reduced into one number, for every ſeverall ſort, and have alſo added them together whereby appeareth that they make 33, & ſo conſequently you ſee the works of the Golden Rule ſet forth. For the ſix Drugges

I

I have added the letters A.B.C. &c. as before.

But I would not wish you to cleave still to these Elementary aides, but accustome Memory to trust her self: so shall occasion of negligence best be avoyded. And as for the proof try it at more leisure, because the time now is short, and you sufficiently instructed in that proof. And there resteth divers things behind yet, of which I would gladly give you some tast, before your departure. Note.

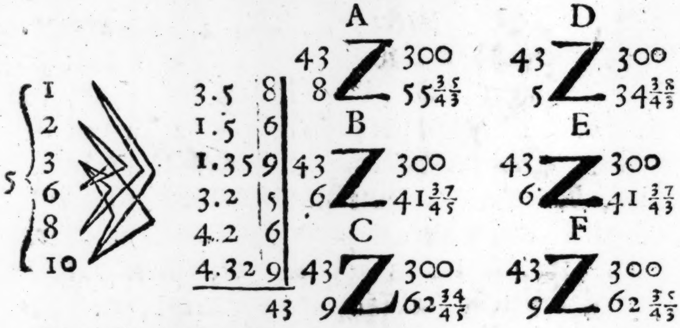
Scholar. But if it may please you to let me see all the variations of this question, before you goe from it, for methinketh I could vary it two or three wayes more yet.

Master. I am content to see you make two or three variations: but I would be loth to stay to see all the variations: for it may be varied above 300 wayes, although many of them would not well serve to this purpose.

Scholar. I thought it impossible to make so many variations.

Master. Marvell not thereat, for some questions of this Rule, may be varied above 1000 wayes; but I would have you forget such fantasie till a time of more leisure. And now goe forward with some variation of this question. Note.

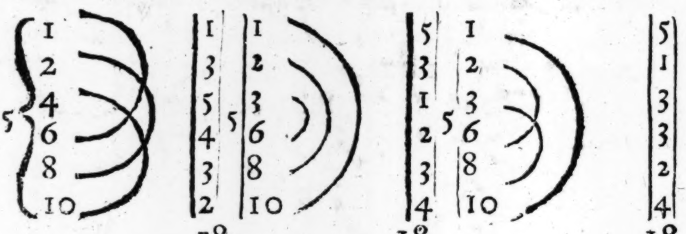
Scholar. For the first variation, I linke the first number 1 with 1 and 10 and 2, I combine with 9 and 10: then joyne I 3 with 6, 8, and 10, as in this forme.



And so doth there appeare the proportion of weight for every kind of Drugge in this mixture. Now for the tryall.

Master. Nay stay there: you shall not neede to make tryall in one example so often, or if you list to doe it by your self, I am content. But now set forth (for declaration that you conceive the Rule) two or three examples of severall Combinations, and then will we pass to some other example, and so end this Rule.

Scholar. As it pleaseth you, so will I doe. And these be the varieties: in which as the combinations are severall, so doth it plainly appear, that the differences by which the proportion of each severall kind is taken are also severall. And yet I see in the three first of these five varieties, and in the one



other

5	1	1.3.5	9	5	1	1.3	4
	2	5.3	8		2	4.5	9
	3	5	5		3	3	3
	6	4	4		6	4.3	7
	8	4.3	7		8	4 5	9
	10	3.3	9		10	3	3
			42				35

other before, the totall summe of the differences to be one, that is to say, 18, whereby I perceive that the variety of their mixture doth depend on the variety of their differences severall, and not of the variety of their totall summe.

Master. So is it. And seeing you conceive it so well, I will make an end of this Rule, onely exhibiting unto you one Question or two of the mixture of Metals, that by it you may divise others like, and exercise your self therein also, because the use of it serveth often in business of charge, not so much for Goldsmiths, as of coynage in Mints. First, I demand of you this question: *If a Mint-master have Gold of 22 Karects, and some of 23 Karects, some of 24: Againe, some 15, some 16, and some of 18 Karects, and would mixe them, so that he might have 100 Ounces of 20 Karects: How much must he take of each sort?*

Scholar. To know that, I answer in order thus:

20	15	2	20	Z	100	20	Z	100
	16	3	2		10	5		25
	18	4	20	Z	100	20	Z	100
	22	5	3		15	4		20
	23	4	20	Z	100	20	Z	100
	24	2	4		20	2		10
		20						

Master. You have wrought the question well, but how chanced you made no doubt of that new name Kareĉt.

Scholar. Because I thought it out of time to demand such questions now seeing you make so much hast to end: and again in this case the proportion of the number is sufficient for my purpose in this work, trusting that another time you will instruct me as well of this, as of sundry other things, which as I have heard you talke of, so I have a great desire to them.

Mast. Your answer is reasonable, and your request and trust (with Gods help) I intend to satisfie, & now to go forward with this matter, let me see your examination of this last work.

Scholar. First for the one part I adde together all the particular sums, as they appear in the work, and they make 100, as here by their Addition doth appeare.

10
15
20
25
20
10
100

And so it seemeth that the sums are well gathered: but for the further triall of them I multiply first 20 which is the common or mean sum of the Kareĉts by 100, which is the sum of the whole Mass which I would have, and it maketh 2000. Then I multiply every particular sum by the Kareĉts that it doth contain, as 10 by 15, and that maketh 150.

150
240
360
550
460
240
2000

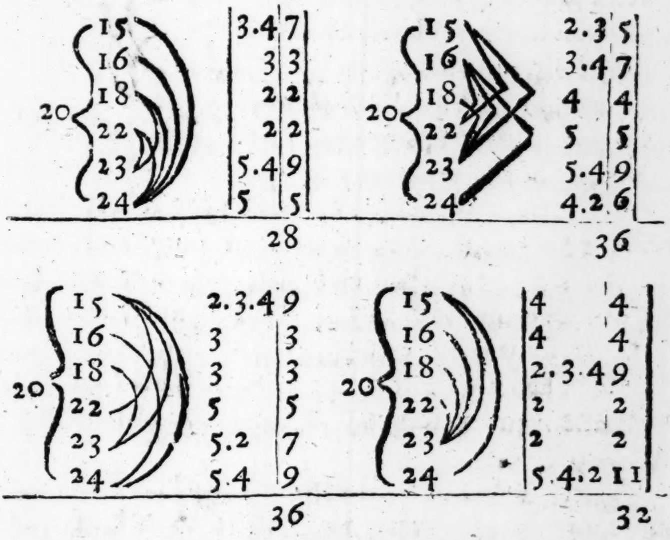
Likewise I multiply 15 by 16, and it yieldeth 240: so 20 by 18, maketh 360. And 25 by 22, yieldeth 550: likewise 20 by 23, bringeth forth 460: and last of all, 10 multiplied by 24, yieldeth 240: which summes all joyned together make 2000, that doth agree with the like summe

before

before, wherefore I may well ſay, that the work is good. And now (if it pleaſe you) I would ſet forth ſome varieties of this queſtion, to prove my wit.

Maſter. Goe to, let me ſee.

Scholar. Here be foure varieties.



And more yet could I make, but not like to the number that you ſpake of in the variation of the other queſtion.

Maſter. That will I teach you at more leiſure, ſeeing it is a thing rather of pleaſure then of any neceſſity.

But now for your exerciſe in this Rule, one other queſtion I will propoſe. A Mint-maſter hath ſix Ingots of ſilver, of ſundry fineneſs, ſome of foure Ounces fine, and ſome of five Ounces, ſome of ſix, and other of eight, ſome of 11, *and other of* 12, *and his deſire is to mixe* 500 *pounds weight, ſo that in the whole maſs* A queſtion of mixing of ſilver.

every

every pound weight ſhould beare 9 Ounces of fine ſilver: How much ſhall he take ſay you of every ſort of ſilver?

Scholar. To find out that, I ſet the numbers thus in order.

9	4	2	2
	5	2	2
	6	3	3
	8	3	3
	11	5.4	9
	12	3.1	4
			23

And gathering the differences it will appeare, that, of the firſt ſort there muſt be $43\frac{11}{23}$ of the ſecond as much: of the third ſort $65\frac{5}{23}$: and of the fourth ſort as much: of the fifth ſort $195\frac{15}{23}$ and of the ſixth ſort $86\frac{22}{23}$, which in the whole will make 500 pound weight, and in ounces after 9 ounces fine 4500, that is of the firſt ſort $173\frac{21}{23}$, and of the ſecond ſort $217\frac{9}{23}$, of the third ſort $391\frac{7}{23}$, of the fourth ſort $521\frac{17}{23}$, of the fifth ſort, $2152\frac{4}{23}$, and of the ſixth ſort $1043\frac{11}{23}$, which all together doe make 4500 ounces, agreeable to the multiplication of 9 by 500.

Master. This is well done of you, therefore now make three or foure varieties, and ſo an end of this Rule.

Scholar. Theſe foure varieties I ſet for examples.

9	4	3	3
	5	3	3
	6	3	3
	8	2	2
	11	1	1
	12	5.4.3	12
			24

9	4	2.3	5
	5	2	2
	6	2	2
	8	2	2
	11	5.4.3.1	13
	12	5	5
			29

Maſter.

9	4	2.3	5	9	4	3	3
	5	3	3		5	3	3
	6	2	2		6	2.3	5
	8	2	2		8	2.3	5
	11	5.3.1	9		11	3.1	4
	12	5.4	9		12	5 4.3.1	13
			30				33

Maſter. And by theſe it appeareth, that you can find out more, with which I will not now meddle, ſave onely (for to ſhew you an eaſie help drawing the lines of combination) I will ſet forth 2 varieties here.

9	4	2	2	9	4	3	3
	5	2.3	5		5	2.3	5
	6	3.2	5		6	2.3	5
	8	3	3		8	2.2	5
	11	5.4.3	12		11	4.3	8
	12	4.3.1	8		12	5.4.2.1	12
			35				38

And this ſhall ſuffice now for the Rule of Alligation of mixture: for by theſe examples may you eaſily conjecture ſuch other as do appertain to it, as well for the due working, as for variety of drawing the lines of Combination.

Scholar. Sir, albeit it pleaſed you erewhile to put me from my muſing at the many varieties that may fall in theſe Combinations, and termed them phantaſies, yet my phantaſie giveth me, that the conſideration of this ſhould in many other examples and caſes of importance be very needfull, and the knowledge of it moſt profitable: Therefore ye may well think, that at another time convenient

I

I will request you to aid me herein.

Master. Truth it is, that this consideration may fall in practice as well Politick as Philosophicall, and sundry wayes in them be applied: Therefore when time shall fall fit for the discussing of this consideration, you shall not want my helping hand.

The Rule of Falshood.

The occasion of the name.

NOw *will I briefly also teach you the* Rule of Falshood, *which beareth his name, not for that it teacheth any fraud or falshood, but for that by false numbers taken at all adventures, it teacheth how to find those* true numbers *you seek for.*

Scholar. So might any other Rule be called the Rule of Falshood, for they work by wrong numbers, and by them find out the right numbers: so doth the Rule of Alligation, the Rule of Fellowship, and the Golden Rule, partly.

Mast. In the Golden Rule, the Rule of Fellowship, and the Rule of Alligation, although the numbers that you work by, be not the true numbers that you seek for, yet are they numbers in just proportion, and are found by orderly work, whereas in this Rule the numbers are not taken in any proportion, nor found by orderly work, but taken at all adventures.

And therefore I sometimes being merry with my friends, and talking of such questions, do call unto them such Children or idiots, as hapned to be in the place, and so take their answer, declaring that

that I would make them solve those questions, that seemed so doubtfull.

And indeed I did answer to the question, and work the Triall thereof also by those answers which they happened at all adventures to make: which numbers seeing they be taken as manifest false, therefore is this Rule called the Rule of false Positions, and for briefness, The Rule of Falshood: which Rule for readiness of remembrance, I have comprised in the few verses following, in form of an obscure Riddle.

Ghess at this work as hap doth lead,
By chance to truth you may proceed,
And first work by the question,
Although no truth therein be done.
Such falshood is so good a ground,
That truth by it will soon be found,
From many bate too many moe,
From too few take too few also:
With too much joyn too few again:
To too few adde too many plain:
In cross wise multiply contrary kind,
And all truth by falshood for to find.

The sense of these Verses, and the summe of this Rule is this.

When any *question is proposed* appertaining to this *Rule*, first imagine any number that you list, which you shall name the *first position*, and put it in stead of the true number, and then work with it as the *question* importeth: and if you have missed, then is the last number of that work either too great or too little: [that shall you note as hereafter shall be taught The exposition of the Rule.

taught you, and you ſhall call it the *firſt errour*.

Then begin again, and take another number, which ſhall be called the *ſecond poſition*, and work by the *queſtion*: if you have miſſed again, note the exceſs or default as it is, and call that the *ſecond errour*. Then multiply croſs-wiſe the *firſt poſition* by the *ſecond errour*, and again, the *ſecond poſition* by the *firſt errour*, and note their totalls ſeverally by the names of totalls: Then marke whether the two errours were both alike, that is to ſay, both too much, or both too little: or whether they be unlike, that is, the one too much, and the other too little: for if they be like, then ſhall you *ſubtract* the one totall from the other (I mean the leſſer from the greater) and the remainer ſhall be your *dividend*: ſo muſt you abate the leſſer errour out of the greater, and the reſidue ſhall be the *diviſor*. Now divide the *dividend* by that *diviſor*, and the *quotient* will ſhew you the true number that you ſeek for. But and if the errours be unlike, then muſt you adde both thoſe totalls (which you noted) together, and take that whole number for the *dividend*, ſo ſhall you adde both errours together, and that whole number ſhall be the *diviſor*, and the *quotient* of that diviſion ſhall give you the true number that the *queſtion* ſeeketh for, and this is the whole Rule.

Scholar. This Rule ſeemeth ſo unlike any other, that without ſome example I ſhall not eaſily underſtand it.

Maſter. With a good will: propoſe half a ſcore ſundry queſtions and examples of variety, for the better underſtanding of the work hereof: and for the firſt take this example. *A maſon*

was

was bound to build a wall in 40 *dayes, and it was covenanted so with him, that every day that he wrought, he should have for his wages* 2 shillings 1 penny, *and every day that he wrought not, he should be amerced* 2 shillings 6 pence, *so that when the wall was made and the reckoning taken of the dayes that he wrought, and of the other that he wrought not, the Mason had clearly but* five shillings five pence for the work. *Now doe I demand how many dayes he did work of those* 40, *and how many he did not work.*

A question of Masonry, the first example.

Scholar. I pray you expresse the order of the work, that I may partly by imitation, and partly by comparing it with the Rule, be able again to doe the like.

Master. This order shall you keep in the work of this Rule: first take some number (as you list) at adventure; as for example, I say he played 12 dayes, and wrought 28 dayes. Now cast you the wages of every day and see whether it will agree with the summe of 5 shillings 5 pence.

Scholar. The 28 dayes that he wrought after 25 pence the day, yieldeth 700 pence: Then 12 dayes that he wrought not, at 30 pence each day, doth amount to 360 pence, which if I abate out of 700 pence, there resteth 340: but you say he had not so much.

Master. He had but 65 pence, and by this supposition he should have had 340: therefore is this summe too much by 275, which summe I must set down after this sort as you see here, where first I have made a cross (commonly called S. Andrews cross) and

12

X

275 †

at

at the over corner of the left hand I have set the first position 12: and at the other corner under it I have set 275 which is the first errour, with this figure †, which betokeneth too much, as this line ——— plaine without a cross line betokeneth too little.

On the right hand of the cross I have left two like roomes for the second position and his errour. Therefore to prosecute the work, I suppose he played 16 dayes, and wrought 24.

Scholar. I was a while in doubt why you named the dayes of his working, seeing they be not set in the figure: and I doubted how you knew them, or else whether you did suppose them at all adventures, as you did the dayes that he played: but now I gather, that seeing 40 dayes is the whole time limited, then the dayes that he played being supposed, the rest of 40 must needs be the dayes that he wrought, and therefore 28 followed 12 of necessity, and 24 followeth 16 also of necessity, but yet I scarce perceive why you set not in the figures as well 28 as 12.

Master. It forceth not which of them I take, so that in the second position I take the numbers of the same nature that is here both of working dayes, or both of idle, but now examine you this second position.

Scholar. If he played 16 dayes, then abating 16 times 30 pence, the sum will be 480 pence, and for 24 dayes that he wrought, every day yielding 25 pence, the totall is 600 pence: so that abating 480 out of 600, there resteth 120, and as you say, it should be but 65: therefore it is too much by 55: that

that must be set on the right hand of the figure, at the neather part, and over it on the same side 16, which is the second position, thus.

12 ╳ 16
275† 55†

And as I gather by your words, it were all one if I did set 28 in stead of 12, and 24 in stead of 16.

Master. So were it. But this shall you mark, that of what nature soever the two positions be, of the same nature is the quotient. Therefore when the positions in this question are 12 and 16, which both being numbers of the playing dayes, the quotient shall declare the true number of playing dayes: whereas if the positions had been 28 and 24, which are supposed to be the working dayes, then would the quotient declare the true number of the working dayes, and not of playing dayes, as it will doe now. And therefore to continue the work of this question, and to finde the true number of playing dayes, I most multiply crosse-wise the first position by 55 that is the second errour, and the totall will be 660. Then I multiply 275 by 16, and it yeildeth 4400. Now because the errours are alike, that is to say, both too much, I must subtract 660 out of 4400, and so remaineth 3740, which is the dividend. Againe, I must subtract the lesser errour 55 out of 275, that is the greater errour, and there will remaine 220 which will be the divisor: then dividing 3740 by 220 the quotient will be 17. Wherefore I say now constantly, that 17 is the true number of dayes that the Mason played: and then it followeth that he wrought 23 dayes: and so is the question answered.

Now for the order of triall of this work, there needeth none other triall but onely this, to work with this The proof of this rule

at the over corner of the left hand I have set the first position 12: and at the other corner under it I have set 275 which is the first errour, with this figure †, which betokeneth too much, as this line ——— plaine without a cross line betokeneth too little.

On the right hand of the cross I have left two like roomes for the second position and his errour. Therefore to prosecute the work, I suppose he played 16 dayes, and wrought 24.

Scholar. I was a while in doubt why you named the dayes of his working, seeing they be not set in the figure: and I doubted how you knew them, or else whether you did suppose them at all adventures, as you did the dayes that he played: but now I gather, that seeing 40 dayes is the whole time limited, then the dayes that he played being supposed, the rest of 40 must needs be the dayes that he wrought, and therefore 28 followed 12 of necessity, and 24 followeth 16 also of necessity, but yet I scarce perceive why you set not in the figures as well 28 as 12.

Master. It forceth not which of them I take, so that in the second position I take the numbers of the same nature that is here both of working dayes, or both of idle, but now examine you this second position.

Scholar. If he played 16 dayes, then abating 16 times 30 pence, the sum will be 480 pence, and for 24 dayes that he wrought, every day yielding 25 pence, the totall is 600 pence: so that abating 480 out of 600, there resteth 120, and as you say, it should be but 65: therefore it is too much by 55: that

that must be set on the right hand of the figure, at the neather part, and over it on the same side 16, which is the second position, thus. 12 16

And as I gather by your words, it were all one if I did set 28 in stead of 12, and 24 in stead of 16. 275†55†

Master. So were it. But this shall you mark, that of what nature soever the two positions be, of the same nature is the quotient. Therefore when the positions in this question are 12 and 16, which both being numbers of the playing dayes, the quotient shall declare the true number of playing dayes: whereas if the positions had been 28 and 24, which are supposed to be the working dayes, then would the quotient declare the true number of the working dayes, and not of playing dayes, as it will doe now. And therefore to continue the work of this question, and to finde the true number of playing dayes, I most multiply crosse-wise the first position by 55 that is the second errour, and the totall will be 660. Then I multiply 275 by 16, and it yeildeth 4400. Now because the errours are alike, that is to say, both too much, I must subtract 660 out of 4400, and so remaineth 3740, which is the dividend. Againe, I must subtract the lesser errour 55 out of 275, that is the greater errour, and there will remaine 220 which will be the divisor: then dividing 3740 by 220 the quotient will be 17. Wherefore I say now constantly, that 17 is the true number of dayes that the Mason played: and then it followeth that he wrought 23 dayes: and so is the question answered.

Now for the order of triall of this work, there needeth none other triall but onely this, to work with this The proof of this rule

number according to the question, and if it agree, then appeareth the number to be that you would have.

And here now seeing he wrought 23 dayes, and must have for every day 25 pence the whole summe commeth to 575. Then again, seeing he played 17 dayes and must abate 30 pence for every day, the whole summe of the abatement will be 510. Therefore I subtract 510 out of 575, and there will remain 65, which maketh 5 shillings 5 pence, the cleere wages of the Mason for his work according to the question.

Scholar. Now I trust I understand the work and the Rule so well (and the better by this proof) that I can be able to doe the like: And for a proof, I take the same question, all save the last number, where I will suppose that he had 10 shillings for his wages cleare. And now to guesse of the number of the dayes he wrought, I suppose first that he wrought 20 dayes: then say I if he wrought 20 dayes, his wages must be 500 d. then did he play other 20 dayes, for which must be abated 600 d. and then he looseth 100 d. And so am I at a stay, for it is not like to your former worke.

Master. You should have required of me some question, and not have taken a question of your owne phantasying, untill you were more expert in this Art, for so might you as well happen to an impossible question, as on a possible: but now to go forward, consider that this number is too little by 220, seeing he should gain by your supposition 120 pence, and in this position he looseth 100, those both make 220, which you shall set down for the first errour, with this signe—, betokening too little

as

as here in this forme following doth ap- appeare.

20

—220

And now for the rest go forward your selfe once againe.

Scholar. As my errour hath uttered my folly, so it hath procured me better understanding.

Now therefore considering this position not to solve the question, I take another, supposing that he wrought 30 dayes. Then for his wages he must be allowed 750 pence, and for the 10 dayes which he wrought not, he must abate 300 pence, and so remaineth cleere 450 pence, but it should be onely 120 pence, therefore it is too much by 330, which I set down in the figure with the former position and his errour, and the figure appeareth thus:

20 30

—220-- 330+

Now first, I multiply in cross wayes 20 by 330, and it will be 6600.

Then again I multiply 30 by 220, and it will be also 6600. Wherefore if I shall subtract the one out of the other, there will remain nothing to be the Dividend.

Master. In this you forget your selfe again; for in as much as the signes in the errours be unlike, therefore must you work by Addition, adding together those two totalls to make the Dividend, and also adding the two errours to make the Divisor. And because you shall no more forget this part of that Rule, take this brief remembrance.

Unlike require Addition;
And like desire Subtraction.

Scholar. You mean, that if the errours have like signes, then must the Dividend, and the Divisor

bee made by Subtraction, as is taught before; And if those signes bee unlike (as in this last example they bee) then must I by Addition gather the dividend and the divisor. Therefore must I adde 6600 to 6600, and it will bee 13200, which will be the dividend. Then againe I adde 220 to 330 and it will be 550, which must be the divisor: wherefore dividing 13200 by 550, the quotient will be 24, whereby I know that the Mason wrought 24 dayes, and then it followed, that he played 16 dayes.

Master. Examine your work, whether it be agreeable to the question or no.

Scholar. For 24 dayes work the wages must be 600 pence, and for 16 dayes which the Mason wrought not, there must be abated 480 pence, and then remaineth cleare to the Mason 120, as the question importeth: wherefore it is evident that 24 is the true number of dayes that he wrought.

Master. Although you seem now to understand this work, yet to acquaint your mind the better with the new Trade of this Rule, I think it good to propose to you 5 or 6 examples more before I make an end of it.

Scholar. Sir, I thank you that you doe so consider my commodity and profit in knowledg, for undoubtedly it is practice and exercise that maketh men prompt & expert in every kind of knowledge.

Master. You say well, so that they follow some certain precepts to govern and rule their practice by, else may practice procure custome of errour, and a repugnance to exactness of knowledge: namely, as long as the errour is not plainly known to the vulgar sort. But returne to your work.

There

A question of wares, the second example.

There is a servant that hath bought of Velvet and Damask for his Master 40 yards, the Velvet at 20 shillings *a yard, and the Damask at* 12 shillings, *and when he cometh home, his Master demandeth of him, how much he hath bought of each sort: I cannot tell (saith he) exactly: but this I know that I paid for Damask* 48 shillings *more then I paid for Velvet: now must you guesse how many yards there is of each sort.*

Scholar. Although the guesse seemeth difficult, yet I will prove what I can doe: for I remember your saying, that it forceth not how fond or false the guesse bee, so it be somewhat to the question, and not an answer of a contrary matter.

Therefore first I imagine that he bought 20 yards of Damask, for which he should pay after the former price 240 shillings: then must he needs have of Velvet other 20 yards, (to make up the 40 yards) and that would cost 400 shillings. So that the totall of the price of the Damask is less then the summe paid for Velvet 160 shillings, and should bee more by 48. Therefore the first errour is 208 too little. Then begin I againe, and suppose he bought of Damask 30 yards, that cost 360 shillings, then had he but 10 yards of Velvet, which cost 200 shillings: and now the price of the Damask is greater then the price of the Velvet by 160 shilliogs, and should be but 48, therefore is the second errour 112 too much, which I set in forme of figure, as here doth appeare. Then doe I multiply in cross wayes 208 by 30, and the summe will be 6240. Also I multiply 112 by 20, and there will amount 2240. And in as much

20 30

208 — 112†

much as the signes of the errours be unlike, I know I must work by Addition, therefore adde I these two totalls together, and they make 8480, which is the Dividend: then adde I also the two errours together, 208, and 112, and they make 320, which is the Divisor: wherefore dividing 8480 by 320, the quotient will be 26 ½. which is the true summe of yards of Damask that he bought, and in Velvet 13 yards ½, and that appeareth by examination, thus: 26 ½ yards of Damask at 12 shillings the yard maketh 318 shillings: then in Velvet he had but 13 yards and ½ and cost 270 shillings, at 20 shillings the yard. Now subtract 270 out of 318, and there will remain 48, which is the number of shillings that the Damask did cost more then the Velvet.

Master. Now shall you have a question of another kind.

A question of debt, the third example.

There are three men that doe owe mony to me, and I have forgotten what the totall summe is, and what the particulars be.

Scholar. Why, then it is impossible to know the debt.

Master. Peace, you are too hasty, there is more help in it then yet you see, I have three severall notes, whereby it appeareth that I did conferre their debts together, and found the debt of the first and the second to amount to 47 pound, the debt of the first man and the third man did make 71 pound, and the second man his debt with the third, did rise to 88 pound. Now can you tell what every man did owe, and what was the whole summe?

Scholar. Nay, in good faith, but as I perceive that it must be found by conjecture, so will I guesse

at

at it, ſuppoſing that the firſt man did owe 20 pound and the ſecond man 30, and the third——

Maſter. Nay ſtay there you are too farre gone already: you may not ſuppoſe a ſeverall ſumme for every man, for it is enough to ſuppoſe one ſumme for the firſt man, and let the other riſe as the queſtion importeth. Therefore ſeeing you ſet the firſt man his debt to bee 20 pound, the ſecond man cannot owe 30 pound, for the declaration is, that their debts added together did make 47 pound, ſo muſt the ſecond man his debt be but 27 pound: Now the ſecond debt with the third muſt make 88: therefore ſubtract 27 out of 88, and there will remain 61, as the third man his debt. Then ſaith the declaration, that the firſt and third mans debts doe make 71: but by this ſuppoſition they make 81, that is 10 too much, which I muſt ſet for the firſt errour. Now worke you the ſecond poſition.

Scholar. I ſuppoſe the firſt mans debt to be 24 pound: then muſt the ſecond mans debt (by your declaration) be but 23 poumd, ſeeing both they make but 47 pound. And the ſecond man his debt with the third, doe make 88 pound, and the ſecond man oweth but 23: therefore the third man muſt owe 65 pound. Now the third mans debt with the firſt, ſhould make by the declaration 71 pound, and they doe make 89 pound, that is 18 pound too much, and that is the ſecond errour, which I ſet down with the firſt, and their poſitions in this forme, and then I doe multiply in croſs wayes 20 by 18, and it is 360. And 10 by 24 maketh 240. Alſo becauſe the ſignes of the errours be

20	24
10†	18†

P4 like,

like, I must work by Subtraction: therefore I subtract 240 out of 360, and there resteth 120, which is the Dividend; then doe I subtract 10 out of 18 by the same reason, and so is the Divisor 8, which is found 15 times in 120: therefore I say that the first man did owe 15 li. and then the second man must owe 32 li. for those two doe make 47 li. and the third mans debt is 56: for so much remaineth if I abate, 15 out of 71, or if I take 32 out of 88.

The fourth example.

Master. *For the fourth example, take this easie question for the variety in work. Two men having severall summes, which I know not, doe thus talke together: the first saith to the second, if you give me two* shillings *of your mony, then shall I have three times so much mony as you. The second man answereth, It were more reason that our summes were made equall, and so will it be if you give mee* 3 shil-lings *of your mony. Now guesse what each of them had.*

Scholar. I imagine that the first had 9 s.

Note.

Master. Consider evermore in your imagination that you take a likely summe, as in this question, take such a summe, that having 2 added unto it, may be divided into three parts even.

Scholar. Why? I remember you said before, it forceth not how fondly soever I guessed.

Master. As for the possibility of the solution, it is truth: but for easinesse in worke, the aptest numbers are most convenient.

Scholar. I thought no less, and therefore I took 9 as an apt number to be parted into three: but I perceive I should have considered the aptnesse of that partition after the Addition of two unto

unto it, and then 7 had been more meet.

Master. That is truth, and then should the second man his summe be 5: for although he have now but the third part of 9, that is 3, yet you must remember that he lent the first man 2, & so had he 5.

Scholar. Then to go forward: if the second man had three of the first man, then should he have 8 and the first man but 4: so hath he double to the first man, yet he said in the question they should have equall: wherefore it appeareth that he hath 4 too much.

Therefore I note that errour with his supposition, and guesse again that he hath 10 shillings: whereunto I adde 2 shillings borrowed of the second man, and then he hath 12 shillings, so the second man hath remaining but 4, whereunto if I adde the 2 that he lent to the first man, so had he but 6 shillings, at the begining.

Then take 3 shillings from the first man, and give to the second, and then hath the first man but 7, and the second hath 9, which are not equall, but there are 2 too many, wherefore I set down both the positions with their errours, as before you see, and multiply a cross, so cometh there 40 and 14: and because the signes be like, I take 14 out of 40, and so resteth 26 to be divided: then likewise I take 2, out of 4, and there resteth 2, by which I divide 26, and the quotient will be 13, which is the summe that the first man had. And so appeareth that 2 being added thereto, the summe will be 15, so hath the second man but 5, and before he had 7: then take 3 from the first, and put to this 7, and so have

7 10

4† 2†

each

each of them 10, and that is equall as the queſtion would.

Maſter. *For the fifth example take this queſtion. One man ſaid to another, I think you had this yeare two thouſand Lambs: ſo had I ſaid the other; but what with paying of tythe of them, and then the ſeverall loſſes, they are much abated: for at one time I loſt halfe as many as I have now left, and at another time the third part of ſo many, and the third time* $\frac{1}{4}$ *ſo many. Now gueſſe you how many are left.*

The fifth example: a qeuſtion of Lamebs.

Scholar. Becauſe here is mention made of certaine parts, I muſt take a number that may have all theſe parts, that is to ſay, $\frac{1}{2}$ $\frac{1}{3}$ and $\frac{1}{4}$ which will be 24, howbeit 12 hath the ſame parts. Therefore I take firſt 12 to be the number that doth remain, ſo hath he loſt 6, 4, and 3 that is 13, and the whole 25, but it ſhould be 2000.

Maſter. You are deceived yet ſtill, you have forgotten the 10 part, which muſt be defalked, that is 200, ſo there remaineth but 1800, and now go on again.

Scholar. Then to find the errour, I take 25 out of 1800, and there remaineth 1775 too few, which I ſet for the errour. Then for the ſecond poſition I take 24, whoſe halfe is 12, the third part 8, and the quarter 6, whereby riſeth 50, which is too little by 1750 therefore I ſet down both the poſitions, with their errours thus:

12	24
—1775	2750—

And multiply in croſs wayes 1775 by 24, whereof cometh 42600. Alſo I multiply 1750 by 12, and there ariſeth 21000. And becauſe the ſignes are like

like, I doe subtract the one from the other, and so remaineth the Dividend 21600. Then doe I subtract 1750 out of 1775, and there resteth 25, by which I divide 21600, and the quotient is 864, whereof the halfe 432, and the third part is 288, the quarter is 216; which all being added together, will make 1800: And if you adde thereto the tenth which was abated before, then will the the whole summe be 2000.

864
432
288
216
1800

And now doth there come a question to my memory which was demanded of me, but I was not able to answer to it: And now me thinketh I could solve it.

Master. Propose your question.

A question of sheep and tillage: the sixth example.

Scholar. *There is supposed a Law made, that (for furthering of tillage) every man that doth keep sheep, shall for every ten sheep ear and sow one Acre of ground: and for his allowance in sheep pasture, there is appointed for every foure sheep one Acre of pasture. Now is there a rich Sheep-master which hath 7000 Acres of ground and would gladly keep as many as he might: by that Statute I demand how many sheep he shall keep.*

Master. Answer to the question your selfe.

Scholar. First, I suppose he may keep 500 sheep, and for them he shall have in Pasture after the rate of four sheep to an Acre, 125 Acres, and in Arable ground 50 Acres, that is, 175 in all: but this errour is too little by 6825. Therefore I guesse againe that he may keep 1000 sheep, that is, in Pasture 250 Acres, and in tillage 100 Acres, which make 350, that is too little by 6650, Both these errours with their positions, I set down as

you

you see, and multiply them cross 6825 by 1000, and it maketh 6825000, also I 500 1000 multiply 6650 by 500, and there cometh 3325000, which summe I subtract out of the former, 6825—6650— and there remaineth 3500000 for the Dividend: likewise I subtract the lesser errour out of the greater, and there resteth 175, by which I divide 3500000 (the Dividend aforesaid) and the quotient will be 20000, so that by this rate he that hath 7000 Acres of ground, may keep 20000 sheep.

Another way of working.

Master. You have done well, notwithstanding both this last question, and the next before might bee wrought without the second position by the Rule of proportion, as thus: When in this question you found in the first errour that for 500 sheep there must be 175 Acres, then might you reduce it to the Golden Rule, thus:

175	500
7000	20000

If 175 Acres will admit in allowance 500 sheep then 7000 will have 20000. And so by one position, with the help of the Golden Rule may you answer that question.

Likewise for the question of Lambs, when you had found that 12 came of 25, you might have set the figure as followeth, and have said:

25	12
1800	864

If 25 doe leave but 12, what shall 1800 leave? and it would appeare to be 864.

Scholar. Sir, I thank you for this aide, for it doth much shorten the work of this Rule.

Another way yet.

Master. Yet againe I will shew you another way to answer to this last question without the Rule of

of falſe poſition, and that by the Rule of Fellowſhip, for it appeareth in the propoſing of the queſtion, that ten ſheep muſt have in paſture 2 Acres and $\frac{1}{2}$, and for them muſt there be eared but one Acre; ſo it followeth, that for 2 Acres eared, there muſt bee 5 ſet to paſture: and if you put them both into one ſumme, they will make 7. Therefore look what portion 7 being this totall, doth bear to 5 and to 2, ſuch proportion ſhall any totall in this queſtion beare to the paſture ground, and the eared ground.

Scholar. This ſerveth wondrous aptly. Therefore to prove it, I demand this by the former ſuppoſition: If a man have 300 Acres, how much ſhall he leave in paſture, and how much ſhall he turn to tillage? You ſay, that as 7 is to 5, ſo ſhall 300 be to the Acres of paſture: and as 7 is to 2, ſo is 300 to the Acres of tillage, whereof for both I have ſet examples here following, whereby appeareth that of paſture, there ſhall be 214 $\frac{2}{7}$ Acres, and of Tillage 85 $\frac{5}{7}$, which both ſums added together, doe make 300.

7 Z 5
300 Z 214 $\frac{2}{7}$

7 Z 2
300 Z 85 $\frac{5}{7}$

Maſter. *Now take another Example: A man hath three ſilver Cups with one Cover, the Cover weigheth 18 ounces, the ſecond Cup weigheth even half the weight of the firſt and the third. Now if the Cover be put to the firſt Cup, they weigh juſt as much as all the three Cups do weigh: and if the Cover be joyned with the ſecond Cup, they weigh as much as the ſecond twice, and the third: and if the Cover be put to the third Cup they will make twice as much as the firſt and ſecond Cup. Now try you what was the juſt weight of every Cup.*

Another queſtion: the ſeventh example.

Scholar

Scholar. I doe ſet the weight of the firſt Cup to be nine Ounces, then in as much as theſe two (that is to ſay, the cover and the firſt Cup) doe weigh the weight of the three Cups, I ſee that the three Cups muſt weigh 27 ounces, for ſo much is 18 and 9. Alſo becauſe the firſt and the third doe weigh double ſo much as the ſecond, therefore it is the third part of that weight, that is 9, and then would it follow, that the third Cup alſo ſhould weigh 9 ounces; but then the queſtion ſaith that the Cover being joyned to the ſecond Cup, they weigh as much as the ſecond twice, and the third once, that ſhould be 27, and ſo it doth; that being joyned with the third Cup, they ſhould weigh twice as much as the firſt and the ſecond, that ſhould be 36, and they weigh but 27, ſo is that errour 9 too little. Then beginne I again, and ſay, that the firſt Cup doth weigh twelve ounces which I joyne with the Cover, and they make thirty ounces: then ſeeing the ſecond is of that weight, it muſt needs weigh ten ounces, and the third muſt weigh 8 ounces, ſeeing the firſt and the third muſt weigh 20 ounces. Now put I the Cover to the ſecond Cup, and they weigh 28 ounces, which ſhould be even ſo: then joyne I the cover with the third Cup, & ſo ſhould it weigh twice the firſt and the ſecond, that is 44 ounces, and they weigh but 26, that is 18 too little: thoſe errours with their poſitions I ſet down, and multiply in croſſe wayes 9 by 12, whereof cometh 108: Alſo 9 by 18, and that yeildeth 162: and in as much as the ſignes be like, I abate the leſſer out of the greater, and there doth

9 12

9— 18—

remain 54. Then doe I also abate the lesser errour from the greater, and so remaineth 9, by which I divide 54, and the quotient is 6, which I take for the true weight of the first Cup, which being joyned with the Cover, must weigh as much as the three Cups, so doe they weigh but 24 ounces. Then seeing the second Cup is the third part of that weight, for the other two Cups (you say) must weigh double his weight, the weight of the second Cup is 8 ounces, and so the weight of the third Cup must be 10 ounces. Now put the Cover to the second Cup, and it will make 26 ounces: that must be the weight of the second twice, and the third once, that is, twice 8, and once 10, and so is it. Againe put the Cover to the third Cup of 10 ounces, and they must weigh twice as much as the first and the second, that is, 28, and so is all agreeable.

Master. Then answer to this Question.

There is n Cisterne with four Cocks, containing 72 barrels of water: and if the greatest Cock be opened, the water will avoid clean in six hours; at the second Cock it will ask eight houres: at the third Cock it will avoid in no less then nine houres: and at the smallest it will require twelve houres. Now I demand in what space will it avoid, all the Cocks being set open? A question of water: the eight example

Scholar. First I imagine it will avoid in two houres.

Master. Then must there avoid by the first Cock $\frac{1}{3}$ of the water, that is 24 Barrels, and by the second Cock $\frac{1}{4}$, that is 18, and by the third Cock $\frac{2}{9}$, that is 16 Barrels, and by the smallest Cock $\frac{1}{6}$,

that

that is 12 Barrels, all which ſummes put together, do make 70, as by their Addition it doth appear, but it ſhould be 72, therefore the errour is too few.

Scholar. Then will I begin again by 24
your favour, becauſe I think I underſtand the 18
work, and put three houres for the due time: 16
ſo ſhall there run out at the greateſt Cock $\frac{1}{2}$, 12
that is, 36 Barrels, and at the ſecond hole $\frac{3}{8}$, —
that is 27, and at the third Cock $\frac{1}{3}$, that is 70
24, and at the ſmalleſt hole $\frac{1}{4}$, that is 18 Barrels, which all together do make 105, and ſhould be but 72, ſo is it too much by 33: therefore do I ſet the errours in order of the figure with their poſitions, and work by multiplication in croſs, ſaying, 2 times 3 is 6, and 2 times 33 make 66, and becauſe the ſignes are unlike, I muſt adde theſe 2 totalls together, which make 72: alſo I adde the two errours, and they make 35, by which I divide 72, and the Quotient riſeth 2 $\frac{2}{35}$, whereby I ſee that all the Cocks being ſet open, the water will avoid in two houres, and $\frac{2}{35}$ of an hour.

2 3

2---33 †

Maſter. This excercise maketh you to grow expert in the Rule. Therefore I will enure you ſomewhat more with a queſtion or two.

A queſtion of partners. The ninth example.

There were two men that had been partners, and had in account between them 300 *Duckets; whereof the one ſhould have for his part* 180, *and the other* 120*: but in the parting of them, they fell at variance; ſo that each of them catched as many as he could: yet afterward being reconciled they agreed that he which hath gotten moſt part of them, ſhould lay down* $\frac{3}{4}$ *of them again, and he that had gotten leaſt, ſhould lay down* $\frac{1}{3}$ *of thoſe*

which

which he had taken, and then parting them into two equall parts, each man to have halfe thereof, and so had they their just portions as they ought to have: now I demand of you what each of them had gotten by the scrambling.

Scholar. I suppose he that had least, got 108 Duckets, then the other had 192: wherefore in laying down again of the 192, there was put down $\frac{3}{4}$, that is, 144, and so had he left but 48. Also of the 108, there was laid down 36, that is $\frac{1}{3}$, and so he had left 72. Then I put together 144 and 36, and it maketh 180, which I part into two parts even, and so commeth 90 to be given to each of them: which summe put to 72 maketh 162; and joyned to 148, it maketh 238: and now I doubt how I shall go forward.

Master. You need not to take but one of them, which you list, the greater or the smaller, for all commeth to one purpose: and so may you compare it that you take, to any of the other summes, remembring that you make comparison to the same in the second work: as for example of the first part. If you compare 138 with the lesser summe one, that is, 120, so is it 18 too much, and if you compare it with the greater summe, then is it 42 too little. Again, if you compare 162 to the greater summe, the errour will be 18, as it was in the other: but it will have a contrary signe: and if you compare it with the lesser summe, it will be 42 too mnch: so that the errour both wayes is either 18 or 42: and as for the signes it little forceth, for in them is nothing considered here, but likeness and unlikeness, which in this case doth neither Note.

further or hinder: But now go on with the work.

Scholar. If it be so, then am I out of my greatest doubt. Then I joyne that 90 (which I found as the half of the latter partition) unto 48, which is left with the one man, and so hath he 138 which (I may say) is 18 too many, for the least should be but 120 that errour doe I note, and then make a new position, supposing the one man to have 204, and the other to have 96, wherefore of the 204, there must be laid down 153, and so remaineth with him 51. Also of the 96, there must be laid down $\frac{1}{3}$, that is 32, and so resteth with that man 64: Now of the 153 and 32, I make one summe, as 185, which I must divide into two equall parts, and so each man shall have 92 $\frac{1}{2}$, whereunto if I adde their former portions reserved, then the one shall have 156 $\frac{1}{2}$, and the other hath 143 $\frac{1}{2}$, Wherefore take the lesser summe now againe, as I did before, that is, 143 $\frac{1}{2}$ and I find that he hath too many by 23 $\frac{1}{2}$ for he should have but 120, and so have I for my two positions two errors, which I set down as here may be seen, each error under his position, and then by the Rule I doe multiply in cross wayes 108 by 23 $\frac{1}{2}$, and there riseth 2538, which I note, then again I multiply 96 by 18, and thereof amounteth 1728.

108		96
	X	
18 †		23 $\frac{1}{2}$ †

Now because the signes are both like, that is, both too many, I must work by Subtraction, and so abating 1728, out of 2538, there will rest for the Dividend 810: then for the Divisor I subtract 18 out of 23 $\frac{1}{2}$, and there remaineth

eth $5\frac{1}{2}$, by which I divide 810, and the quotient will be $147\frac{3}{11}$, which is the just portion of him that had the least summe. And if I doe subtract it out of 300, being the totall summe, then will there remain $152\frac{8}{11}$, as the portion that the other did get.

Master. For the proof of this work, you may chuse whether you will examine those numbers according to the forme of the questions or else work by other two positions, for to find the second number: and if those positions bring the same numbers that did amount by the first two positions, then doth each work confirm other.

Scholar. By your patience, I will prove both wayes, not onely to seek their agreement but also to accustome my mind to those works: for I perceive it is exercise that must be the chief engraver of these Rules in my memory.

Master. You consider it well: then go to.

Scholar. First, I will by two other positions try to find the portion of him which had most.

Master. Although you may doe it with any positions, yet to see the agreement of your work the better, take the same positions that you did before, comparing them now to the greater, as you did before unto the lesser.

Scholar. Then I suppose that he that had most, had 192, so had the other 108. Now if I take $\frac{3}{4}$ out of 192, that will be 144 and there will rest to that man but 48. And from the second which had 108, if I take $\frac{1}{3}$, that is 36, there will remain to him 72: then joyning 144 with 36, it will make 108, the half whereof being 90, if I adde to

each of thoſe two mens portions remaining with them, the one ſhall have 138, and the other 162, of which two I take the greater (that is 162) and ſee it to be 18 too few; for it ſhould be 180, that errour I note under this poſition. Then for the ſecond poſition I take (as I did before) 204 for the one, and ſo reſteth 96 for the other: then take I $\frac{3}{4}$ of 204 and it will be 153, and there reſteth to him 51. Alſo of the 96 I take $\frac{1}{3}$ that is, 32, and there remaineth to him 64; now put I that 32 to 153, and it yieldeth 185: which being parted in equall values, maketh $92\frac{1}{2}$ to be added to each mans remainder, and ſo the one hath $143\frac{1}{2}$, and the other $156\frac{1}{2}$: wherefore I take the greateſt ſumme, and it is $23\frac{1}{2}$: too little, that do I note alſo, and ſet both theſe errours under their poſitions, as in this Example following doth appear.

And then multiplying 192 by $23\frac{1}{2}$, there doth ariſe, 4512.

Again, I multiply 204 by 18, and it maketh 3672, which I doe ſubtract out of 4512, becauſe the ſignes be like, and there reſteth 840 for the dividend, then ſubtracting 18 out of $28\frac{1}{2}$ there will remain $5\frac{1}{2}$, which I muſt take for the Diviſor. And ſo dividing 840 by $5\frac{1}{2}$ the quotient will be $152\frac{8}{11}$, whereby I have found an agreeable ſumme to that which I found by the former poſitions, for him that had moſt, which I doe ſubtract out of 300, that is the totall, there will reſt $147\frac{3}{11}$, which was the portion of him that had the leaſt part.

192	204
— 18	— $23\frac{1}{2}$

Maſter. So by divers poſitions, you ſee, that one doth

doth confirme the work of the other. Now examine those two numbers by the forme of the question, and so shall you prove your work good also.

Scholar. If that he which gat most had 152 $\frac{8}{11}$, then must he lay down $\frac{3}{4}$ of this summe, that is 114 $\frac{6}{11}$, and so shall remain with him but onely 38 $\frac{2}{11}$. The other which had least, that is 147 $\frac{3}{11}$, must put down of his summe $\frac{1}{3}$, that is 49 $\frac{1}{11}$, and so doth there remain with him yet 98 $\frac{2}{11}$. Then do I adde together 114 $\frac{6}{11}$, and 49 $\frac{1}{11}$, and it will make 163 $\frac{7}{11}$ which I must part into equall parts, and that will be 81 $\frac{9}{11}$ to be given to each of them: putting 81 $\frac{9}{11}$ unto 38 $\frac{2}{11}$, there doth amount 120 just, which is the true Portion of him that should have the lesser summe: and adding 81 $\frac{9}{11}$, to 98 $\frac{2}{11}$, the totall will be 180, the true portion of the other. And so is the work by this proof also tryed to be good. And this I marke by the way, that in their scrambling, he got most (as it chanceth often) that ought to have had least by just partition.

Master. Let your study be to learn truth and just Art of proportion, and to distribute and part according thereunto, as often as occasion shall be ministred. And here would I make an end of this Rule, save that I remember one pleasant question which I cannot overpass, which I will declare somewhat largely, because you shall as well understand some reason in the pleasant invention, as apt proceeding in the witty working thereof.

The tenth example, of Gold and Silver.

Hiero *King of the Syracusans in Sicilia had caused to be made a Crown of Gold of a wonderfull weight, to be offered for his good succes in wars: in making whereof the Goldsmith fraudulently took out a certain portion*

Portion of Gold, and put in Silver for it, so that there was nothing abated of the full weight, although there was much of the value diminished.

Which thing at length being uttered (as no evill can alwayes lie hid) the King was sore moved: and being desirous to know the truth without breaking of the Crown, proposed the doubt to Archimedes, unto whose wit nothing seemed unpossible, which although presently he could not answer unto, yet he had good hope to devise some policy for that invention, and so musing thereon, as he chanced to enter into a Baln full of water to wash him, he observed, that as his body entred into the Baln, the water did runne over the Tub, whereby his ready wit of such small effects conjecturing greater works, conceived by and by a reason of solution to the Kings question, and therefore rejoycing exceedingly, more then if he had gotten the Crown it self, forgat that he was naked, and so ranne home, crying, as hee ranne, ἕυρηκα, ἕυρηκα, I have found, I have found. And thereupon caused two massie pieces, one of Gold, and another of Silver, to be prepared, of the same weight that the said Crown was of: and considering that Gold is heavier of nature then Silver, and therefore Gold of like weight with Silver, must needs occupie less roome, by reason it is more compact and sound in substance, he was assured that putting the masse of Gold into a vessell brimfull of water, there would not so much water runne out, as when he should put in the silver masse of the like weight. Wherefore he tried both, and noted not onely the quantities of the water at each time, but

but also the difference or excess of the one above the other, whereby he learned what proportion in quantity is betwéen Gold and Silver of equall weight. And then putting the Crown it self into the vessell of water brim-full (as before) marked how much water did run out then, and comparing it with the water that ranne out when the Gold was put in, noted how much it did excéed that: and likewise comparing it to the water that ranne out of the Silver, marking how much it was less then that, and by those proportions found out the just quantity of Gold that was taken out of the Crown, and how much Silver was put in stead of it: but séeing Vitruvius which writeth this History, doth not declare the particular work of this tryall, it shall be no inconvenience to suppose an example for declaration sake, wherein although the true & just proportion be not expressed, yet the forme of triall shall be truly set forth. And for an example, I suppose the weight of the Crown to be 8 pound, and so of each the other two Masses. And when the Mase of Gold was put into the water, I imagine that there ranne out two pound of water: and when the masse of silver was put in, I suppose there ranne out 3 pound $\frac{1}{2}$. Again, when the Crown was put in, there ran out two pound $\frac{1}{4}$: Now to know what quantity of Silver was in the Crown, work by the Rule of false position, and imagine that there was two pound of Silver, then must there be six pound of Gold, then say thus by the Rule of Proportion. If eight pound of Gold do expell two pound of water, what shall six ponnd expell? and it will be 1 pound $\frac{1}{2}$. Again, for the Silver; if eight pound of Silver

 expell

Portion of Gold, and put in Silver for it, so that there was nothing abated of the full weight, although there was much of the value diminished.

Which thing at length being uttered (as no evill can alwayes lie hid) the King was sore moved: and being desirous to know the truth without breaking of the Crown, proposed the doubt to Archimedes, unto whose wit nothing seemed unpossible, which although presently he could not answer unto, yet he had good hope to devise some policy for that invention, and so musing thereon, as he chanced to enter into a Baln full of water to wash him, he observed, that as his body entred into the Baln, the water did runne over the Tub, whereby his ready wit of such small effects conjecturing greater works, conceived by and by a reason of solution to the Kings question, and therefore rejoycing exceedingly, more then if he had gotten the Crown it self, forgat that he was naked, and so ranne home, crying, as hee ranne, ἕυρηκα, ἕυρηκα, I have found, I have found. And thereupon caused two massie pieces, one of Gold, and another of Silver, to be prepared, of the same weight that the said Crown was of: and considering that Gold is heavier of nature then Silver, and therefore Gold of like weight with Silver, must needs occupie less roome, by reason it is more compact and sound in substance, he was assured that putting the masse of Gold into a vessell brimfull of water, there would not so much water runne out, as when he should put in the silver masse of the like weight. Wherefore he tried both, and noted not onely the quantities of the water at each time, but

but also the difference or excess of the one above the other, whereby he learned what proportion in quantity is betwéen Gold and Silver of equall weight. And then putting the Crown it self into the vessell of water brim-full (as before) marked how much water did run out then, and comparing it with the water that ranne out when the Gold was put in, noted how much it did excéed that: and likewise comparing it to the water that ranne out of the Silver, marking how much it was less then that, and by those proportions found out the just quantity of Gold that was taken out of the Crown, and how much Silver was put in stead of it: but séeing Vitruvius which writeth this History, doth not declare the particular work of this tryall, it shall be no inconvenience to suppose an example for declaration sake, wherein although the true & just proportion be not expressed, yet the forme of triall shall be truly set forth. And for an example, I suppose the weight of the Crown to be 8 pound, and so of each the other two Masses. And when the Mase of Gold was put into the water, I imagine that there ranne out two pound of water: and when the masse of silver was put in, I suppose there ranne out 3 pound $\frac{1}{2}$. Again, when the Crown was put in, there ran out two pound $\frac{1}{4}$: Now to know what quantity of Silver was in the Crown, work by the Rule of false position, and imagine that there was two pound of Silver, then must there be six pound of Gold, then say thus by the Rule of Proportion. If eight pound of Gold do expell two pound of water, what shall six ponnd expell? and it will be 1 pound $\frac{1}{2}$. Again, for the Silver; if eight pound of Silver

expell three pound $\frac{1}{2}$ of water, what shall two pound of silver put out ? it will be $\frac{7}{8}$, now adde those two weights of water together, and they will make two pound $\frac{3}{8}$, and it should be by the supposition two pound $\frac{2}{4}$, so is it too much by $\frac{1}{8}$.

Scholar. Now do I understand the work as I think, therefore I pray you let me work the rest of the question. And because this first supposition did erre, I note that position & his errour, & take a new position, esteeming the Silver to be but one pound, so must there be in Gold 7 pound. Then say I, if eight pound of Gold do yield two pound of water, what shall seven pound yield ? and it will be 1 pound $\frac{3}{4}$. Again, if 8 l. of Silver expell 3 pound $\frac{1}{2}$ of water, what shall 1 pound expell ? and it will be $\frac{7}{16}$. Now must I adde those two summes together, and they make two pound $\frac{3}{16}$, and they should make 2 pound $\frac{1}{4}$, so is it too little by $\frac{1}{16}$. Therefore I set the positions with their errours in order as here followeth: And then I multiply in cross wayes 2 by $\frac{1}{16}$ and it maketh $\frac{1}{8}$: Likewise 1 multiplyed by $\frac{1}{8}$ maketh $\frac{1}{4}$. And because the signes be unlike, I must adde these two summes which make $\frac{1}{8}$: and that is the dividend.

2 1

X

$\frac{1}{8}$ + --- $\frac{1}{16}$

Again, I must adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be $\frac{3}{16}$, that is the Divisor. Now I shall divide $\frac{1}{4}$, by $\frac{3}{16}$ and the quotient will be $\frac{16}{12}$, that is, 1 $\frac{1}{3}$: whereby I know that there was put 1 pound and $\frac{1}{3}$ of Silver into the Crown, and so much Gold taken out for it.

Master. Prove it now by examination, according to the question.

Scholar. If there were 1 pound $\frac{1}{3}$ of Silver, then was

was there of Gold 6 pound $\frac{2}{3}$. Now say I by the Rule of proportion: if 8 pound of Gold expell two pound of water, what shall 6 pound $\frac{2}{3}$ expell, it will be 1 pound $\frac{2}{3}$.

8 Z 2
$6\frac{2}{3}$ Z $1\frac{2}{3}$

8 Z $3\frac{1}{2}$
$1\frac{1}{3}$ Z $\frac{7}{12}$

Again, if 8 pound of Silver expell three pound $\frac{1}{2}$ of water, what shall 1 $\frac{1}{3}$ expell? It will be $\frac{7}{12}$. Now must I adde together 1 pound $\frac{2}{3}$ and $\frac{7}{12}$, and they will make 2 pound $\frac{9}{36}$, that is, 2 pound $\frac{1}{4}$, according to the supposition of the question: whereby I perceive the work to be well done. And I cannot but much rejoyce of this excellent invention, so my desire is kindled vehemently to be perfectly instructed in every part thereof, and namely in this point, whether the proportion between water and Gold be such that for 8 l. of Gold put into a vessell full of water, there shall runne out two pound of water, and for as much Silver, whether 3 pound $\frac{1}{2}$ of water would avoid.

Master. I perceive your meaning, and conjecture your imagination to be thus, that if you knew the exact proportion betwen Gold and Silver, and water, both in their weight and quantities, then could you easily finde out the mixtures of them, which thing I have reserved for another work that intreateth of such matters especially. And at this time you must consider that you learne Arithmetick, which intreateth of the manner to solve doubtfull questions touching number, without regard what matter is signified by that number: else were it necessary in Arithmetick, to teach all Arts, seeing in it may be moved questions of all Arts.

But seeing you are so desierous to know these things,

A question of the proportion of gold, silver, and quick-silver unto water.

things, I will tell you in such a sort, that you shall practise your Art in finding it, and propose it in forme of a question. Gold beareth a greater proportion to water then Silver doth, and their two proportions be in proportion together, as 48 to 25. But to help you somewhat in this Riddle, you shall note that the proportion of Quick-silver unto water, is the just middle number proportionall in progression Geometricall between the proportion of Gold and Silver unto water.

And this proportion is $\frac{220}{21}$. Now if you will know the just numbers of these 3 proportions, then must you find out 3 numbers in Progression Geometricall, whereof the middlemost must be $\frac{220}{21}$, & the first must be unto the last, as 25 to 48. And thus I will leave you to find those numbers when you be at leasure.

Scholar. Yet Sir, I thank you heartily for thus much, for now I see the possibility to find them out. Howbeit, because this question seemeth strange, if it might please you to instruct me somewhat in the order of working for it, I should the more easily find the true working.

Master. You desire too much if you will study for nothing: Therefore to occasion you to study the better, I will leave this doubt wholy to your own search: But as touching the generality of the Rule, Archimedes needed not to take two Masses of gold and Silver equall in weight with the Crown, for the proportion might as well be found in any other weight, yea, although the Masse of Gold were of one weight, and the Masse of Silver of another. As for example: if the Crown were of 8 pound weight as I did suppose, and I have not

so

so much other fine Gold, but onely one pound, and trying that by water, and finding that it doth expell but $\frac{3}{4}$ of an ounce of water, yet then by it I may inferre, that 8 pound of Gold would expell 6 ounces of water. And likewise of Silver, whereof if I had but two pound, and find that it doth expell three ounces of water, then might I affirme, that 8 pound would expell 12 ounces, that is, one pound weight: and so is as it good as if the three Masses were all of one weight. And thus for this time I will make an end of this other part of Arithmetick.

Scholar. Although I cannot sufficiently thank you for this, yet your promise made me to look for the Art of Extraction of Roots, whereof hitherto I have learned nothing.

Master. I will not break my promise, but intend (God willing) to performe it within this three or foure moneths, if I perceive this my paines to be well taken in the mean season. And you shall not repent the tarrying for it: for it shall be increased by the tarrying. And in the mean time you shall take this Addition, not for the second part of Arithmetick which I promised, but for an augmentation of the first part, unto which I would have annexed the Extraction of Roots square and cubie, namely for Examples of the Statute of Assise of Wood, but that in the second part I must write of divers other Roots, and thought it best to reserve those Rules also with their Examples unto the same second part.

Scholar.

Scholar. Sir, although I cannot recompence your goodness, yet I shall alwayes doe mine endeavour to occasion you not to repent your benefit on me thus imployed.

Master. That recompence is sufficient for your part.

FINIS.

The third Part,

OR,

Addition to this BOOKE,
Entreateth of briefe Rules, called
Rules of Practise, of Rare, pleasant,
and commodious effects abridged into
a briefer method then hitherto hath
been published.

With divers other necessary
Rules, Tables, and Questions, not
onely profitable for Merchants, but
also for Gentlemen, and all other Oc-
cupiers whatsoever, as by the Con-
tents of this Booke may
appear.

Set forth by JOHN MELLIS,
School-master.

The firſt Chapter of this Addition entreateth of brief Rules, called Rules of practiſe, with divers neceſſary queſtions, profitable not onely for *Merchants, but alſo for all other Occupiers whatſoever.*

THE working of Multiplication in practiſe, is no other thing then a certain manner of Multiplying of one kind by another: whereupon is brought forth the Product of the propoſed number, which is accompliſhed by the meanes of Diviſion in taking the *half*, the *third*, the *fourth*, the *fifth*, or ſuch other parts of the ſumme, which is to be multiplied. Rule. 1.

And for the better underſtanding of ſuch converſions, you ſhall underſtand that in the manner and uſe of theſe Rules of Practiſe, you ought firſt to know the even or aliquot parts of a ſhilling, which in this Table following doth appear.

$$\textit{Item}\left\{\begin{matrix}6\\4\\3\\2\\1\end{matrix}\right\}\textit{pence is the}\left\{\begin{matrix}\frac{1}{2}\\\frac{1}{3}\\\frac{1}{4}\\\frac{1}{6}\\\frac{1}{12}\end{matrix}\right\}\textit{of a ſhilling.}$$

Wherein as you ſee according to the order of theſe rules of Practiſe: at 6 pence the yard of any thing, you muſt take $\frac{1}{2}$ of your number which is to be multiplied, and the product that commeth thereof ſhall

be

be shillings, if any unite doe remain it is 6 pence.

For 4 d. take the $\frac{1}{3}$ of the number that is to be multiplied, and the product also produceth shillings, if any unites doe remain, each one shall be worth in value 4 pence. The like is to be understood of the other 3, &c.

I *Example.*

At 6 d the yard, what	379 yards?
	189 s — 6 d

II

At 4 d the yard, what	104 yards?
	34 s — 8 d

III

At 3 d the yard, what	5014 yards?
	1253 s — 6 d

IV

At 2 d the yard, what	532 yards?
	88 s — 8 d

V

At 1 d the yard, what	409?
	34 s — 1 d

Here you may see in the first example, that 379 yards at 6 d. the yard, are worth 189 s. 6 d. in taking the $\frac{1}{2}$ of 379. And in the second example the 104 yards at 4 pence the yard, are worth 34 s. 8 d. in taking the $\frac{1}{3}$ of 104. Likewise in the third example, 5014 yards at 3 d. the yard, bringing fourth 1253 s. 6 d. in taking the $\frac{1}{4}$ of 5014. Also in the fourth Example at 2 d. the yard, maketh 88 s. 8 d.

And lastly, in the fifth Example: 409 yards at 1 d. the yard amounteth to 34 s. 1 d. in taking the $\frac{1}{12}$ of

409. and

409. and so is to be done also of all other questions the like, when the number of the pence is any of the even or aliquot parts of 12 d.

Item, to bring the Products of these shillings, and all other the like, into pounds, is very easie in dividing of it in your minde by 20. for it is to be understood that as often as 20 is found in that Product, so many pounds doth it contain: which with facility to performe, alwayes strike off the figure towards your right hand, with a right-down dash of your pen, for the o that appertaineth to the 20. and then begin at the left hand, in taking the half off the rest. And if that at the last any unite doe remain, the same shall be joyned with the figure that is cut off, which shall represent the odde shillings contained in that work.

As for example, in your third question at 3 d. the yard, which amounteth to 1253 s. 6 d. the Product whereof maketh 62 li. 13 s. 6 d. as here you may see is easily performed by this example.

1
125 | 3
62 -- 13 — 6

Also for the working of one peny the yard, it is something harsh and hard to take the $\frac{1}{12}$ of some Products: therefore to ease that hard work, you shall first bring your delivered summe into groats by taking $\frac{1}{4}$ part of the Product, and if any unites remain of that $\frac{1}{4}$ part, as sometimes there may, they are pence, & must be signified with a line from the *groats* with their title of *pence*; and because that 60 *groats* maketh a *Pound* or twenty shillings, *strike* off the first figure toward your right hand, for the o that appertaineth to 60 (as you did even now for the o that belongeth to 20:) Then in taking the $\frac{1}{9}$ of that product, if there doe remain any unites, the same shall

you joyne with the figure that you cut off, esteeming them as *groats*, which keep in your minde, and by taking the $\frac{1}{3}$ part of them, you shall turn them into *shillings*, and so have you done: *As for example*, by a Question or two hereafter proposed, shall more plainly by the work appear.

At 1 d, the yard, what 54368 yards?

13592 *groats*.

$\frac{1}{6}$ li.——226 - 10 s. 8 d.

Here in taking the $\frac{1}{6}$ part of 1359, in coming to the last work, the $\frac{1}{6}$ part of 39 being taken, the *remainder* is 3, which joyned with the two that was cut off, maketh 32 *groats*, which converted into *shillings*, by taking the $\frac{1}{3}$ part, maketh as appeareth 10 *shillings* 8 d. Many other wayes there are, but none more apt for a *young learner* to understand then this: wherefore this one way well impressed in memory is better then 20 wayes doubtfully understood.

At 1 peny the yard, what 4533 yards?

113 | 3 *groats* - 1 d

$\frac{1}{6}$ li. 18——17 -- 9 d

At 1 peny the yard, what 64768 yards?

1619 | 2 *groats*

li. 269 -- 17 -- 4 d.

NOW followeth also to be understood, that if the number of pence be not an aliquot part of 12, *you must reduce them into some aliquot part of* 12: *and after the aforesaid manner, you shall make of them two or three Products as need shall require, and adde them together into one summe. And here for thy furtherance appeareth a note of the order of their parts,*

parts, as they are to be taken.

for pence	*take*	*and*	*or*	*and*
5	3	2	4 &	1
7	4	3	6 &	1
8	4	4	6 &	2
9	6	3	4 4	1
10	6	4	4 4	2
11	6	4 & 1	4 4	3

Here in the first note of this *Table* at 5 d. you shall first take for 3 d. the $\frac{1}{4}$ of the number that is to be multiplyed: and likewise for 2 d. the $\frac{1}{6}$ of the same number, adding together both the Products. But if you will work by 4 and 1, you must for 4 d, first take $\frac{1}{3}$ of the number that is to be multiplyed: and for 1 d, take the $\frac{1}{12}$ of the whole summe, or rather, which is better, for 1 peny you may take the $\frac{1}{4}$ of the product which did come of the 4 pence. because that 1 d is the $\frac{1}{4}$ of 4 pence. The totall summes of these two numbers shall be the solution to the *Question.* And in like manner it is to be done of all others, as by these Examples following shall appear.

I.

At 5 d. the yard, what	758 yards?
3 d	189 —— 6 d
2 d	126 —— 4 d
shillings,	315 —— 10 d.

Otherwise,

At 5 d. the yard, what	758 yards?
4 d	252 —— 8 d
1 d	63 —— 2 d
shillings,	315 —— 10 d.

II.

At 7 pence the Ell, what	563 Ells ?
4 d	187 — 8 d
3 d	140 — 9 d
shillings,	328 — 5 d

III.

At 8 d. the pound, what	112 pound ?
4 d	37 — 4 d
4 d	37 — 4 d
shillings,	74 — 8 d

Otherwise,

At 8 d. the pound, what	112 pound ?
6 d	56 — 0
2 d	18 — 8
shillings,	74 — 8 d

IV.

At 9 pence the Elle, what	356 Ells ?
6 d	178 — 0
3 d	89 — 0
shillings,	267 — 0 d

V.

At 10 pence the peece, what	795 peeces ?
6 d	397 — 6
4 d	265 — 0
shillings,	662 — 6

VI.

VI.

At 11 pence the pound, what	7576 pounds?
6 d	3788——-0
4 d	2525——-4
1 d	631——-4
	6944——8 d
	Pounds 347——4 s.——8 d

1 Here in this firſt example, where it is demanded (at 5 d the yard) what will 758 coſt? Firſt, for 3 d, I take the $\frac{1}{4}$ of 758: and thereof cometh 189 ſ. 6 d: Then for 2 d I take the $\frac{1}{6}$ of the ſame 758, which amounteth to 126 ſ. 4 d. Theſe two ſummes added together, doe make 315 *ſhillings* 10 *pence*: and ſo much are the 758 *yards* worth at 5 d. the *yard*.

Item alſo for the ſame again: Firſt for 4 d, I take the $\frac{1}{3}$ of 758: and thereof cometh 252 ſ. 8 d: then for 1 peny I take the $\frac{1}{4}$ of the ſame 758, that is to ſay, of 252 ſ. 8 d, and it yeeldeth me 63 ſ. 2 d: which both added together make 315 ſ—10 d, as before.

2 *Item, for* 7 d. *there is taken the* $\frac{1}{3}$ *and the* $\frac{1}{4}$ of the whole ſumme which is to be multiplyed, and adde them together; that is to ſay, firſt, for 4 pence there is taken $\frac{1}{3}$ of 563. which comes to 187 ſ. 8 d, as appeareth by the work, and for 3 d there is taken the $\frac{1}{4}$ of the whole ſumme, which amounteth to 140 s—9 d. Both which products added together, doe make 328 s.—5 d. and ſo much comes 563 *Elles* to at 7 d. the *Elle*.

3 *Item, for the firſt* 8 d. *there is taken for* 4 d. the $\frac{1}{4}$ of the whole ſumme, and another $\frac{1}{4}$ for the other 4 d, which added together, as in the example doth

evidently appear, amounteth to 74 s.-8 d.

Again, for the second work of 112 l. there is taken first the $\frac{1}{2}$ of the whole summe for 6 d. which comes to 56 s. then for that 2 d. you have to take $\frac{1}{6}$ of the whole summe, or if you will, the $\frac{1}{3}$ of the product that came of 6 d. either of which maketh 18 s.-8 d. these two summes being added together do make 74 s. 8 d. as in the third example appeareth.

4. *Item for* 9 d, *there is taken for 6 pence* the $\frac{1}{2}$ of the whole summe, and the $\frac{1}{4}$ of the whole summe for 3 d. or otherwise for the 3 d. you may take the $\frac{1}{2}$ of the product that came of 6 d. because 3 pence is the $\frac{1}{2}$ of 6 d. which added together, as plainly appeareth in the *fourth example*, amounteth to 267 s. 0 d.

Item for 10 d. *first there is taken for 6 pence* the $\frac{1}{2}$ of the whole sum, which amounteth to 397 s. 6 d. then for 4 d. there is found 265 s. both which added together, make 662 *shillings*, 6 d, as appeareth in the *fifth example*. It may also be wrought, *as appeareth by the second note in the Table*, by 4 d twice taken, and the $\frac{1}{2}$ of the product of 4 d. or else by the $\frac{1}{6}$ of the whole summe, *&c.*

Item for 11 d. *there is first taken the* $\frac{1}{2}$ *for* 6 d. then the $\frac{1}{3}$ of the whole summe for 4 d. lastly, the $\frac{1}{4}$ of the last product for 1 d. All which 3 summes added together make in shillings 6944 s - 8 d. and in pounds 347 4 s. 8 d.

Item, likewise by the same reason, when you will multiply (by shillings) any number that is under 20. *you shall have in the Product pounds, if you know the even or aliquot parts of* 20. *which are here in this little Table set down to sight.*

Item,

$$\text{Item, s.}\left\{\begin{matrix}10\\5\\4\\2\\1\end{matrix}\right\}\text{is the}\left\{\begin{matrix}\frac{1}{2}\\\frac{1}{4}\\\frac{1}{5}\\\frac{1}{10}\\\frac{1}{20}\end{matrix}\right\}\text{of one pound,}$$

So that for 10 s. *which is the* $\frac{1}{2}$ *of a pound,* you may take the $\frac{1}{2}$ of the number which is to be multiplyed, and you ſhall have in your product pounds: if an unite doe remain, it ſhall be worth ten *ſhillings.*

Likewiſe for 5 *ſhillings you muſt take the* $\frac{1}{4}$ of the number which is to be multiplyed; and if there do remain any *unites*, they ſhall be fourth parts of a *Pound*, every unite being in value five *ſhillings.*

For 4 *ſhillings take the* $\frac{1}{5}$ *of the number* which is to be multiplyed: and if there doe remain any *unites*, they ſhall be fifth parts of a *pound*, each unite being in value 4 *ſhillings.*

For 2 *ſhillings you muſt take the* $\frac{1}{10}$ *of the* number to be multiplyed; wherefore to take the $\frac{1}{10}$ of any number, you muſt cut off the laſt figure of the ſame number (which is neareſt your right hand) from all the other figures with a ſmall right down line or daſh with a Pen, and ſo have you done: for all the other figures which doe remain toward your left hand from the ſame figure that you do ſeparate, ſhall be *pounds*; and that figure ſo ſeparated towards your right hand, ſhall be ſo many pieces of 2 s. the which figure you muſt double to make thereof the true number of ſhillings, as by the Example ſhall appear.

Finally, *for* 1 *ſhilling needeth ſmall work*, for it is ſo many ſhillings as be propoſed in the ſumme, which to bring into pounds, hath been already taught in the *firſt rule.*

Example.

At 10 s. the piece, What		6543 pieces?
$\frac{1}{2}$	li.	3271 10 s.
At 5 s. the Elle, What		4373 Elles?
$\frac{1}{4}$	li.	1093 5 s.
At 4 s. the yard, What		7839 yards?
$\frac{1}{5}$	li.	1567 16 s.
At 2 s. the pound weight, What		752/7 pound?
$\frac{1}{10}$	li.	752 —— 14 s.
At 1 s. the piece, What		775/3 pieces?
$\frac{1}{20}$	li.	387 13 s.

NExt followeth in order to be understood, that if the number of shillings be not some even or aliquot part of 20, you must then convert the same number of shillings into the aliquot parts of twenty, and thereof make two or three products as need shall require: which done, adde them together, and bring them into Pounds. And here for thy furtherance, I have set down a note of the order of their parts, as they are to be taken.

s.					s.		
3	of	2 & 1	or	5 & 1	13	of	10. 2 & 1
6		4 & 2		5. 2. 1	14		10. & 4
7		5 & 2		4. 4. 1	15		10. & 5
8		4 & 4			16		10. 5. 1
9		5 & 4			17		10. 5. 2
11		10 & 1			18		10. 4. 4
12		10 & 2			19		10. 5. 4

For 3 s. *according to the tenure that you see* is expressed in the Table, you must first take for 2 *shillings* the

the $\frac{1}{10}$ of the number that is to be multiplied; then for one *shilling* you must take the $\frac{1}{2}$ of the product which did come of the same $\frac{1}{10}$ part: which two summes added together produce the effect desired.

Item, for 6 shillings according to the note set forth in the Table, first for 4 s. I take the $\frac{1}{5}$ of the number that is to be multiplied: Then for 2 s. the $\frac{1}{2}$ of the product that came of 4 s. and adde them together.

Or else as appeareth also in the Table, for 5 shillings you may take the $\frac{1}{4}$ and the $\frac{1}{5}$ part of the product that came of 5 *shillings*, and adde them together.

Item, for 7 s. *first for* 5 s. *take* $\frac{1}{4}$ *of the product* that is to be multiplied; then for 2 s. take the $\frac{1}{10}$ of the number that is to be multiplied, and adde them together, *&c.*

Item, for 8 s. *according to reason, and the* intent of the *Table*, for the first 4 s. take the $\frac{1}{5}$ of the product, and the same number again for the other 4 s. and adde them together.

Item, for 9 shillings: first for 5 shillings, take the $\frac{1}{4}$, then for 4 shillings take the $\frac{1}{5}$: and adde them together.

Otherwise, as you see by the intent of the Table, work twice for 9 shillings as was taught even now for 8: and then take the $\frac{1}{4}$ of the last product for the 1 shilling: but 5 and 4 is the shorter.

Item, for 11 s. *first dispatch* 10 s. for which you must take the $\frac{1}{2}$ of the product: then lastly, for one shilling take the $\frac{1}{10}$ part of the summe produced of the $\frac{1}{2}$ of the product, and adde them together.

Item, for 12 *shillings, where I will end with the first part of my Table.* For take the $\frac{1}{2}$ for 10 *shillings.* And

And then for 2 *ſhillings*, take the $\frac{1}{5}$ of the ſumme that came of 10 *ſhillings*, take and adde them together, or elſe if you pleaſe for 2 *ſhillings* you may take the $\frac{1}{10}$. of the whole given number.

To write more of the manner of taking the true parts, I omit. The deſirous practitioners will (no doubt) conceive it. Alſo the *Table* is ſome aid to help the unperfect, whereupon by and by I will ſet down three or four of theſe notes in *Examples*, and the reſt I will leave to thine own induſtry and practice, to labour upon.

This is the order moſt commonly uſed in practice, when the number of ſhillings is not an *aliquot* part of a pound. But *(loving Reader)* after I have touched the even or *aliquot* parts of a pound that falleth out in pence and ſhillings. I will deliver two new Rules that ſhall drown this common order quite and clean: wherein ſhall be comprehended in one line or working both of even and odde parts of ſhillings under 20. without regard whether it be an *aliquot*, or not an *aliquot* part; which two Rules (when they come in place) I commit to thy friendly judgement in working.

Now follow the examples upon the notes aforeſaid.

At 6 ſhillings the yard, what	3215 yards?
4 ſhillings	643
2 ſhillings	321 —— 10
li.	964 -- 10 s.

Other-

Otherwise by Multiplication of 6.

3215
1929|0
6 shillings
li ——— 964 ——— 10 *shillings.*

At 7 ſhillings the Ell, what 4563 Ells?
5 ſhillings 1140 —— 15
2 ſhillings 456 —— 6
li. 1597 —— 1 ſhilling.

Otherwise by Multiplication of 7.

4563
7 s 3194|1
1597 —— 1

At 8 s. the piece, what 7563 pieces?
4 s 1512 —— 12
4 s 1512 —— 12
pounds 3025 —— 4s.

Otherwise by Multiplication of 8.

7563
8 s 6050|4
pounds 3025 ——— 4 ſhillings.

At 13 s. the piece, what 401 pieces?
10 s 200 —— 10
2 s 40 —— 2
1 s 20 —— 1
pounds 260 —— 13

Other-

Otherwise by Multiplication.

	401
13 s.	1203
	401
	5213
pounds	260——13 s.

These and such like questions of compound number, which I have here in this fourth rule for orders sake set down, for that it hath been heretofore a common course of work, I account but superfluous. For in the eighth and ninth Rules of this my simple Addition shall appear, that the given price of any even or odde number of shillings, either under or above 20, shall be wrought at one or two workings at the most, how difficult soever the question be.

3 Rule. To reduce pence into pounds at one operation.

Item, *there resteth yet a kinde of practice, how to bring pence into pounds at the first working: whereupon you must understand that* 240 *pence maketh one pound, or* 20 s. *In consideration whereof I cut off the last figure or* 0, *and there remaineth but* 24 (*of which* 24) 8 d. *is the* $\frac{1}{3}$ *part thereof,* 6 d. *is the* $\frac{1}{4}$ *part,* 4 d. *is the* $\frac{1}{6}$ *part, and* 2 *pence is the* $\frac{1}{12}$ *part thereof.*

Whereupon if it were demanded what 1486 yards, or pounds of any thing cometh to, at 8 pence the yard; in pricking or cutting off the first figure towards your right hand, for the 0 that appertaineth 240. There is remaining of the said summe 148, whereout I take the $\frac{1}{3}$ part, and it cometh to 49 li. and there resteth 1, which 1 I put to the 6. that I prick or cut off, and it maketh 16 pieces of 8 pence, which I double to make into groats, & they make 32, whereof

whereof the $\frac{1}{3}$ maketh 10 s. and there remaineth $\frac{2}{3}$ s. which is 8 d, whereby it followeth, that the 1486 yards at 8 pence the yard maketh 49 li. 10 s. 8 d. as by the example ſhall appear.

Item, for 6 d. take the $\frac{1}{4}$ part of the number from the prickt figure; and if any unites remain, they are ſo many ſix-pences, whereof taking the $\frac{1}{2}$ they are ſhillings, if there doth remain yet one, it is in value ſix-pence.

Item, for 4 d. take the $\frac{1}{2}$ part of the number from the prickt figure; if any unites doe remain, they are ſo many groats, which to convert into ſhillings, take the $\frac{1}{3}$ part. And if any yet remain, they are thirds of ſhillings, each one in value being worth 4 pence.

Item, for 3 pence take the $\frac{1}{4}$ part from the prickt figure, if any unites remain, they are ſo many pieces of 3 pence, whereof in taking the $\frac{1}{4}$ part, maketh ſhillings: if any thing yet remain, they are the fourth parts of ſhillings, each one being in value 3 pence.

Item, for 2 pence, as appeareth alſo by the Table, take the $\frac{1}{6}$ part of the number from the prickt figure: if any thing remain, they are ſo many pieces of 2 pence, which by taking the $\frac{1}{6}$ part, you ſhall turn into ſhillings, and if any unites remain, they are ſo many 6 parts of ſhillings, or pieces of two pence, whether you will.

If one coſt 8 pence, what	1486 ?
maketh pounds	49---10--8 d.
If one coſt 6 pence, what	7865 ?
maketh pounds	196--12--6d.

At

At 4 pence the yard, what	8736 yard ?
maketh pounds	145——12--0 d.
If one cost 3 pence, what	9874 worth ?
maketh pounds	123 — 8 --6 d.
At 2 d. the Ell, what	7894 Ells to ?
maketh pounds	65 —15---8 d.

6 Rule. *BUT if your number of pence be not an aliquot or even part of 24, then must you bring them into the aliquot parts of 24. and make thereof diverse products which must be added together, as by the question hereafter following shall appear.*

Item for 5 d. first take for 3 d. then for 2 d. and adde them together, according to the instruction of the *second Rule*: or else first take for 4 d. then 1 d.

Item, for 7 d. first take for 4 d. then for 3 d. and adde them together.

Item for 9 d. first take for 6 d. then for 3 d. and adde them together.

Item, for 10 d. first take for 6 d. then for 4 d. and adde them together.

Item, for 11 d. first take for 8 d. then for 3 d. and adde them together: as by these Examples.

Examples.

1. If one yard cost 5 d. what	759|6
4 pence	126——12
1	31——13
maketh pounds	158——5

Other-

Otherwise.

1 —— 5 —— 759 | 6

3 pence 94 —— 19

2 pence 63 —— —— 6

maketh pounds 158 —— —— 5 s.

2. If one cost 7 d. what 98 | 7 worth?

4 pence 16 —— 9

3 pence 12 —— 6 —— 9

maketh pounds 28 —— 15 – 9 d.

Otherwise.

1 —— 7 —— 98 | 7

6 pence 24 —— 13 —— 6

1 peny 4 —— 2 —— 3

maketh pounds 28 —— 15 —— 9 d.

3. If one cost 9 d. what 98 | 7 worth?

6 pence 24 —— 13 —— 6

3 pence 12 —— 6 —— 9

maketh pounds 37 —— 0 —— 3 d.

Otherwise.

1 —— 9 —— 98 | 7

6 pence 24 —— 13 —— 6

3 pence 12 —— 6 —— 9

maketh pounds 37 —— 00 —— 3

4. If one cost 10 pence, what 98 | 7?

6 pence 24 —— 13 —— 6

4 pence 16 —— 9 —— 0

maketh pounds 41 —— 2 —— 6 d.

5. If

5. If one coſt 11 pence, what	98 | 7?
8 pence	32——18--0
3 pence	12——6--9
maketh pounds	45——6--9

But if you have any ſhillings and pence to be multiplied together, then are you to take for the ſhillings according to the inſtruction of the third Rule: and for the pence according to the firſt Rule before mentioned: unleſſe you can ſpie the advantage thereof, and thereby helpe your ſelfe; as appeareth in this ſecond example, where firſt I work for 6 d. which is to be rebated out of the given number, and I have 719 li. 11 s. my deſire.

At 19 s. 9 d. the yard, what 738 yards?

	738		*Otherwiſe by Rebating.*
10 s.	369——0		
5 s	184——10		738
4 s	147——12	6 d	18——9 s
6 d	18——9	li.--7--19——11 s	
pounds	719——--11 s.		

The like again is done by Rebating, as by theſe two examples appeareth.

At 18 s. the Ell, what	418 Ells?
2 s	41——16
pounds	376——4 s.
At 16 s. the Ell, what	517 Ells?
4 s	103——8
pounds	413——12 s.

And

And now I will touch a little the even part of a pound, that falleth out in pence and ſhillings, whereof for thoſe parts you ſhall take ſuch like parts out of the given number that is to be multiplied, as the price of that given number beareth in proportion to a pound, which alſo for their better aid is here ſet down.

1 s.	8 d.	*is the*	$\frac{1}{12}$	*part of a pound.*
2	6		$\frac{1}{8}$	
3	4		$\frac{1}{6}$	
6	8		$\frac{1}{3}$	

Item, firſt for 1 ſhilling 8 pence, take the $\frac{1}{12}$ part of the given number; and if any thing do remain they are twelve parts of a pound, each one being in value 1 ſhilling 8 pence.

Item, for 2 ſhillings 6 pence, take the $\frac{1}{8}$ part of the number that is to be multiplyed; and if any thing do remain, they are eight parts of 1 pound, each one being in value 2 ſhillings ſix pence.

Item, for 3 ſhillings 4 pence, as appeareth by the Table, you muſt take the $\frac{1}{6}$ part of the given number; and if any thing doe remaine, they are 6 parts of a pound, each one being in value 3 ſhillings 4 pence.

Item, for 6 ſhillings 8 pence take the $\frac{1}{3}$ part of the number that is to be multiplyed; and if any unites doe remain, they are thirds of a pound, every one being worth 6 ſhillings 8 pence.

Other infinite numbers there are, that may be reduced by abbreviation into the proportionate parts of a *pound*, as 16 ſhillings 8 pence maketh $\frac{5}{6}$: which 16 ſhillings 8 pence is eaſily reduced into groats, by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats.

5. If one cost 11 pence, what	98 | 7?
8 pence	32——18..0
3 pence	12——.6..9
maketh pounds	45——.6..9

But if you have any shillings and pence to be multiplied together, then are you to take for the shillings according to the instruction of the third Rule: and for the pence according to the first Rule before mentioned: unlesse you can spie the advantage thereof, and thereby helpe your selfe; as appeareth in this second example, where first I work for 6 d. which is to be rebated out of the given number, and I have 719 li. 11 s. my desire.

At 19 s. 9 d. the yard, what 738 yards?

	738		*Otherwise by Rebating.*
10 s.	369——0		
5 s	184——10		738
4 s	147——12	6 d	18——9 s
6 d	18——9	li.—7—19	——11 s
pounds	719————11 s.		

The like again is done by Rebating, as by these two examples appeareth.

At 18 s. the Ell, what	418 Ells?
2 s	41——16
pounds	376——4 s.
At 16 s. the Ell, what	517 Ells?
4 s	103————8
pounds	413——12 s.

And

And now I will touch a little the even part of a pound, that falleth out in pence and shillings, whereof for those parts you shall take such like parts out of the given number that is to be multiplied, as the price of that given number beareth in proportion to a pound, which also for their better aid is here set down.

s.	d.			
1 s.	8 d.	*is the*	$\frac{1}{12}$	*part of a pound.*
2	6		$\frac{1}{8}$	
3	4		$\frac{1}{6}$	
6	8		$\frac{1}{3}$	

Item, first for 1 shilling 8 pence, take the $\frac{1}{12}$ part of the given number; and if any thing do remain they are twelve parts of a pound, each one being in value 1 shilling 8 pence.

Item, for 2 shillings 6 pence, take the $\frac{1}{8}$ part of the number that is to be multiplyed; and if any thing do remain, they are eight parts of 1 pound, each one being in value 2 shillings six pence.

Item, for 3 shillings 4 pence, as appeareth by the Table, you must take the $\frac{1}{6}$ part of the given number; and if any thing doe remaine, they are 6 parts of a pound, each one being in value 3 shillings 4 pence.

Item, for 6 shillings 8 pence take the $\frac{1}{3}$ part of the number that is to be multiplyed; and if any unites doe remain, they are thirds of a pound, every one being worth 6 shillings 8 pence.

Other infinite numbers there are, that may be reduced by abbreviation into the proportionate parts of a *pound*, as 16 shillings 8 pence maketh $\frac{5}{6}$: which 16 shillings 8 pence is easily reduced into groats, by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats.

Then set 60 the groats of a pound under 50: cutting of the two Cyphers as is here performed.

16	8
3	
5|0	
6|0	

And then have you brought 16 *shillings* 8 pence into the known parts of a *pound*, which maketh

But yet gentle Reader, for thy further instruction, I have hereunto annexed in a *Table*, how pence and shillings bear proportion to a pound, which I commit to thy friendly benevolence; it will be some aid unto the ungrounded Practitioner: but I count him the best workman that can presently reduce his given price into the known and proportionate parts of a pound.

A

A Table of the Aliquot parts of a pound or 20 shillings.

s.	d.	l.	s.	d.	l.
0	2	$\frac{1}{120}$	8	4	$\frac{5}{12}$
0	3	$\frac{1}{80}$	8	9	$\frac{7}{16}$
0	4	$\frac{1}{60}$	9	0	$\frac{9}{20}$
0	6	$\frac{1}{40}$	10	0	$\frac{1}{2}$
0	8	$\frac{1}{30}$	11	0	$\frac{11}{20}$
1	0	$\frac{1}{20}$	11	3	$\frac{9}{16}$
1	3	$\frac{1}{16}$	12	0	$\frac{3}{5}$
1	8	$\frac{1}{12}$	12	6	$\frac{5}{8}$
2	0	$\frac{1}{10}$	13	0	$\frac{13}{20}$
2	6	$\frac{1}{8}$	13	4	$\frac{2}{3}$
3	0	$\frac{3}{20}$	13	9	$\frac{11}{16}$
3	4	$\frac{1}{6}$	14	0	$\frac{7}{10}$
3	9	$\frac{3}{16}$	15	0	$\frac{3}{4}$
4	0	$\frac{1}{5}$	16	0	$\frac{4}{5}$
5	0	$\frac{1}{4}$	16	8	$\frac{5}{6}$
6	0	$\frac{3}{10}$	17	0	$\frac{17}{20}$
6	6	$\frac{13}{40}$	17	6	$\frac{7}{8}$
6	8	$\frac{1}{3}$	18	0	$\frac{9}{10}$
7	0	$\frac{7}{20}$	18	4	$\frac{11}{12}$
7	6	$\frac{3}{8}$	18	9	$\frac{15}{16}$
8	0	$\frac{2}{5}$	19	0	$\frac{19}{20}$

Here follow four examples upon the four Notes delivered.

At 1 s. 8 d. the yard, what 3884 yards?

maketh pounds 323---13--4 d.

At 2 s. 6 d. the yard, what 4563 yards?

maketh pounds 570---7---6 d.

At 6 s. 8 d. the Ell, what 7562 Ells?

maketh pounds 2520--13--4 d.

Now by custome you are able to work by all sorts of summes, being delivered in shillings and pence, as one shilling one peny, two shillings two pence, three shillings three pence, and so of all other: wishing you to have some consideration of your questions, when they are set down, for there are many subtile abbreviations, and great advantages to be gotten, and easily to be perceived.

As of 3 s.———————8 d. of 2 s. and 1 s. 8. d.

Of 4 s.———————2 d. of 3 s.————4 d. and 10 d. which 10 d. is $\frac{1}{4}$ of 3 s.———————4 d.

Of 5 s.———————8 d. of 4 s. 1 s.———4 d.

Of 5 s. 10 d. of 5 s. and 10 d. which 10 d. is $\frac{1}{6}$ of 5 s.

And by this mean, when you have taken one product, you may oftentimes upon the same take another more briefly then upon the summe which is to be multiplied, &c.

8 Rule. *NOW (Gentle Reader) that you have seen the vertue of the even or aliquot parts of a pound in shillings alone, and also in the aliquot parts of shillings and pence: according to my promise, hereafter followeth*

followeth a brief and easier method for any even number of shillings, either under or above 20, then ever yet hath been published. Notwithstanding M. Humphrey Baker, *whose travell is worthy commendation, and whom for knowledge sake I reverence, hath in some part touched this first part, though not in this method. The work of the Rule both pleasant, ready, and brief, as by the variety of the examples delivered thereupon shall appear. And first I will set forth a question, thereby the better to expresse or teach you the order thereof: which is this.*

If one cost *6* s. what		8574?
	I *6* s.	8574
maketh pounds		2572 ——--4 s.

To the understanding of this example, after you have set down your given number in form of the Rule of 3, with a line drawn under it, you shall presently set a prick under your first figure 4 toward your right hand, drawing from the prick, as heretofore hath been practised, a little short line, thereto set down the shillings anon, which done, multiply the first figure 4 by *6*, the value of your price; (which here you see standeth in sight above the line) it maketh 24. which is one pound foure shillings. The one pound keep to carry to the next place, and the foure shillings set down at the end of the prescribed line towards your right hand. Thus have you done now with *6* above the line, and also with 4 in the first place (for the prick under 4 doth signifie that 4 hath done his office.) Then secondarily for a general Rule take but the $\frac{1}{2}$ of the given price, which here is the number that shall now continue the rest of the multiplication, and end the work, whereupon I multiply

Mr. *John Mellis* his first Rule.

Note a general rule.

3 into 7, ſtanding in the ſecond place it maketh 21, and with the 1 pound I kept in 22; ſet down 2 and keep 2 in minde, working according to the Rule of multiplication, delivering the tenths in minde in their due place; which done, the product from the prick to your left hand repreſenteth the pounds, and the other at the end of the ſhillings, as appeareth by the examples.

If one yard coſt 2 s. what		7536?
1	2 s.	7536
maketh pounds		753——12 s.
If one yard coſt 4 s. what		8792?
1	4 s.	8792
maketh pounds		1758——8 s.
If one piece coſt 6 s. what		9537?
1	6 s.	9537
maketh pounds		2861——2 s.
If one piece coſt 8 s. what		7509?
1	8 s.	7509
maketh pounds		3003---12 s.
If one coſt 12 s. what		5794?
1	12 s.	5794
maketh pounds		3476——8 s.
If one coſt 14 s. what		3705?
1	14 s.	3705
maketh pounds		2593----10 s.
If one coſt 18 s. what		5703?
1	18 s.	5703
maketh pounds		5132----14 s.

If

If one coft 22 s. what		953?
1	22 s.	953
maketh pounds		1048——6 s.

Let theſe ſuffice (gentle Reader) for an entrance into even numbers. And now I will ſhew the like rule for any odde or uneven part of a pound.

TO help you to the underſtanding of theſe other queſtions that hereafter follow: where in my firſt Example the given number is 6487 at 3 s. the yard: I multiply 3 above the line into 7, it maketh 21. The one ſhilling is ſet down, and the 1 pound I keep. Now am I to take the ½ of three, which becauſe it is an odde number I cannot.

Therefore I ſhall keep and continue my multiplication by three ſtill, and work by the ½ of the reſt of the given figures or number, to wit, 648. And firſt the ½ of 8, which is 4 multiplyed into 3, maketh 12, there to joyn the 1 li. *in minde, it maketh 13. ſet down 3, keep one. Then again multiply by two the ½ of four it maketh ſix, and with one in minde it maketh 7. Then laſtly, take the ½ of ſix, which is 3. ſaying, 3 times 3 is 9, which 9 ſet down, and ſo is the queſtion anſwered, as appeareth by the practice, and examples following.* Mr. *John Mellis* his ſecond Rule.

At 3 s. the yard, what		6487?
1	3 s.	6487
maketh pounds		973———1 s.

If one yard coſt 5 s. what		4269?
1	5 s.	4269
maketh pounds		1067———5 s.

At 7 s. the Ell, what	6489?	
1 7 s.	6489	
maketh pounds	2271	3 s.
If one Ell cost 9 s. what	2807?	
1 9 s.	2807	
maketh pounds	1263	3 s.
At 11 s. the Pistolet, what	8263?	
1 11 s.	8263	
maketh pounds	4544	13 s.
If one piece cost 13 s. what	4629?	
1 13 s.	4629	
maketh pounds	3008	17 s.

But now note (gentle Reader) when the given price falleth upon an odde number, as 3, 5, 7, 11, 13, &c. then it is to be presupposed that the given sum to be multiplyed, must be a sum made of even numbers, 2, 4, 6, 8, 10, &c. else cannot the question be wrought at one line or working.

Providing alwayes that it may bear an odde figure in the first place towards your right hand, as appeareth in these six examples, which last were wrought, and such like, &c. which may bear an odde number for the price, and be done at one line or working very well.

But if the given price be an odde number, and the sum to be multiplyed odde numbers also, then can it not be done at one working, but requireth the aid of two workings, for odde with odde will not agree; which notwithstanding to bring to passe, take this for a generall Rule. First, worke for the even number, contained in that question, or given price, according

A generall Rule.

ding as you have learned, and then afterwards for the one odde ſhilling, take the $\frac{1}{2}$ of the ſumme given to be multiplyed, omitting the firſt prickt place, as was taught for the working of one ſhilling in my firſt Rule of Practice, and adde thoſe two together, and you ſhall have your deſire.

Examples.

At 3 s. the yard, what	7539 yards?
2 s.	753——-18
1 s.	376——-19
maketh pounds	1130——-17 s.
At 7 s. the Ell, what	7539?
5 s.	7539
2 s.	2261——-14
	376——-19
maketh pounds	6238——-13
At 13 s. the yard, what	7534?
10 s.	3767——0
2 s.	753——8
1 s.	376——-14
maketh pounds	4897——2

And thus have I abridged into theſe two rules how to bring any number of ſhillings whatſoever they be, into pounds, with a briefer Method then ever yet hath been publiſhed, which I commend unto thy friendly cenſure and judgement in the uſe and practice thereof. Note this well.

If one cost 6 s. 5d. what	1231 ?
6 s.	369 —— 6
4 d.	20 —— 10 -- 4
1 d.	5 —— 2 -- 7
maketh pounds	394 —— 18 -- 11
At 14 s. 2 d. what	2825 ?
14 s.	1977 --- 10
2 d.	23 --- 10 -- 10
maketh pounds	2001 —— 0 -- 10
At 16 s. 4 d. what	2531 ?
16 s.	2024 —— 16
4 d.	24 —— 3 - 8d.
maketh pounds	2066 ---- 19 -- 8
At 3 s. the Pistolet, what	8325 ?
maketh pounds	1248 —— 15 s.
At 7 s. the Crown, what	6529 ?
	2285 —— - 3 s.
At 9 s. the piece, what	6567 ?
maketh pounds	2955 —— - 3 s.

These three last questions may seeme something harder, yet they are easy enough, if you mark them well: If I should explain them, then are they too easy. Therefore I leave them to whet the minds of the desirous.

10. Rule. ITem *when any one of the summes, which is to be multiplyed, is composed of many Denominations, and the given number but of one figure alone; then shall you multiply all the Denominations of the other summe*

ſumme by the ſame one figure, beginning firſt with that ſumme which is leaſt in value toward your right hand, and bring the product of thoſe pence into ſhillings, and the product of the ſhillings into pounds, as by this example appeareth.

At 3 l. 7 s. 4 d. a yard, what are 9 worth?	
maketh pounds	30--6s-od.

BUT *if in any of the ſummes that are to be multiplyed, there be a broken number; Firſt work for the whole according to the inſtructions that you have learned, and then take ſuch part of the given price, as that broken number beareth in proportion to the price, as in the examples following. After you have wrought for* 3 s. *and for* 6 d. *then are you to take the* $\frac{1}{2}$ *of* 3 s. 6 d. *for the* $\frac{1}{2}$ *yard, and adde that to the ſumme: So adding all the* 3 *products together, which make* 43 li. 2 s. 9 d. *the juſt price of* 245 $\frac{1}{2}$ *Ells, and thus muſt you doe of all other.* 11. Rule.

At 3 s. 6 d. the Ell, what	245 $\frac{1}{2}$?
3 s.	36——18
6 d.	6——3
$\frac{1}{2}$	1-9
maketh	43——9--5 $\frac{3}{4}$

At 16 s. 4 d. the piece, what	14 $\frac{3}{4}$?
16 s.	11——4—0
4 d.	0——4—8
$\frac{3}{4}$	12---3
maketh pounds	12——0——11

If

If one piece cost 4 li. 3 s. 6 $\frac{1}{2}$ d. what 12 pieces?

4 li.	48		
3 s.	1	16	
6 d.		6	6
$\frac{1}{2}$			
maketh pounds	50	2	6

The proofe.

If 12 pieces cost 50 li. 2. s. 6 d. what one piece? maketh pounds 4 — 3 — 6 $\frac{1}{2}$

14. Rule. ITem, *touching the manner how to understand the order of this question, and others the like, first seek how many times* 12 *is contained in* 50, *which is* 4 *times, and so resteth* 2 *Pound, which* 2 *Pound converted into shillings, and joyned with the other* 2 *shillings, maketh* 42 *shillings: wherein is found* 12 *three times, resteth* 6 s. *which turned into pence, putting thereto the* 6 *pence in the first place, it maketh* 78, *wherein* 12 *is found six times, resteth* 6 d. *which containeth* 12, *but* $\frac{1}{2}$ *a time, put that* $\frac{1}{2}$ *to the* 6 d. *and then the solution is* 4 li. 3 s. 6 $\frac{1}{2}$ d. *as appeareth by the Practice thereof.*

15. Rule. Item, *The like is to be done of any thing that is bought or sold after five score to the hundred, or the Quintall. As for example.*

If 100 pound cost 27 li. 13 s. 4 d, what one pound?

27 li.

27 li.——13 s.——4 d.
2

s.5 | 53
 | 12
1 | 10
5 3

d.6 | 4 | 0
 | 10 | 0. or $\frac{2}{5}$

Maketh 5 s. 6$\frac{2}{5}$ d.

But to worke it more neatly, it is by a little understanding ended thus.

27 li.——13s.——4 d.
20

s. 5 | 53
 | 12
d.6 | 40
 | 100

Maketh 5 s. 6 $\frac{2}{5}$ d.

I have wrought this at length for the aid of the young learner, because he should understand how all the multiplication is set down.

Item, *to the understanding of this and such like questions, the right down line is all the guide, which is pulled down close by* 20. *as you see in the example, where* 27 li. 13 s. *is reduced all into* s. *and maketh* 553 s.

The 5 *towards the left hand being separated with the hanging or right down line, is the just number of shillings, that answereth the question. Nextly,* 53 s. *is multiplyed by* 12, *to reduce them to pence, putting to the* 4 d. *it yeeldeth for the multiplication of the first figure two* 110. *the one beyond the line towards the left hand, is* 1 *peny towards the rest of the price: then* 53 *also multiplyed by* 1 *yeeldeth* 53 *: but the* 5 *behinde the line towards the left hand, is also* 5 *pence more, towards the price, which* 1 *and* 5 *I adde together under the line, it maketh* 6 d. *So is there found now as appeareth by the Titles of shillings and pence,* 5 s. 6 *pence.*

Finally, I come now on this side the line towards the

the right hand, and under 12 I finde first 10, *and then* 3, *which added together make* 40. *under which* 40 *you must put the* 100, *and it maketh* $\frac{40}{100}$ *which abbreviated, cometh to* $\frac{2}{5}$. *So the just price of one pound after* 5 *score to the hundred, maketh* 5 s. 6 $\frac{2}{5}$ d. One example more, and so I will leave this Rule.

If 100 cost 10 $\frac{3}{4}$ d. what	9874?	
6 d	246——17	
4 d	164——11——4	
$\frac{2}{4}$ d	20——11——5	
$\frac{1}{4}$ d	10——5——8 $\frac{1}{2}$.	

Maketh	li.	4	42	5	parts of a peny,	5 $\frac{1}{2}$
			20			
	s.	8	45			
			12			
			45 $\frac{1}{2}$ 91			
	d.	5	100 | 100			

Also the like may be done of the usuall weights here in *England*, (which is 112 for every hundred weight) in case you know the aliquot parts of a hundred weight, which are these, 56 li. 28 li. 14 li. and 7 li. For 56 li. is the $\frac{1}{2}$ of 112 li. 28. li. is the $\frac{1}{4}$ of 112 li. 14 li. is the $\frac{1}{8}$ and 7. li. is $\frac{1}{16}$ part.

Therefore for 56 li. take the $\frac{1}{2}$ of the summe of the money that 112 li. weight is worth.

For 28 li. take the $\frac{1}{4}$ of the summe of money that 112 pound weight is worth.

For 14 li. take the $\frac{1}{8}$ of the summe that 112 li. is worth.

And for 7 li. the $\frac{1}{16}$ of the summe of money that 112 li. is worth.

As

As for example; at 17 li. 19 s. the hundred pounds weig ht, that is to say, the 112 li. what shall 3 quarters and 7 pound cost?

1. C.	17 li.	19 s.	3 q. — 7 li.
2 quarterns	8	19	6
1 quartern	4	9	9
7 pounds	1	2	5 $\frac{1}{4}$
Maketh pounds	14	11	8 $\frac{1}{4}$.

The second Chapter treateth of the Reduction of divers measures to others value by Rules of Practice.

NOW *will I shew a few examples of practice in reducing of Measures, as* Ells, Yards, Braces, Pawnes of Genee, &c. *Much more would I have touched, but that I feare the Book will rise to too great a Volume.* 18 Rule.

In 864 Ells of *Antwerpe*, how many yards of *London*?

864	864
432	216
216	648

maketh 648 yards of *London*.

ITem, *in these and such like questions of* Flemmish *measure, to be brought into yards* English, *first take the* $\frac{1}{2}$ *of the given number, as appeareth in the first example*

ample towards your left hand. Then take half of that product, or the $\frac{1}{4}$ of the given number, and adde these two products together, as they shall be yards English: *as by the example you may perceive.*

The second example toward your right hand is yet briefer then the first, whose worke is this; Take the $\frac{1}{4}$ of the delivered number, and that product subtract out of the given number, and the rest sheweth your desire. Of these two wayes use which you think best.

The Proof.

In 648 yards *London*,
How many Ells of *Antwerpe?*

	648	
	216	
maketh	864	Ells of *Antwerpe.*

15 Rule. ITem, *for the understanding of this worke, first take the $\frac{1}{3}$ part of the yards of* London, *which found adde that $\frac{1}{3}$ part and the yards together, as appeareth by the Practice, and the product sheweth the Ells of* Antwerpe.

Item, in 20 yards of *London*,
How many Ells of *Antwerpe?*
maketh $426\frac{2}{3}$ Ells,

320 yards		Proof,
$106\frac{2}{3}$		$426\frac{2}{3}$ Ells.
$426\frac{2}{3}$ Ells	$\frac{2}{3}$	$106\frac{2}{3}$
		320 yards,

Other Reductions,

16 Rule. ITem, *you shall understand, that forasmuch as six braces of* Millain, *make five Ells of* Antwerpe, *whereupon according to the* Rules of Practice, *you may reduce the one into the other, by the like reason aforesaid,*

ſaid, in taking the $\frac{1}{6}$ part, and then ſubtract the ſame, to make Ells of Antwerpe. *And again by the contrary taking the $\frac{1}{5}$ part, with adding the given number, to turn the Ells to Braces. As for example.*

In 876 Braces, how many Ells of *Antwerpe* ?

876	The contrary.
146	730 Ells *Flemmiſh.*
Ells 730 *Antwerpe.*	146 Braces.
	876 Braces.

Ells 730 *Antwerpe.*
182$\frac{1}{2}$
Yards 547$\frac{1}{2}$ *Engliſh.*

Thus appeareth that 876 Braces by Practice make 730 Ells *Flemmiſh*, which Ells *Flemmiſh* reduce into *Engliſh yards.*

So again upon the ſame firſt queſtion of Braces, I would know how many yards *Engliſh* they make.

After the rate that 100 Braces are worth 62 $\frac{1}{2}$ yards.
876 Braces.

438
109$\frac{1}{2}$

I anſwer, 547$\frac{1}{2}$ yards.

ITem, *To the underſtanding of this work, and ſuch like, firſt take the $\frac{1}{2}$ of the given Braces, and after take the $\frac{1}{4}$ of that halfe, or the $\frac{1}{2}$ of the given number, and adde them together, and the Products are alſo yards* Engliſh.

ITem, *three Ells of* Rochell *make 5 Ells at* Lisbone. *So likewiſe 3 Ells at* Lions *make 5 Ells at* Antwerpe.

 To

ample towards your left hand. Then take half of that product, or the $\frac{1}{4}$ *of the given number, and adde these two products together, as they shall be yards* English: *as by the example you may perceive.*

The second example toward your right hand is yet briefer then the first, whose worke is this; Take the $\frac{1}{4}$ *of the delivered number, and that product subtract out of the given number, and the rest sheweth your desire. Of these two wayes use which you think best.*

The Proof.

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How many Ells of *Antwerpe?*

	648	
	216	
maketh	864	Ells of *Antwerpe.*

15 Rule. ITem, *for the understanding of this worke, first take the* $\frac{1}{3}$ *part of the yards of* London, *which found adde that* $\frac{1}{3}$ *part and the yards together, as appeareth by the Practice, and the product sheweth the Ells of* Antwerpe.

Item, in 20 yards of *London*,
How many Ells of *Antwerpe?*

maketh	$426\frac{2}{3}$ Ells,		
	320 yards		Proof,
	$106\frac{2}{2}$		$426\frac{2}{3}$ Ells.
	$426\frac{2}{3}$ Ells	$\frac{2}{3}$	$106\frac{2}{3}$
			320 yards,

Other Reductions,

16 Rule. ITem, *you shall understand, that forasmuch as six braces of* Millain, *make five Ells of* Antwerpe, *whereupon according to the* Rules of Practice, *you may reduce the one into the other, by the like reason aforesaid,*

ſaid, in taking the $\frac{1}{6}$ *part, and then ſubtract the ſame, to make Ells of* Antwerpe. *And again by the contrary taking the* $\frac{1}{5}$ *part, with adding the given number, to turn the Ells to Braces. As for example.*

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876
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Ells 730 *Antwerpe.*

The contrary.
730 Ells *Flemmiſh.*
146 Braces.
876 Braces.

Ells 730 *Antwerpe.*
182 $\frac{1}{2}$
Yards 547 $\frac{1}{2}$ *Engliſh.*

Thus appeareth that 876 Braces by Practice make 730 Ells *Flemmiſh*, which Ells *Flemmiſh* reduce into *Engliſh yards.*

So again upon the ſame firſt queſtion of Braces, I would know how many yards *Engliſh* they make.

After the rate that 100 Braces are
worth 62 $\frac{1}{2}$ yards.
876 Braces.
438
109 $\frac{1}{2}$
I anſwer, 547 $\frac{1}{2}$ yards.

Item, *To the underſtanding of this work, and ſuch like, firſt take the* $\frac{1}{2}$ *of the given Braces, and after take the* $\frac{1}{4}$ *of that halfe, or the* $\frac{1}{8}$ *of the given number, and adde them together, and the Products are alſo yards* Engliſh.

Item, *three Ells of* Rochell *make* 5 *Ells at* Lisbone. *So likewiſe* 3 *Ells at* Lions *make* 5 *Ells at* Antwerpe.

To worke these and such like, double the Ells of Lions, *and the Ells of* Rochell, *and from their Product subtract the* $\frac{11}{21}$ *and the rest shall be the Ells of* Antwerp, *or the Ells of* Lisbone.

I. *Example.*

In 63 Ells of *Lions*, how many Ells of *Antwerpe?*

63
63
126
$\frac{1}{2}$ 21

Ans. 105 Ells *Ant.*

In 100 Ells of *Rochel*, how many Ells of *Lisbone?*

110
200
$\frac{1}{2}$ 23$\frac{2}{3}$.

Ans. 166$\frac{2}{3}$ Ells of *Lisb.*

Touching the proof or returne of these and such like questions, for a generall Rule, you shall first take the $\frac{1}{5}$ of the given number, and adde that $\frac{1}{5}$ and the given number together, and the $\frac{1}{2}$ of that product shall be your desire.

II. *Example.*

In 105 Ells of *Antwerpe*, how many Ells of *Lions?*

105
$\frac{1}{5}$) 21
$\frac{1}{2}$) 126

Ans. 63 Ells of *Lions.*

In 166$\frac{2}{3}$ Ells of *Lisbone*, how many Ells of *Rochell?*

166$\frac{2}{3}$
$\frac{1}{5}$) 33$\frac{1}{3}$
$\frac{1}{2}$) 200

Ans. 100 of *Roch.*

Questions

Queſtions of Factorage and Intereſt, briefly and truly reſolved by the Rule of Practice without Time.

1 *Queſtion.*

AT 5 ſhillings *per Centum*, what comes 8860 li. 15 s. 4 d. unto?

Anſwer. Note that 5 s. is the fourth of 20 s. I take the $\frac{1}{4}$ part of 8860 li. 15 s. 4 d, which makes 2215 li. 3 s. 10 d. Now the Root is 100, which you ſhould divide by, ſo cutting the two laſt figures away of the pounds, you have 22 li. then multiply 15 li. by 23's, ſo adde the 3 unto, you ſhall have 303 s, cut away the two laſt figures, there reſteth 03 s. Laſtly, there remains 3 s, which I multiply by 12 to bring into pence, and ſo I finde od. and $\frac{45}{100}$ remaining, which being abbreviated make $\frac{23}{50}$ parts of a peny, ſo I finde that there is gained 22 li. 3 s. od. $\frac{23}{50}$ parts of a peny.

li. 22. 15. 03. 10.
| 20
s. 3 | 03
| 12
| 06
od | 4
| 4 23
| 100 | 50

2. *Queſt.* At 10 s. *per centum*, what comes 1448 li. 16 s. 8 d. unto?

Anſw. Note that 10 s. is the $\frac{1}{2}$ of 20 s. I take the $\frac{1}{2}$ 1448 li. 16 s. 8 d. which makes 724 li. 8 s. 4 d. cut off the two laſt figures, and there reſteth 7 li. then multiply the 24 li. by 20 s. and adde the 8 s. and it maketh 488 s. cut

li. 7. | 24. 8. 4.
| 20.
s. 4 | 88
| 12
180
88
d. 106 | 0 | 3
| 100 | 5

 the

the two laſt figures off, and there reſteth 4 s. then multiply 88 s. by 12 d. and take in 4 d. and there reſteth 1060 d. cut off the two laſt figures, and there reſteth 10 d. and $\frac{60}{100}$ which is $\frac{3}{5}$ of a peny: ſo the whole ſumme is 7 li. 4 s. 10 $\frac{3}{5}$ which is the anſwer to the queſtion.

3. *Queſt.* At 15 ſhillings *per centum*, what comes 1008 l. 12 s. 0 d. unto?

Anſwer. Note 15. that is $\frac{1}{2}$ and $\frac{1}{4}$ of 20. take the $\frac{1}{2}$ of 1008 li. 12 ſhillings, there reſteth 504 li. 6 s. then take the $\frac{1}{4}$ adde them together, the totall will be 756 li. 9 s. cut off the 2 laſt figures, reſteth 7 li. then multiply by 20 s. & take in your 9 s. it maketh 1129 s. cut off the two laſt figures, there reſteth 11 s. then multiply by 12 d. there cometh 348 d. cut off the laſt two figures, there reſteth 3 d. and $\frac{48}{100}$ which being abbreviated maketh $\frac{12}{25}$ parts of a peny, ſo ſhall you finde 7 li. 11 s. 3 d. $\frac{12}{25}$. which is the anſwer to the queſtion.

1008 l. 12 s. 0 d. unto?
504. 6 —— 0
252. 3 —— 0
7 | 56. 9 —— 0
20
11 | 29
12
58
29
3 | 48 | 24 | 12
100 | 50 | 25

4. *Queſt.* At 1 li. *per centum*, what comes 8 | 68 li. 13 s. 4 d. unto?

Anſwer. Cut away the two laſt figures, and multiply by 20 and 12, and take in your ſhillings and pence. And you ſhall finde 8 li. 13 s. 8 pence $\frac{4}{5}$ as doth appeare by this work.

li. 8 | 68 li. 13 s. 4 d. unto?
20
s. 13 | 73
12
150
73
d. 8 | 8 | 0 | 4
100 | 5

5 *Queſt.*

5. *Quest.* At 2 li. *per centum*, what comes 5608 li. 6s. 8d. unto?

2

112|16|13. 4

20

3|33

12

70

33

400

Answ. Multiply the whole summe by two lib. thus, then cut off the two last figures of your pounds, as you did before, and you shall finde 112 pound, then multiply by 20 and by 12, taking in your shillings and pence, and you shall finde 112 li. 3 s. 4 d. which is either for Factor or Broker, &c.

6. *Quest.* At 3 pound *per centum*, what comes 800 li. 18 s. 2d. unto?

3

24|02. 14. 6.

20

0|54

12

114

54

6|54|27

100 50

Answer. Multiply the sum by 3 pound, thus; then cut off the two last figures, and you shall find 24 pound; then multiply by 20, and by 12. taking in your shillings and d. and you shall finde 0s. 6 d. $\frac{27}{50}$ parts of a peny, which is something above a half-peny.

7. *Quest.* At four *per centum*, what comes 298 li. 15s. 9d. unto?

4

11|95. 3. 0.

20

19|03

12

06

03

036 18 9

100 50 25

Answer. Multiply by 4 li. thus, cut off the two last figures; multiply by 20, and by 12. taking in your shillings and pence, and you shall finde 11 li. 19s. 0d. $\frac{9}{25}$ parts of a peny, which is something above a farthing.

8 *Queſt.* At 5 li. $\frac{3}{4}$ *per centum*, what comes 3658 li. 16 s. 8 d. unto? 5

Anſwer. Multiply by 5 li. thus, then take the $\frac{1}{2}$ of the whole ſumme and place the figures even, then take the $\frac{1}{2}$ of that $\frac{1}{2}$ and adde all three ſums together, cut off the two laſt figures, then multiply by 20. and by 12. taking in your ſhillings and pence, and you ſhall finde 210 l. 7 s. 7 d. $\frac{9}{10}$ parts of a peny, which is the anſwer to the queſtion.

18294 03 4
1829 08 4
914 14 2
210|38 05 10
|20
7|65
|12
130
66
7|9|0
10|0

9 *Queſt.* At 6 l. $\frac{1}{2}$ *per centum*, what comes 5684 li. 12 s. 6 d. unto? 6

Anſwer. Multiply by 6 li. and then take $\frac{1}{2}$ of the whole ſumme, adde them both together, then multiply by 20, and by 12, taking in your odde ſhillings and pence, and you ſhall finde 369 li. 10 s. 0 d. $\frac{3}{20}$ parts of a peny, which is the anſwer to your queſtion.

34107 15 0
2842 06 3
369|50 01 3
|20
05
01
15|3
100|20

10 *Queſt.* At 7 li. $\frac{1}{2}$ *per centum*, what comes 3868 li. 13 s. 4 d. unto? 7

Anſwer, Multiply by 7 li. then take the $\frac{1}{2}$ adde them together, cut off the two laſt figures, then multiply by 20, you ſhall finde 290 li. 3 s. The anſwer to the queſtion.

27080. 13. 4
1934. 06. 8
290|150 0
|20
3,00

11. *Queſt.*

11 *Queſt.* At 8. li. *per Centum*, what comes 2560l. 17s. 9d. unto?

Anſwer, Multiply 8 li. cut off the two laſt figures, multiply by 20. and by 12. and you ſhall finde 204 li. 17 s. 5 d. $\frac{1}{25}$ parts of a peny.

```
2560l. 17s. 9d.
         8
---------------
204|87. 02. 0
   |20
   -----
 17|42
   |12
   --------
    84
   42
---------
   5|04| 2| 1
-----------
  100|50|25
```

Queſtions of Intereſt with Time, wrought by Practice.

1. *Queſtion.*

AT 6 *per Centum*, what comes unto for 1 month 468 li. 16 s. 8 d?

Anſwer. Multiply by 6 li. there cometh 2813 li. 00 s. 0 d. then take for 1 moneth the $\frac{1}{12}$ of the Totall, and you ſhall finde 234 li. 8 s. 4 d. of the two laſt figures of the li. Multiply by 20 and by 12, taking in your odd money, and you ſhall finde 2 li. 6 s. 10 d. $\frac{3}{5}$ parts of a peny, which is the anſwer to the queſtion.

```
468 li.  16 s.  8 d?
2813.    00.    0
--------------------
li. 2|34. 08. 4
     |20
     -----
s.  6|88
     |12
     -------
     180
     88
d. 10|6 |0 |3
----------------
     10|0 |5
```

2. *Quest.* At 7 pound $\frac{1}{2}$ *per centum*, what comes unto for 2 months?

Answer. Multiply by 7 pound, then take $\frac{1}{2}$, adde them two together: then for your two months take the $\frac{1}{6}$ of the Totall, multiply by 20, and 12. taking in your odde shillings and pence, and you shall finde 47 pounds 10 shillings 1 peny $\frac{9}{10}$ parts of a peny, which is the answer to the question.

3800 li. 12 s. 8 d?
7
26604. 08 8
1900. 06 4
28504. 15 0
47|50 15 10
|20
10|15
|12
30
1 6
1| 9|0
10|0

3. *Question.* At 8 pound *per centum*, what comes unto for 3 months

Answer. Multiply by 8 pound, then take for your 3 months $\frac{1}{4}$ of the Totall, multiply by 20, and by 12, adding in your odde shillings and pence, and you shall finde 197 pound 5 shillings 11 pence, $\frac{3}{25}$ parts of a peny, your demand.

9864 li. 16 s. 4 d?
8
78918. 10 8
197|29. 12 8
|20
592
12
192
93
11|12|6|3
100|50|25

4. *Quest.* At 6 pound $\frac{1}{2}$ *per centum*, what comes unto for 4 months

Answer. Multiply by 6 li. Then take $\frac{1}{2}$, adde both together, then for your four months take $\frac{1}{3}$ part of the whole, cut away your two last figures, multiply by 20, and by 12, adde in your odde shillings and pence, and you shall finde 131 pounds 14 shillings 11 pence $\frac{19}{50}$ parts of a peny, your demand.

```
6080 li. 13s. od?
               6
-----------------
36483 | 18   0
 3040 | 06   6
-----------------
 39524  04   6
131|74  14  10
   |20
14 |94
   |12
   188
    94
11 |38 /19
   |100|50
```

5. *Quest.* At 8 *per centum*, what comes unto for 5 months

Answer. Multiply by 8 li. then for 5 months take $\frac{1}{4}$ and $\frac{1}{6}$ of the Totall, cut off the 2 last figures of your pounds, multiply by 20 and by 12, adde in your odde shillings and pence, and you shall finde 100 pound 13 shillings 4 pence, your demand.

```
3020 li. oos. ood?
               8
------------------
24160  00  00
------------------
 6040  00  00
 4026  13  04
------------------
100|66  13  04
   |20
------------------
13 |33
   |12
------------------
   |70
   |33
------------------
  4|00
```

6. *Quest.*

6 *Question.* At 8 *per Centum*, what comes unto for 6 months

8060 li. 12 s. 0 d?
8
64484. 16 s.
322|42. 08. 0.
20
8|48
12
96
48
5|100 76|50 38|25 19

Answer. Multiply by 8 li. then for your six moneths take the $\frac{1}{2}$ of the Totall, cut off the two last figures of your pounds. Multiply by 12. taking in your odde shillings and pence, and you shall finde 322 li. 8 s. 5 d. $\frac{19}{25}$ parts of a peny, your desire.

7 *Quest.* At 8 li. *per Centum*, what comes unto for 7 moneths

5896. 00.0 d?
8
47168.0.0
15722. 0. 0
11792. 13.4
275|14.13.4
20
2|93
12
190
93
11|2 |0 |1
10|0 |5

Answer. Multiply by 8 li. then for your 7 moneths take $\frac{1}{3}$ and $\frac{1}{4}$ of the Totall, cut off the two last figures of your pounds, then multiply by 20, and 12, taking in your odde money, and you shall finde 275 li. 2 s. 11 d. $\frac{1}{5}$ your desire.

8. *Quest.*

8 *Question.* At 8 *per centum*, what comes unto for 8 months

Answer. Multiply by 8 li. then for 8 months take $\frac{2}{3}$ of the totall, cut off the two last figures of your pounds, then multiply by 20, and by 12, adde in your odde money, and you shall finde 116 li. 5 shillings, 9 pence, $\frac{3}{25}$ your desire.

3680 li.	08 s.	0 d.
		8
29443.	04	0
9814.	08	0
9814.	08	0
116 | 28	16	0

| 20
5 | 76
| 12
152
76
9 | 12 | 6 | 3 |
10 | 100 | 25 |

9 *Quest.* At 8 li. *per centum*, what comes unto for 9 months

Answer. Multiply by 8 pound, then for your nine moneths take $\frac{1}{2}$ and $\frac{1}{4}$ of the whole summe, cut off the two last figures of the pounds, then multiply by 29, and by 12. taking in your odde shillings and pence, and you shall finde 221 pounds, 1 shilling, 11 pence, $\frac{7}{25}$ which is something above a farthing.

3684 li.	19 s.	0 d.
		8
29479.	12	0
14739.	16.	0
7369.	18.	0
221 | 09.	14	0

| 20
1 | 94
| 12
188
94
11 | 28 | 14 | 7
100 | 50 | 25

10. *Quest.*

10. *Queſt.* At $6\frac{3}{4}$ *per centum*, what comes unto for 10 months

Anſwer. Multiply by 6 pound, then take the $\frac{1}{2}$ and $\frac{1}{4}$ of 100 pound, adde all three ſummes together, then for the 10 months take $\frac{1}{2}$ and $\frac{1}{3}$ of the Totall, adde them together, cut off the two laſt figures of the pounds, multiply by 20, and 12, adding in your ſhillings and pence, cutting off the laſt figures of your ſhillings and pence, you ſhall finde 5 pound 12 ſhillings, 6 pence, your deſire.

```
100. li. 0s. 0d?.
              6
-----------------
600.    0    0
 50.    0    0
 25.    0    0
-----------------
675.    0    0
-----------------
337.   10    0
225.   00.   0
-----------------
   5|62.10.  0.
    |20.
-----------------
  12|50
    |12
-----------------
   100
    50
-----------------
    6|00
```

11. *Queſt.* At 8 pound *per centum*, what comes unto for 11 months

Anſwer. Multiply by 8 pound, then for 11 months take $\frac{2}{3}$ and $\frac{1}{4}$ from the Totall, adde all three ſummes together; cut off the two laſt figures of your pounds, multiply by 20 and by 12, adding in of your ſhillings and pence, cutting off the two laſt figures of your ſhillings and pence, and you ſhall finde 65 pound 0 ſhillings 7 d. $\frac{17}{25}$ parts of a peny, your deſire.

```
886 li. 16s. 0d?
              8
-----------------
 7094  08    0
-----------------
 2364  16    0
 2364  16    0
 1773  12    0
65|03  04    0
  |20
-----------------
 064
  |12
-----------------
 128
 64
-----------------
7|68|34|17
-----------------
100/50/25
```

12. *Queſt.*

12 *Quest*. At 8 pound *per centum*, what comes unto for 12 months

Answer. Multiply by 8 pound, cut off the two last figures of the pounds, multiply by 20, and by 12, adding in your shillings & pence, cut off the two last figures of your shillings, & the two last of your pence, and you shall finde 726 l. 8 s. 11 d. $\frac{17}{25}$ parts of a peny, your desire.

	9080 li.	12 s.	2 d ?
			8
726	44	17	4
	20		
8	97		
	12		
11	68	34	17
	100	50	25

The third Chapter teacheth of the Order and work of the Rule of *Three* in broken numbers after the Trade of *Merchants*, digressing something from Master *Record*, which is comprehended in three Rules.

NOW *that I have somewhat intreated of the* Rules of Practice; *I will give a few instructions, after my simple order, for the working of the* Rule of Three *in broken numbers: wherein I shall need to say the lesse, because I hope the studious Learner, that hath travelled any thing in the* Grounds of Arts, *is not unfurnished of knowledge capable to understand me.*

But before I deliver any instructions for broken Numbers, I will propose a question which shall be wrought three sundry wayes, thereby to shew as it were, three Degrees of Comparison, how farre the Rule of Three in broken, for more speed of worke, differeth from the whole; which

I

I rather set down for a view, that the studious herein may be more desirous to attaine broken, leaving any more to discourse in Dialogue forme, but onely to give instructions where need is, and in the rest to put forth the questions, with their answers.

My first question is thus,

If one yard cost 6 s. 8 d. what are 789 worth at that rate?

The first way.

1 ——— 6 s ——— 8 d ——— 789
12 ——— 80
——— ———
80 ——— 63 120d.

Here the Product of the summe are pence, according to the nature of the middle number.

11
1370 1
63120 (5260 (263
12222 2220
111

I answer ——— 263 li.

The second way.

1 ——— $6\frac{2}{3}$ s ——— 789
——— 20
3 20 ———
15780s.

Here the Product of the summes are shillings according to the nature of the middle number.

1
15786 (5260 (263
3333 2220
li.

The third way.

1 ——— $\frac{1}{3}$ ——— 789
3 1 1
789

Here

Here the Product is pounds, according to the title of the ſecond number.

$\cancel{1}$
$\cancel{7}\cancel{8}\cancel{9}$ (263
$\cancel{3}\cancel{3}\cancel{3}$

I anſwer, 263 li.

Now that you have ſeen the three former vertues of the Rule of three, whoſe Products have firſt brought forth pence, next ſhillings, and laſtly pounds, I will deliver three notes in order following: and with them a dozen queſtions, that ſhall ſhew the work of the Rule of three in broken Numbers or Fractions.

1. The firſt four ſhall be ſundry queſtions of a Fraction coming in the ſecond place. Note theſe three well.
2. The ſecond four ſhall be of two Fractions coming in the ſecond or third place.
3. The third four of Fractions in all three places.

Notes upon the firſt Rule for a Fraction coming in the ſecond place.

My firſt Queſtion is this.

If one yard coſt me 3 ſhillings 4 pence, *what are* 756 *worth at that price?* 1 Rule.

In ſetting down the queſtion to perform the work, I turn four pence into the part of a ſhilling, which is $\frac{1}{3}$ and then the queſtion ſtandeth thus: The firſt variety.

$$1 \text{ ———— } 3\tfrac{1}{3} \text{ ———— } 756$$

To the ready working of this queſtion, and all ſuch other like, my firſt note is this, which take for a generall Rule, That when any one Fraction ſhall come,

A generall Rule. come either in the ſecond or third place, that the Denominator of that Fraction or Fractions muſt alwayes be brought unto the Number or Numerator of the firſt place ; and thereby multiply the one into the other.

And this benefit is alwayes gotten by the vertue of bringing the Denominator of the ſecond Numbers Fractions unto the firſt place : For the Fraction in the middle number is now releaſed and the Product that cometh of the Multiplication, is of the nature and like the denomination of the whole number in the ſecond place, which here are ſhillings. Note this.

Whereupon now, to work the Queſtion, I bring 3, the Denominator of the Fraction in the ſecond place, unto the firſt number 1, with a line ſet under thus $\frac{1}{\ }$ and the third under it thus, $\frac{1}{3}$ ſaying, once 3 is 3 my Diviſor : that done, reduce 3 $\frac{1}{3}$ ſaying, 3 times 3 is 9, and the other 1 over 3 make 10, my ſecond number in the *Rule of Three*, by which 10 I doe multiply my laſt number 756, as appeareth by the work thereof, and it yieldeth 7560 ſhillings my Dividend.

Then dividing 7560 by 3, my Diviſor, it yieldeth in quotient 2520 ſhillings, which maketh 126 pounds, as appeareth here moſt plainly, both by the Example, and the work.

At 3 s. 4 d. the yard, what 756 yards?

1	$3\frac{1}{3}$	756
3	10	10
		7560 s.

7560 | 2520 (126.
3333 | 2220

I anſwer 126 li.

Yet

Yet otherwise upon the ſame queſtion, altering the price now into the proportion it beareth to a pound, for the 3 s. 4 d. *is* $\frac{1}{6}$ *part of a pound: which Example firſt ſtandeth thus as appeareth on the left hand, and afterwards wrought as appeareth on the right hand.*

$$1 \text{———} \tfrac{1}{6} \text{———} 756 \qquad \begin{array}{c} 1 \text{———} \tfrac{1}{6} \text{———} 756 \\ 6 \qquad\quad 1 \qquad\quad 1 \\ \hline 126 \text{ pounds.} \end{array}$$

As ſoon as I have carried 6, the Denominator of my middle number, unto my firſt place, as before hath been taught, I pull down 1, the numerator of 6, with a line under 6, thus $\frac{6}{1}$, and that one in cuſtome I pull down in ſight; being the figure that I will multiply my third or laſt number by, according to the tenour of the *Rule of Three*. And becauſe one can neither multiply nor yet divide (though here it is ſet down in form of multiplication, the rather for your underſtanding) the Product of the multiplication according to the declaration of this my firſt Rule or Note, is converted into the Title of my ſecond number, which here are pounds. Now followeth the diviſion performed in my Diviſor 6, to make an end of that queſtion. The ſecond variety.

$\cancel{1}\cancel{3}$
$\cancel{7}\cancel{5}\cancel{6}$ (126. *which maketh* 126 li. *as before.*
$\cancel{6}\cancel{6}\cancel{6}$

And thus much for the variety in working that queſtion.

And now followeth another.

My second Question.

If one yard of Cotten cost 8 $\frac{1}{4}$ d. what 859?

$$
\begin{array}{ccc}
1 & 8\frac{1}{4} & 859 \\
\hline
4 & 33 & 33 \\
 & \overline{2577} & \\
 & 2577 & \\
 & \overline{28347} &
\end{array}
$$

```
          1
    1|321        1|1       li.      s.     d.
 28347 (7086  (590      (29 ——— 10 ——— 6 3/4
  4444   1222   220
```

This Question was also wrought like the first, and bringeth forth 29 li. 10 s. 6 $\frac{3}{4}$ d. *the price of* 859 *yards.*

My third Question.

If seven pounds of any thing cost 3 li.—10 s. what comes 987 pounds to?

$$
\begin{array}{ccc}
 & \text{li.} & \\
7 & 3\frac{1}{2} & 987 \\
2 & 7 & 7 \\
\hline
14 & & 6909
\end{array}
$$

```
 00
 141
 234|7
 6909 (493 7/14 | 1/2
  1444
   11
```

I answer, 493 li.—10 s.

Notes

Notes upon my second Rule for two Fractions coming in the second and third place.

My first Question is this.

IF one Ell cost 13 s.——4 d. what halfe a quarter or $\frac{1}{8}$ of an Ell?

Answer, *First bring* 13 s.——4 d. *into the parts of a pound, which is* $\frac{2}{3}$, *and then will the question stand thus.*

1 ———— $\frac{2}{3}$ li. ———— $\frac{1}{8}$

ITem, for the performance of this worke, doe as before was taught in the first Rule: first *bring* 3 *the Denominator of the second Fraction unto your first number* 1, setting a line under it thus, $\underline{1}$ saying once 3 is 3. that done, bring 8 the *Denominator of the third Fraction*, setting it under 3, and multiply them together, saying, 3 times 8 maketh 24. which 24 is your *Divisor*. (Now have you done with the *Denominator* 8) therefore you shall put a line under thus, $\underline{3}$. and the like line also under $\underline{8}$, setting or pulling down under them their own *numerators*, that is, 2 under 3, and also 1 under 8, as appeareth in the Example: which *numerators* for a generall rule are evermore to be pulled down of custome in sight, to multiply the one by the other, according to the tenour of the *Rule of Three*. Then I multiply the one by the other, saying, once 2 is 2, which signifieth 2 pound, being of the nature and like denomination of the middle number, which 2 pound is to be reduced into shillings, otherwise it cannot be divided by my first number 24.

Then dividing 40 by 24, the quotient bringeth

forth 1 $\frac{2}{3}$. So much is $\frac{1}{8}$ of an Ell worth after that rate. Otherwise, although 2 pound could not be divided by 24, yet it might have been abbreviated to $\frac{1}{12}$ pound: which is worth 1 s. 8 d. as before.

li.

1 ———	- ——— $\frac{2}{3}$ ———	——— $\frac{1}{2}$	(1
3	2	1	$\cancel{2}$ (6
8	20		$\cancel{40}$ (1 $\frac{2}{3}$ s.
24	40		$\cancel{24}$

Second Question.

IF *one pound of any weight cost* 13 *shillings* 4 *pence, what are* $\frac{7}{8}$ *of the pound worth after that rate?*

Answer. Reduce the 13 shillings 4 pence into the parts of a pound: which is $\frac{2}{3}$, and then will the question stand thus:

li.

1 —— $\frac{2}{3}$ —— $\frac{7}{8}$

ITem, *for the understanding of this, if you marke well the last Example, this and the rest lieth open, and needs small instruction. For as you did last, so again, bring the* Denominator *of the second and third* Fraction *unto the first figure* 1, *multiplying the one into the other; which maketh also* 24, *your* Divisor.

Note. Then making a line under 3 thus, $\underline{3}$ and a line under 8 thus $\underline{8}$ and pulling down their *Numerators* under each figure, that is 2 under 3, and 7 under 8, which as I said before for a generall rule, I pull down of custome in sight, to be the two Numbers that of duty ought to be multiplyed together; which done, I bring 2, being the lesser figure, under 7, multi-

multiplying them together, it maketh 14. which are of the nature of the middle number, that is to wit, pounds, which 14 cannot aptly be divided among 4. therefore are reduced into *shillings*, as is plainly to be seen in the example: then 280 *shillings* parted among 24. yieldeth for his *quotient* 11 s. 8 d. your desire, and the just price of $\frac{7}{8}$ of an *Ell*. Otherwise 14. though it could not be divided by 24. might by *Mediation* or *Division* in broken *Numbers* have been divided or abbreviated to $\frac{7}{12}$, which in effect being reduced to his known parts, maketh 11 s. 8 d. as before. But my good will and meaning is to aid young beginners: therefore have I reduced the 14 *pound* into *shillings*, which is the easier way.

Now followeth the example.

1 ——	2 ——	7	1ı
3	3	8	$\cancel{2}$
8	2	7	$\cancel{4}$ (6
24		3	$\cancel{2}\cancel{8}\cancel{0}$ (11$\frac{2}{3}$s.
		14	$\cancel{2}\cancel{1}\cancel{4}$
		20	$\cancel{2}$
		280 s.	I answer, 11$\frac{2}{3}$s.

The third example.

If 1 yard cost me 2 s.——6 d. what 345$\frac{1}{4}$ yards?

Answer. First put 6 d. into the parts of a shilling, and then the question standeth thus:

1——2$\frac{1}{2}$——345$\frac{1}{4}$.

Item, to the ready understanding of this, and all

ſuch like, according as before hath been declared, bring the *Denominators* of the ſecond and third *Fractions* unto the firſt place, multiplying them the one into the other, all which make 8 for the common *Diviſor*. Then next reduce your ſecond number, ſaying, two times 2 is 4. and 1 is 5. as was taught in the Example aforeſaid. Laſtly, reduce your third number 345 $\frac{1}{4}$ all into fourths, and they make 1381. which 1381 is to be multiplyed by 5. according to the tenour of the *Rule of Three*: which done, maketh 6905 ſhillings, and divided by 8. your *Diviſor* yieldeth in Quotient 863$\frac{1}{8}$ ſhillings, which maketh in pounds 43 pounds, 3 ſhillings, 1$\frac{1}{2}$: and ſo much are the 345$\frac{1}{4}$ yards worth at that price.

The ſame queſtion wrought again by two ſhillings 6 pence, is now converted into the parts of a pound, and ſtandeth thus:

$$1 \text{ —— } \tfrac{1}{8} \text{ —— } 345\tfrac{1}{4}.$$

Item, After I have brought here my ſecond and third *denominator* unto my firſt place, and found 32 to be my *diviſor*; having thus finiſhed my firſt place with all things unto him belonging (which is meant of bringing and multiplying the *denominators* of the ſecond and third *Fractions* into him) I then goe in hand to ſee what is to do in my ſecond place, where preſently of cuſtome I pull down my numerator 1 under 8. being the figure in ſight that ſhall multiply my third number.

Then laſtly, I reduce 345 $\frac{1}{4}$ all into fourths, as afore was practiſed, which maketh 1381. the which 1381 I am to multiply by 1 my ſecond number: they are nothing increaſed, but by the *Metamorphoſis*

of

of my work they are now 1381. pound, being of the nature of the middle number, as I have often shewed you, which divided by 32, my Divisor yeeldeth 43 pound, and $\frac{5}{32}$, which $\frac{5}{32}$ of a pound reduced into known numbers, make 3 shillings $1\frac{1}{2}$ pence, as before.

Example.

$$
\begin{array}{ccccccc}
 & & & & \cancel{1} & & \\
1 & \text{———} & \frac{1}{8} & \text{———} \; 345\frac{1}{4} & \cancel{1}\cancel{0}\cancel{5} & & \\
32 & & 1 & & \cancel{1}\cancel{3}81 & (43 & \frac{5}{32} \\
 & & & 1381 & \cancel{3}\cancel{2}\cancel{2} & & \\
 & & & & \cancel{3} & &
\end{array}
$$

NOW follow four other questions, which are in all three places broken numbers; or whole and broken together.

Item, First, for the finding out of your *Divisor*, you shall take this for a most certaine and generall Rule, That you must multiply the *Numerator* of the first *number* in the question by the *Denominator* of the second; and that *Product* again, by the *Denominator* of the third: And the totall thereof shall be your *Divisor*.

Secondly, for a generall rule to finde out your *Dividend*, *multiply* the *Denominator* of the first number by the *Numerator* of the second, and the whole thereof by the *Numerator* of the third. And the totall thereof shall evermore be your *Dividend*.

Now for an example, I propose this question, thereby to make my meaning more plaine, and to shew you, as I have done in the rest, the manner and order of the work.

If $\frac{2}{3}$ of any weight or measure cost $\frac{5}{6}$ of a pound, or 20 shil-

20 s, *what are* $\frac{3}{8}$ *of the like weight or measure worth after that rate?*

Example.

$\frac{2}{3}$ —— $\frac{5}{6}$ —— $\frac{3}{8}$

ITem, for the more plaine understanding hereof, and all other the like in broken Numbers: First, you shall pull down two, the *Numerator* of the first Number or Fraction, with a line under, thus, $3\frac{2}{}$: that done, according as you have learned before, bring 6, the *Denominator* of the second *Fraction*, and set it under two, multiplying the one into the other, which maketh 12. Then lastly, bring 8. the *Denominator* of the third *Fraction*, and set it under 12. multiplying that 12 by 8. which amounteth to *96*. or else for more brief, multiply *6* by 8, saying, six times 8 makes 48. which 48 set under 2. and multiply the one into the other, it maketh *96*. as before. And this *96* is the first number in the *Rule of Three*. That shall alwayes for a most generall Rule be your *Divisor*.

Secondly, to work for your *dividend*, you shall, (as it hath beene sufficiently declared before) pull down 5. the *Numerator* of your second *Fraction*, and set it under *6*, with a line under thus $\underline{6}$.

That done (as you know) you are to pull down 3, the *Numerator* of the third *Fraction*, and set it under 8, with a line under it, thus, $\underline{8}$, multiplying the one into the other, according to the Tenour of the *Rule of Three*; which maketh 15. Then according to my Note, forget not to bring the *Denominator* of the first *Fraction*, which is 3, under 15. and multiply them together, which maketh 45. which 45 is your *dividend*, and are of the nature of

denomi-

denomination of the middle number, as I have taught you before: and therefore are 44 li. which aptly cannot be divided by 96. Therefore you ſhall reduce the 45 li. into s. as you ſee performed in the Example, which amounteth to 900s. which divided by 96 your *Diviſor* it yieldeth 9 s. and $\frac{16}{20}$ of a ſhilling, which in leſſer terms is $\frac{3}{8}$: which $\frac{3}{8}$ in money maketh $4\frac{1}{2}$d. and ſo much will the aforeſaid $\frac{3}{8}$ coſt, as by the work following ſhall appear.

The Example.

$\frac{2}{3}$	$\frac{5}{6}$	$\frac{3}{8}$	
2	5	3	
6		5	13
12		15	$\cancel{9}$ 16
8		3	$\cancel{9}\cancel{0}\cancel{0}$ ($9\frac{16}{20}$ | $\frac{3}{8}$ s.
96		45	$\cancel{9}\cancel{6}$
		20	
		900	

Otherwiſe though 45 could not be divided by 96. yet by Diviſion in broken numbers it might have been abbreviated to $\frac{15}{32}$ of a pound, which reduced into known parts, will make 9 s. $4\frac{1}{4}$d. as before.

Now my ſecond Example ſhall be the proof of this queſtion.

If $\frac{5}{8}$ *yards coſt* $\frac{15}{32}$ *of a pound, or* 20 *ſhillings, what ſhall* $\frac{2}{3}$ *coſt?*

Anſwer. Work as was taught you before, and you ſhall have your deſire.

Here

$$\frac{3}{8} \text{———} \frac{15}{32} \text{———} \frac{2}{3}$$

3	15	2
32	2	
96	30	
3	8	
288	240	

Here as appeareth by the work, the Multiplication being ended, 240 is to be divided by 288, which to some perchance may seem hard, yet notwithstanding is the work good. Therefore abbreviate 240. by 288. as you see here is practised: and the end of your abbreviation shall come to $\frac{5}{6}$ your desire, $\frac{240}{288}$ $\frac{30}{36}$ $\frac{5}{6}$.

Otherwise, $\begin{array}{c|c|c|c|c} 240 & 120 & 60 & 30 & 5 \\ 288 & 144 & 72 & 36 & 6 \end{array}$

Otherwise, $\begin{array}{c|c|c} 340 & 40 & 5 \\ 288 & 48 & 6 \end{array}$

The third Question.

If $\frac{3}{4}$ *ells cost* 13s.—4d. *what* 156 $\frac{1}{2}$ *ells ?*

Answer. To work this question the shortest way, reduce 13 shillings 4 pence into the parts of a pound, which is $\frac{2}{3}$.

Then as you did afore, after you have set down the question, the *Numerator* of the first *Fraction* 3 is pulled down under 4. and *Denominators* of the other two *Fractions* multiplyed into him, which maketh 18. your *Divisor*.

Then the *Numerators* of the second *Fraction* is pulled downe under 3 in custome now in sight, ready to *multiply* my third number, by which is performed as soon as the last numbers 156$\frac{1}{2}$ is reduced into halfs.

Then

Then laſtly, I multiply that product by 4. the *Denominator* of the *Fraction*: it yeeldeth 2504. which I divide by 18. and my quotient is 139 pound, and $\frac{2}{18}$ or $\frac{1}{9}$ of a pound remaining, which is worth 2 s—$2\frac{2}{3}$d. And ſo much will $156\frac{1}{2}$ Ells coſt, as by the work following doth appear.

3	2	$156\frac{1}{2}$	1	
4	3	313	47	
3	2	2	1762	li.
6		626	2504	$139\frac{1}{9}$
18		4	1888	
		2504	11	

The fourth queſtion.

If $2\frac{1}{2}$ *Ells coſt* $1\frac{2}{3}$ *pounds, what cometh* $29\frac{1}{4}$ *Ells to?*

Item, to the workmanſhip of this queſtion, firſt reduce your ſecond *number* in ſaying, three times 1 is 3, and two is 5. Then bring the *multiplication* of the *Denominators* of the ſecond and third *Fractions* which maketh 12. and *multiply* that 12 by 5 your firſt *numerator*, and it maketh 60. which is your *diviſor*.

Then the *Reduction* of the ſecond Number, which is 5, *multiplyed* by 117. the Product of the laſt *numbers* reduction, make 585, which 585 yet reſteth to be *multiplyed* by 2, the *Denominator* of the Fraction in the firſt place, yeeldeth 1170, which divided by your *Diviſor*, 60 yeeldeth 19 pound, 10 ſhillings, as appeareth by the work thereof.

Thus having now touched the 12 queſtions whereof I firſt pretended, which with diligence and oft practice;

practice, I trust are sufficient to aid the desirous unto the working of any broken numbers. I will now treat of divers necessary rules incident unto traffick, as hereafter followeth.

The fourth Chapter teacheth of losse and gaine in the Trade of Merchandise.

IF one yard cost 6 s.——8 d. and the same is sold againe for 8 s.———6 d. the question is what is gained in one hundred pounds laying out on such commodities.

Answer. The *Rule of Three* direct, applyed two manner of wayes to doe the same: the one is to say, If $6\frac{2}{3}$ give $8\frac{1}{2}$, what giveth 100? *Multiply* and Divide, and looke what your quotient bringeth forth, above your laying out, is the neat gaines and solution to your question: If you follow the worke, your solution will bring forth 127 li.———10 s. which is 27 li.——10. more then your principall, and so much is gained in the 100 pounds laying out.

Item, to work it the other way, which I take the nearest, seek the difference betwixt the just price and the other price, which is one shilling ten pence, then say by the *Rule of Three*,

If $6\frac{2}{3}$s. gain $1\frac{5}{6}$s. what shall a 100 pound gain?

Multiply and divide, and you shall finde 27 li. 10 s. and so much is gained in 100 li. laying out.

You may use which of these two wayes you think good.

The

The proof.

If a yard of cloth be delivered for 8 s. 6 d. whereupon was gained after the rate of 27 li. 10 s. in 100 pounds laying out: the question is, what the yard cost at the first hand?

Answer. Put your gain 27 li.—10 s. to 100 pounds, all maketh 127 li.—— 10 s. Then say, If 127 li. 10 s. give but a 100 pounds, what giveth $8\frac{1}{2}$s? Work, and you shall finde 6 s. 8 d. the true solution to your question.

Yet another example or proof upon the first Question.

If one yard cost 6 s.—8 d. the question is, at what price the same is to be sold again, for to gain 27 li. 10 s. in 100 pound laying out?

Answer. *Say by the* Rule of Three, *if a* 100 li. *gain* 127 li. 10 s. *what giveth* $6\frac{2}{3}$ s? *Multiply and divide, and you shall finde* 8 s. 6 d. *your true solution.*

If one Ell cost 7 s. 8 d. and be sold again for 8 s. 6 d. *The question is*, What is gained in 20 pound laying out in such commodities.

Answer. *Seek the difference betwixt the just price, and the other price which is ten pence, and then apply the* Rule of Three, *as before is taught, saying, If* $7\frac{2}{3}$ s. *give* $\frac{5}{6}$ *shillings, what giveth* 20 li? *Multiply and divide, and you shall finde* 2 li. $3\frac{11}{23}$ s. *and so much is gained in* 20 li. *laying out.*

The proof also by an example of Losse.

A *Merchant hath bought Holland cloth at* 8 s. 6 d. *the Ell, which proveth not to his expectation, whereupon he is content to lose* 2 li. $3\frac{11}{23}$ *in* 20 *pounds laying*

laying out. The queſtion is, what price ought to be made of the Cloth, abating this loſſe?

Anſwer. Do as before in Gains hath been taught, putting 2 li. $3\frac{11}{23}$ s. to your 20 pound, all together maketh 22 li. — $3\frac{11}{23}$. Then ſay by the *Rule of Three*, If 22 li. $3\frac{11}{23}$ s, give but 20 li. what ſhall come of $8\frac{1}{2}$ s? work, and you ſhall finde 7 s.--8 d. the juſt price that the Ell ought to be ſold for, after the rate of this loſſe.

Thus it appeareth evidently, as in company the *Rule* is appliable as well to gain as loſſe.

If $20\frac{1}{4}$ *yards coſt* 36 li. 10 s. *how ſhall I ſell the ſame again* $\frac{2}{3}$ *of the Principall, or to make of* 3, 4. *which is all one.*

Anſwer. By the *Rule of Three*, If 3 doe give 4, what will $36\frac{1}{2}$ give? Multiply and divide, and you ſhall finde $48\frac{2}{3}$ pounds : then ſay again, if $20\frac{1}{4}$ yards doe give $48\frac{2}{3}$ pounds, as well principall as gain, what will one yard be worth at that price? Multiply and divide, and you ſhall finde 2 li. $\frac{98}{243}$.

If one Ell of Cloth coſt me 8 s. 8 d. *and afterwards I ſell* $10\frac{1}{4}$ *Ells thereof for* 5 li. 13 s. 4 d. *I would know whether I winne or loſe? and how much upon the* 100 *pounds of money.*

Anſwer. See firſt at 8 s. 8 d. the Ell, what $10\frac{1}{2}$ Ells come to, and you ſhall finde 4 li. 11 s. and I ſold the ſame for 5 li.—13 s.—4 d. ſo that I did gain upon the $10\frac{1}{2}$ Ells 22 ſhillings 4 d. Then if you would know how much is gained in 100 pounds, I ſay by the *Rule of Three*, if 4 li.—11 s. did gain 22 s.--4 d. what will 100 pounds gain? Multiply and divide, and you ſhall finde 24 li.—10 s.—10 d. $\frac{10}{91}$ and ſo much is gained in the 100 pound of money.

If $12\frac{1}{2}$ *yards coſt me* 11 *pound five ſhillings, and I ſell*

sell the yard againe for 16 shillings, the question is, whether I doe winne or lose, and how much in or upon the pound of money?

Answer. Look what the 12 $\frac{1}{2}$ yards come to at 16s the yard, and you shall finde ten pound. But they cost 11 pound 5 s. So there is lost upon the whole 1 pound 5 shillings. Then to know how much is lost in the pound, say by the *Rule of Three*, if 11 $\frac{1}{4}$ pound doe lose 1 $\frac{1}{4}$ pound, what will 1 pound lose? Multiply and divide, and you shall find 2 s. 2 d. $\frac{2}{3}$, and so much is lost in the pound of money.

If I sell the 100 weight of any commodity for 4 pound, whereupon I do lose after ten pound in the 100 pound, I demand how much I shall lose or gaine in the 100 pound, in case I had sold the same for 4 pound ten shillings.

Answer. Say, if 90 pound yeeld 100. how much will 4 give? Multiply and divide, and you shall finde 4$\frac{4}{9}$. Then say again, if 4$\frac{4}{9}$ give me 4 $\frac{1}{2}$ what will 100 come to? Multiply and divide, and you shall finde 101 pound $\frac{1}{4}$ which is more then 100 pound by 1 li. 5 s., and so much is gained in the 100 pound.

A Merchant hath sold Currants for the summe of 430 Pound, and he hath gained therein after 10 pound in the 100 Pound. The question is, to know how much he gained in all?

Answer. Say by the *Rule of Three*, If 100 pound doe gain 10 pound, what will 430 pound gain? Multiply and divide, and you shall finde 43, and so much hath he gained in all.

If one yard be worth 28 $\frac{1}{2}$ s. for how much shall 10 yards be sold to gain after 8 pound 6 shillings 8 pence in the 100 pound?

Answer.

Anſwer. Firſt, adde 8 li.—6 s—8 d. to 100. Then ſay, if 100 li. do give 108 $\frac{1}{3}$ s. for principall and gain, what will 28 $\frac{1}{2}$ s. principall yield? Multiply and divide, and you ſhall finde 30 $\frac{7}{8}$ s. Then ſay againe by the *Rule of Three*, if 1 yard do give 30 $\frac{7}{8}$ s. (which is as well the principall as the gain) what ſhall ten yards give? Multiply and divide, and you ſhall finde 15 li. 8 s. 9 d. And for the ſame price ſhall the ten yards be ſold, for to gain after the rate of 8 li.-6 s.-8d. upon the 100.

A Branch or Proof out of this Queſtion.

A Merchant hath ſold clothes for 15 li.--8 s.--9 d. *and he hath gained in the whole the ſumme of* 1 li.—3 s.— 9d. *The queſtion is to know how much he hath gained in the* 100 *pound.*

Anſwer. To know this, firſt rebate the gains from the price, and there will remain 14 li. 5 s. 0 d. Then ſay by the *Rule of Three* direct. If 14 li. $\frac{1}{4}$ give me 1 li. 3 $\frac{3}{4}$, what will 100 li. give? Multiply and divide, and you ſhall finde 8 li. 6 s. 8 d. the effect deſired. The proof is apparent in the queſtion before.

Yet another Branch or Proof of the firſt Queſtion.

If ten yards be delivered for 15 li. 8 s 9 d. *whereupon was gained after the rate of* 8 li. 6 s. 8 d. *upon the* 100 *pound, the queſtion is, what the yard did coſt at the firſt hand?*

Anſwer. Firſt, ſay by the *Rule of three*, if ten with principall and gain yeeld 15 li. 8 s. $\frac{3}{4}$ ſhillings, what ſhall 1 yeeld? Multiply and divide, & you ſhall finde

30

$30\frac{7}{8}$ s. Then say again by the *Rule of Three*, if 108 $\frac{1}{3}$ principall and gain give but 100. what shall $30\frac{7}{8}$ of principall and gain yeeld? Work, and you shall finde $28\frac{1}{2}$ s. And so much did the yard cost at the first peny.

If one yard cost 36 s. *how much shall* 12 *yards be sold for, to gain after the rate of* 10 li. *in the* 100?

Answer. First say, If 100. give 110 li. principall and gain, what will 36 s. give? Multiply and divide, and you shall finde $39\frac{3}{5}$ s. Then say again by the *Rule of Three*, If one yard of principall and gain yeeld $39\frac{3}{5}$ shillings, what shall 12 yards gain? Multiply and divide, and you shall finde 23 li. $15\frac{1}{5}$ s. which $\frac{1}{5}$ s. in known number, is $2\frac{2}{5}$ d. And for the same price shall the 12 yards be sold, to gain after the rate of 10 in the 100.

The Proof.

If 12 *yards be sold for* 23 li. 15 s. $2\frac{2}{5}$ d. *whereupon is gained after* 10 li. *in the* 100. *the question is, what the yard cost at the first peny.*

Answer. First say, If 12 give 23 li. $15\frac{1}{5}$ s. what one yard? Multiply and divide, and you shall finde $39\frac{3}{5}$ s. Then say again by the *Rule of Three*, If 110 pounds give but 100, what shall $39\frac{3}{5}$ s give? Work, and you shall finde 36 s. the just price of the yard at the first hand.

Item, *When one Merchant selleth wares to another, and he giveth to the buyer* 1 li. 6 s. 8 d. *upon the score, or* 20 li. *the question is, how much shall the buyer gain upon the* 100 li. *after that rate?*

Answer. First, adde 1 li. 6 s. 8 d. unto 20 li. and they are $21\frac{1}{3}$. Then say, If 20 pound give $21\frac{1}{3}$ what shall

ſhall 100 give? Multiply and divide, and you ſhall finde $106\frac{2}{3}$. So the buyer getteth after the rate of $6\frac{2}{3}$ li. upon the 100 li..

Gentle Reader, other neceſſary queſtions appertaining to loſſe and Gain, you ſhall have in the eighth Chapter of this Treatiſe.

The fifth Chapter treateth of Loſſe and Gain upon time, wrought by *the double Rule of Three, or by the Rule compoſed:* which is contained in four ſpeciall ſelected branches, or queſtions of divers forms, *each one of them ſpringing from the firſt Queſtion*, and each one of them alſo being a proof to other, *&c.*

IF one yard coſt me 2 s. 8 d. *ready money, & after I ſell the ſame again for* 2s. 10d. *to be paid for it at the end of* 3 *months: the queſtion is, what I gain upon the* 100 li. *in* 12 *months.*

Anſwer. Firſt ſay, if $2\frac{2}{3}$ gain $\frac{1}{3}$, what ſhall 100 li. gain? Multiply and divide, and you ſhall finde $6\frac{1}{4}$ li. Then ſay again, by the *Rule of Three,* if three months gain $6\frac{1}{4}$ pound, what ſhall 12 months gain? Worke, and you ſhall finde 25 li. and ſo much ſhall I gain in 12 months after that rate.

Item, You may alſo work it all at one working by the firſt part of the *Rule of Three* compoſed, ſaying, if $2\frac{2}{3}$ d. in three months do gain $\frac{1}{6}$ of a ſhilling, (which is 2 d.) what will 100 li. gain in 12 months? Which for thy further encouragement, the work of this one example I have here put down, to verifie that I affirm in the firſt part of this *Ground of Arts,* that this Rule,

Rule, and so all others, more rejoyceth in *Broken*, then in *Whole*

s.	months	s.	li.	mo.
$2\frac{2}{3}$ ——	3 ——	$\frac{1}{6}$ ——	100 ——	12
8		1	20	
~~3~~ ~~2~~			2000	
24 ~~72000~~			3	
6 ~~144444~~ (~~500~~ (25			6300	
144 ~~144~~ ~~220~~			12	
~~1~~			72000	

Where the Multiplication and the Division being ended, maketh 25 li. your desire.

If a yard be delivered for 2 s. 10 d. *to be payed at* 3 *months, whereupon was gained after the rate of* 25 li. *in the* 100. *for* 12 *months, the question is now, what the yard cost at the first hand?*

Answer. First say, if 12 months gain 25 li. what shall 3 months gain? Work, and you shall finde $6\frac{1}{4}$ li. Then say again the second time, if $106\frac{1}{4}$ li. give but 100. what shall $2\frac{5}{6}$ s. give? Work, and you shall finde 2 s. 8 d. which is the just price that the yard cost at the first hand.

If one yard of Cloth cost me 2 s. 8 d. *ready money, for what term shall I sell the same again for* 2 s. 10 d. *so that I might gain after the rate of* 25 *pound upon the* 100 *pound in* 12 *months?*

Answer. First say, if $2\frac{2}{3}$ gain $\frac{1}{6}$, what shall 100 li. gain? Multiply and divide, and you shall finde $6\frac{1}{4}$ li. Then say again for the second work, If 25 pound be come of twelve months, what shall come of $6\frac{1}{4}$? Worke, and you shall finde 3 months, the just term of time that the Cloth ought to be delivered

at 2 s. 10 d. to gain 25 li. upon the 100 li. in 12 moneths.

If one yard cost me 2 s. 8 d. *ready money, for what price shall I sell the same again to be paid at the end of* 3 *moneths, so that I may gain after the rate of* 25 *pound in the* 100 *pound for* 12 *moneths?*

Answer. First say, if 12 gain 25 li. what shall 3 moneths gain? Multiply and divide, and you shall finde $6\frac{1}{4}$ li. Then say for the second work, if 100 li. give $106\frac{1}{4}$, what giveth $2\frac{2}{3}$ s? Work, and you shall find 2 s. 10 d. and for that price must the yard be sold to gain after 25 pound in the 100 pound for 12 moneths.

Many other of these questions I might here have delivered, but for fear the Book would rise to too thick a volume, and so make the price so much the dearer, whereby it might not be so portable to my Country-men as I wish it. But these 4 I have of purpose framed in this order, having relation one to another, assuring you that what question soever may be proposed within the compasse of this Rule, you shall finde by one of these 4 to make a solution. And moreover, divers others are yet to be delivered where the Creditor giveth divers dayes of payment, which can never be well wrought, nor yet understood, unlesse you can first finde by Art the just times that all those payments, how different soever they be, ought to be paid at once: whereupon first I thinke good here to give some instructions unto such a Rule; for it is the onely aid for the finishing of such questions as hereafter shall follow.

The

The ſixth Chapter treateth of Rules of Payment, which is a right neceſſary Rule, and one of the chiefeſt handmaids that attendeth upon buying and ſelling, *&c.*

Example.

A Merchant doth owe a ſumme of money, whereof the $\frac{1}{3}$ *is to be paid at ſix months, and the* $\frac{1}{2}$ *at eight months, and the reſt at a year. If he would pay all at one payment, the queſtion is, what time ought to be given him?*

Anſw. I have omitted the quantity of the ſumme, for you ſhall underſtand the Rule is appliable, and yieldeth a true ſolution to what ſumme ſoever ſhall be propoſed: But now for order ſake in teaching, I do imagine the ſumme to be 60 pounds, whereupon the manner of this work is to multiply the proportionate part of the money by the time, as in company. Then 20 being the firſt payment, and the $\frac{1}{3}$ of 60. which $\frac{1}{3}$ multiplyed in broken numbers by 6, his time of payment, maketh $\frac{6}{3}$, which in whole numbers, as appeareth by the Example in the Operation, maketh two months: next 30. which is the $\frac{1}{2}$ multiplyed by his terme 8. yields 4. months, then the reſt, which is 10 li. muſt needs be abbreviated into the proportion it beareth to 60, which is $\frac{1}{6}$, which $\frac{1}{6}$ multiplyed by his time 12 months, produceth $\frac{12}{9}$, maketh two months. All which added together, as appeareth in the Operati-

$\frac{1}{3}$ by $\frac{6}{1}$	2 Months.
$\frac{6}{3}$	
$\frac{1}{2}$ by $\frac{8}{1}$	4 Months.
$\frac{8}{2}$	
$\frac{1}{6}$ by $\frac{12}{1}$	2 Months.
$\frac{12}{8}$	

 on,

on, maketh 8 months, which is the juſt time that all thoſe payments ought to be paid at once.

A Merchant hath 800 li. *to pay, the* $\frac{1}{6}$ *thereof ready money, the* $\frac{1}{4}$ *at two months, the* $\frac{1}{2}$ *at* 4 *months, and the reſt at a year. The queſtion is, if he would pay all at one payment, what time ought to be given him?*

Anſwer. The ready money is never multiplyed; then $\frac{1}{4}$ multiplyed by two months, as you did before, maketh $\frac{1}{2}$; then $\frac{1}{2}$ by 4 produceth 2 months, as appeareth here in the operation. But now for the reſt of the money, you cannot multiply it untill you have ſought what proportion it beareth to 800 pounds. Therefore you muſt ſubtract the ready money, the $\frac{1}{4}$ and $\frac{1}{2}$ out of the principall. The reſt will be *66* $\frac{1}{3}$ li. which you muſt look what part it beareth to the principall, which you ſhall finde to be $\frac{1}{12}$, the ſame you muſt alſo multiply by his time 12 months, and it yieldeth 1 month; ſo all make $3\frac{1}{2}$ months, as appeareth in the operation.

$\frac{1}{4}$	$\frac{2}{1}$	
$\frac{1}{4}$		$0\frac{1}{2}$
$\frac{1}{2}$	$\frac{4}{1}$	
$\frac{4}{2}$		2
$\frac{1}{12}$	$\frac{11}{1}$	
$\frac{1}{1}$	$\frac{2}{2}$	1

A Merchant is to pay 1200 li. *in three terms, that is to wit,* 400 li. *at two weeks, and* 600 li. *at four months, laſtly,* 200 li. *at five months: The queſtion is, in what time they ought to be paid at once.*

Anſwer. Proportionate the parts, and you ſhall finde that 400 is $\frac{1}{3}$ part, and for 600 you ſhall finde $\frac{1}{2}$, and likewiſe 200 is the $\frac{1}{6}$ part, which multiply by their times as before, and you ſhall have $\frac{2}{3}$ weeks, more 8 weeks, and laſtly $3\frac{1}{3}$ weeks, which together maketh 12 weeks, or 3 months, your deſire.

A Merchant is to pay 600 *pound in three termes, whereof* 100 *pound is paid preſent, more* 300 *pound at twenty dayes, and the reſt at five months, accounting*

thirty

thirty dayes to a month. The queſtion is, what time ought theſe payments to be paid at once?

Anſwer. Work, and you ſhall finde 2 months.

The ſeventh Chapter treateth of buying and ſelling in the Trade of Merchandize, wherein is taken part ready money, and divers dayes of Payment given for the reſt, and what is won or loſt in the 100 pound forbearance for twelve months more or leſs, according to the quantity of money, or proportion of time, *&c.*

A *Merchant hath bought ſatins which coſt* 8 s. *the yard ready money, and he ſelleth the ſame again to another man for* 10 s. *the yard, but he giveth 2 dayes for the payment, that is to ſay, 3 months for the one half, and 5 months for the other half. The queſtion is to know how much the ſeller doth gain upon the* 100li. *in* 12 *months after that rate.*

Anſ. Seek firſt, by the Rules of payment, at what time thoſe 2 payments ought to be paid at once, and you ſhall find 4 mõths, at which time the ſecond Merchant ought to have paid the whole entire paymẽt: & therfore ſay by the firſt part of the *Rule* of 3. compoſed, If 8 s. in 4 months do gain 2 s. what will 100 li. gain in 12 months?

```
s.    m.     s.    li.    m.
8 ——— 4 ——— 2 ——— 100 ——— 12
4                  20
———               ————
23                2000
                    2
                  ————
                  4000
                    12
                 48000
 1
 16
 48000    1s.
 322    (1500   (75
 3        220
```

Multiply and divide, and you ſhall finde 75. li. as appeareth in the example, and ſo much doth the firſt Merchant gain upon the 100 li. in 12. months.

A Merchant hath ſold 50. Clothes at $9\frac{1}{2}$ *the piece, to be paid the one* $\frac{1}{2}$ *at four months, the* $\frac{1}{3}$ *at five months, and the* $\frac{1}{6}$ *at 7 months, and the ſellers minde is to take no more but after* 8 li. *in the* 100. *for* 12. *months. The queſtion is now, what the firſt Merchant gaineth in the ſale of theſe Clothes after that rate.*

Anſw. Firſt, look what the 50 Clothes come to at that price, and you ſhall finde 475 li. Then ſecondly, according to your direction in the Rules of payment, ſeeke at what time all the payments are to be performed at once, and you ſhall finde $4\frac{5}{6}$ months. Then thirdly ſay, by the firſt part of the *Rule of* 3. compoſed, If 100li. in 12 months gain 8li. what will 475li. gain in $4\frac{5}{6}$ months? Work, and you ſhall finde 15 li. and $\frac{11}{36}$ of a pound, which is the neat gains that the firſt Merchant hath after the rate aforeſaid.

A Merchant hath bought Holland at 7 s. 3 d. *the Ell ready money, and he ſelleth the ſame again for* 8 s. 4 d. *the Ell, to be paid* $\frac{1}{4}$ *part in ready money, more* $\frac{1}{3}$ *part at 2 months, and the reſt at 4 months. The queſtion is now to know how much the firſt Merchant doth gain upon the* 100li. *in* 12 *months after that rate.*

Anſwer. According to the direction delivered you in the Rule of payment, the ready money is not to be multiplyed. Then working for the other 2 payments to finde out the true proportion at what time they ought to be paid at once, you ſhall finde for $\frac{1}{3}$ at two months, $\frac{2}{3}$ of a month. And the reſt of the mony which is $\frac{5}{16}$ multiplyed by his term 4 months, yieldeth $1\frac{1}{3}$ months; both which added together make $2\frac{1}{3}$ months,

the

the just time that both the payments ought to be performed at once. And therefore say by the first part of the *Rule of* 3. composed, If $74\frac{1}{4}$ in $2\frac{1}{3}$ months do gain $\frac{12}{46}$ of a li. what shall 100 li. gain in 12 months after that rate? Work, and you shall finde $78\frac{172}{603}$ li. And so much doth he gain upon 100 pounds in 12 months.

A Merchant hath bought 30 *clothes at* 6 li. *the piece ready money: Afterward he selleth* 10 *of them for* 7 li. *the piece, for* 3 *months term; and the other* 20 *he selleth for* 8 li. *the piece for* 4 *months term. The question is now, what he gaineth upon* 100 *pounds in* 12 *months.*

Ans. First, find the value of the 30 clothes, which amount to 180 li. Secondly, seek what the ten pieces come to at 7 li. & what the 20 pieces come to at 80 li. the one comes to 70. and the other to 60. both which together make 230. which is 50 li. more then they cost. 3ly, As I have taught you in the rule of payment, proportionate the first and 2d prices unto the proportion they bear unto 230 the product of their 2 prices, and you shall find $\frac{7}{23}$ for the first, and $\frac{16}{23}$ for the latter. Then fourthly, multiply those parts by their times, & you shall have $\frac{21}{23}$ & $\frac{64}{23}$, both which together make 3 whole months, and $\frac{16}{23}$ of a month, which is the just time that both those paymẽts ought to be paid at once. Then say by the first part of the *Rule of* 3. composed, If 180 l. in $3\frac{16}{23}$ months do gain 50 l. what shall 100 gain in 12 mõths? Multiply & divide, & you shall find $90\frac{10}{51}$ l. and so much doth he gain upon 100 l. in 12 months.

A Merchant hath bought Cinnamon which cost him 9 s. *the* li. *ready money. The question is now, at what price he ought to sell the* 100 *weight; to wit,* 112 li. *to be paid the* $\frac{1}{4}$ *at* 2 *months, and the residue at the end of* 3 *months, so that he may gain after the rate of* 10 l. *upon* 100 l. *for* 12 *months.* *Ans.*

Anſ. Seek firſt by the Rule of payment at what term both the payments ought to be paid at once, where the $\frac{1}{4}$ multiplyed by his term 2 months, maketh $\frac{2}{4}$ months.

Likewiſe the next payment, which is $\frac{3}{4}$ multiplyed by his term three months, maketh $2\frac{1}{4}$ months, both which added together make $2\frac{3}{4}$ months, which is the time that both the payments ought to be paid at once. Then ſay by the *Rule of Three*, If 12 months do give me ten pounds, what will $2\frac{3}{4}$ months give? Multiply and divide, and you ſhall finde $2\frac{7}{24}$ pounds. Then ſay again by the *Rule of Three*, If one pound coſt me 9 s. what will 112. pounds coſt? Multiply and divide, and you ſhall finde 50 li. 8 s. Then ſay once again, If 100 pound doe give 102 $\frac{7}{24}$, what will $50\frac{2}{5}$ pounds give? Multiply and divide, and you ſhall finde 51 li. 11 s. 1 $\frac{1}{5}$ d. and for that price ought I to ſell 112. pound of Cinnamon to be paid at the two ſeverall payments aforeſaid, to gain thereby after the rate of ten pounds upon the hundred pound in twelve months.

Brief Rules for our hundred weight here at *London*, which is after 112. pound for the 160.

ITem, *Who that multiplyeth the pence that one pound weight is worth by 7. and divideth the* Product *by 15. ſhall finde how many pounds in money 112 pound weight is worth.*

And contrariwiſe, he that multiplyeth the pounds that 112 pounds weight is worth by 15. and divideth the Product *by 7. ſhall finde how many pence in money the one pounds weight is worth.*

Example.

At 10 pence the pound weight, what is 112 pounds weight worth? *Anſwer.*

Anſwer. Multiply 10 by 7. and thereof cometh 70. the which divide by 15. and you find 4$\frac{2}{3}$ pounds. And thus the 112 pounds is worth 4 li. 13 s. 4 d. after the rate of 10 pence the pound aforeſaid.

At *6* pounds the 112 pounds weight, what is one pound worth?

Anſwer. Multiply *6* by 15, and thereof cometh 90. the which divide by 7. and you ſhall finde 12$\frac{6}{7}$d. So much is one pound worth when the 112 pounds did coſt *6* pounds.

The eighth Chapter treateth of Tares and allowances of Merchandize ſold by weight, and of Loſſes and Gains therein, *&c.*

AT *16* pound the 100 Suttle, what ſhall 895 pound Suttle be worth, in giving 4 pound weight upon every 100. for Treat?

Anſwer. *Adde* 4 *unto* 100, *and you ſhall have* 104. *Then ſay by the Rule of Three, If* 104 *be worth* 16 *pounds, what are* 895 *pounds worth? Multiply and divide, and you ſhall finde* 137 li. 13 s. 10 $\frac{2}{11}$ d. *and ſo much ſhall the* 895 *pounds weight be worth.*

Item, at 3 s. 4 d. the pound weight, what ſhall 754 $\frac{1}{2}$ pound be worth, in giving 4 pounds weight upon every hundred for Treat?

Anſwer. See firſt by the *Rule of Three* what the 100 pound is worth, ſaying, If one coſt 3$\frac{1}{3}$ s. what 100? Multiply and divide, and you ſhall finde 16$\frac{2}{3}$ pounds. Then adde 4 unto one 100. and they are

104.

104. Then ſay again by the *Rule of Three*, If 104 be ſold for $16\frac{2}{3}$ pounds, for how much ſhall $754\frac{1}{2}$ be ſold? Multiply and divide, and you ſhall finde 120 li. 18 s. $3\frac{3}{13}$ d. And for ſo much ſhall the $754\frac{1}{2}$ pound be ſold at 3 s. 4 d. the pound, in giving 4 upon the 100.

Other neceſſary brief Rules there are for the finding of Treats, or caſting up Cheſts of Sugar, &c. which for that it is a myſterie, I omit: if any lack inſtruction that way, they ſhall find me ready to pleaſure them.

Item, If 100 pounds be worth 36s. 8 d. what ſhall 860 pounds be worth in rebating 4 pounds upon every for tare and cloff?

Anſwer. Multiply 860 by 4. and thereof cometh 3440. the which divide by 100. and you ſhall have $34\frac{2}{5}$ pounds, abate $34\frac{2}{5}$ from 860. and there will remain $825\frac{3}{5}$ pounds. Then ſay by the *Rule of* 3. If 100 li. coſt $36\frac{2}{3}$s. what will $825\frac{3}{5}$ coſt after that rate? Multiply and divide, and you ſhall finde 15 li. 2 s. $8\frac{16}{25}$ d. And ſo much ſhall the 860 coſt in rebating 4 li. upon every 100. for tare and cloff.

Item, *Whether doth he loſe more that giveth* 4 li. *upon the* 100. *or he that rebateth* 4 li. *upon the* 100?

Anſwer. Firſt note, that he that giveth 4 pound on 100, giveth 104 for 100. And he which rebateth 4 pounds upon the 100. giveth the 100 for *96*. Therefore ſay by the *Rule of* 3. If 104 be delivered for 100. for how much ſhall the 100 be delivered? Multiply and divide, and you ſhall finde $96\frac{2}{13}$, and he which rebateth 4 in the 100, maketh but *96* pounds of 100, ſo that he loſeth 4 pounds in the 100, and the other which giveth 4 pounds unto the 100, loſeth but $3\frac{11}{13}$ pounds upon the 100. Thus you

you may see, that he which abateth 4 pounds in the 100, loseth more by $\frac{11}{13}$ pound in the 100 pounds, then the other which gave 4 pounds upon the 100, for tare and closse.

If 100 pound of any thing cost me 23 s. 4 d. the question is, how I shall sell the pound, to gain after the rate of ten pounds upon the 100 pound?

Answer. Say by the *Rule of Three*, if 100 pounds give 100 pounds, what shall 23 $\frac{1}{5}$ s. give? Multiply and divide, and you shall finde 1 $\frac{17}{60}$ pounds. Then say again, if 100 pound be worth 1 $\frac{17}{60}$ pounds, what is one pound worth? Multiply and divide, and you shall finde 3 d. $\frac{2}{25}$. And so much is the pound worth in gaining ten pounds upon the 100.

Item, *A Grocer hath bought C. weight of commodity for* 6 li. 10 s. *The question is now to know how many* li. *thereof he shall sell for* 33s. 4d. to gain 20s. *in C. weight.*

Answer. Adde 20 s. unto 6 li. 10 s. and they make 7 li. 10 s. Then say, if 7 $\frac{1}{2}$ pound yeeld me 112 pound, what shall 1 $\frac{2}{3}$ pounds yeeld? Multiply and divide, and you shall finde 24 $\frac{8}{9}$ li. and so many pound ought he to sell to gain 20 s. in his C. weight.

Item, *If one pound weight cost* 3 s. 4 d. *and I sell the same again for* 4 s. *what is gained in a hundred pound of money laid out in that commodity?*

Answer. You may say, If 3 $\frac{1}{3}$ s. give 4. what will 100 pound gain? But then when you have found, you must subtract 100 pounds out of the Product, the rest is your neat gain. Or else to produce the neat gain in your work at the first, subtract the just price out of the over-price, as I taught before in the first beginning of Losse and Gain, and your conclusion shall be all one. Multiply and divide by which of the

the two wayes you think good, and you ſhall finde that he gaineth 20 pounds in the 100 pound.

Item, *If the pound weight which coſt* 4 s. *be ſold again for* 3 s. 4 d. *I demand what is loſt in the* 100 *pounds of money.*

Anſwer. Say if 4 s. loſe $\frac{2}{3}$ s. what ſhall 100 loſe? Multiply and divide, and you ſhall finde 16 li. 13 s. 4 d. and ſo much is loſt upon the 100 of money.

Item, *If C. weight of any commodity coſt* 45 li. *and the buyer repenting would loſe* 5 *pounds in the* 100 *of money, I demand how the pounds may be ſold, his loſſe to be neither more nor leſſe then after the rate aforeſaid of five by the hundred.*

Anſwer. By the *Rule of three*, if 100 loſe 5, what ſhall 45 loſe? Work and you ſhall finde 2 $\frac{1}{4}$ pound, which rebated from the principall 45, reſteth 42 li. 15 s. Laſtly, ſay, if 112 yieldeth but 42 li. 15 s. what one pound? Multiply and divide, and you ſhall finde 7 s. 7 d. $\frac{17}{28}$. And ſo much is the pound worth after that loſſe.

A Grocer hath bought three pieces of Raiſins weighing 175 $\frac{1}{2}$ *pounds,* 182 $\frac{1}{4}$ *pounds,* 191 *pounds: tare for each fraile* 2 $\frac{1}{4}$ *pounds at* 25 $\frac{1}{2}$ s. *the C. weight. The queſtion is, what they amount to in money.*

I anſwer, 6 li.——3 s.——4 $\frac{23}{28}$ d.

A Grocer hath bought three ſacks of Almonds weighing 267 $\frac{1}{2}$ *pound, tare two pound,* 257 $\frac{1}{2}$ *pounds, tare* 2 $\frac{1}{2}$ *pound,* 252 *pound, tare* 3 *pound, at* 2 s. 10$\frac{1}{2}$ d. *the pound, what amount they to in money?*

I anſwer, 110 li.——12 s.——3 $\frac{3}{4}$ d.

The

The ninth Chapter treateth of lengths and breadths of Arras and other Cloths, with other questions incident unto length and breadth.

IF *a piece of Arras be* 7 *Ells and* $\frac{3}{4}$ *long and* 5 *Ells and* $\frac{2}{3}$ *broad, how many Ells square doth the same piece contain?*

Answer. Multiply the length by breadth, that is to say, 7$\frac{3}{4}$ by 5 $\frac{2}{3}$ and thereof will come 43 $\frac{11}{12}$ Ells: so many Ells square doth the same piece contain.

Item, *more, a piece of Arras doth contain* 22 *Ells square, and if the same were in length* 3 $\frac{1}{2}$ *Ells, I demand how many Ells in breadth the same piece doth contain.*

Answer. Divide 22 Ells by 3 $\frac{1}{2}$ and thereof cometh *6* $\frac{2}{7}$. So many Ells doth the same contain in breadth.

Item, *more, a Merchant hath* 3 $\frac{1}{4}$ *Ells of Arras, at* 1 $\frac{2}{5}$ *Ells broad, which he will change with another man for a piece of Arras that is* $\frac{7}{8}$ *Ells square. The question is how many Ells of that squarenesse ought the first Merchant to have?*

Answer. Multiply the first Merchants piece his length by the bredth, and you shall finde it containeth 5 $\frac{5}{12}$ Ells, which $\frac{5}{12}$ Ells you shall divide by $\frac{7}{8}$ and you shall finde 6 $\frac{4}{21}$ Ells, and so many Ells of that squarenesse ought the latter Merchant to give the first.

Item, *A Student hath bought* 3 $\frac{1}{2}$ *yards of broad Cloth at* 7 *quarters broad, to make a Gown, and should line the same throughout with Lamb at a foot square each skin, the question is now, how many skins he ought to have.*

Answer.

Anſwer. Seek firſt the number of yards ſquare that his cloth containeth, which to do, multiply $3\frac{1}{2}$ his length, by $1\frac{3}{4}$ his breadth, and you ſhall finde $6\frac{1}{8}$ yards ſquare: then ſay by the *Rule of three*, if one yard ſquare give 9 foot, what ſhall $6\frac{1}{8}$? Work, and you ſhall finde $55\frac{1}{8}$ skins.

Item, *more, a Lawyer hath a rich piece of ſeeling come home which is 24 foot and 3 inches long, and 7 foot and $2\frac{1}{2}$ inches high: the Joyner is to be paid by the yard ſquare: the queſtion is, how many yards this containeth.*

Anſwer. Multiply his length by his breadth, that is to wit, $24\frac{1}{4}$ foot by $7\frac{5}{24}$ foot, and you ſhall finde $174\frac{77}{96}$ foot ſquare, which 174 you ſhall divide by 9 (for ſo many foot make a yard ſquare) and you ſhall finde 19 yards 3 foot and $\frac{77}{96}$ of a foot, and ſo many yards doth this piece hold.

Item, *I bought a piece of Holland cloth containing 36 Ells $\frac{1}{3}$ Flemmiſh. The queſtion is, how many Ells Engliſh it makes.*

Anſwer. You muſt note, that five Ells *Flemmiſh* doe make but three Ells *Engliſh*.

Therefore ſay by the *Rule of three*, if five Ells *Flemmiſh* make but three Ells of *Engliſh*, how many Ells *Engliſh* will $36\frac{1}{3}$ Ells *Flemmiſh* make? Multiply and divide, and you ſhall finde $21\frac{4}{5}$ and ſo many *Engliſh* doth $36\frac{1}{3}$ Ells *Flemmiſh* contain. The like is to be done of others.

Item, *more, I have bought 342 Ells Flemmiſh of Arras worke, at two Ells broad Flemmiſh, and I would line the ſame with Ell-broad Canvas of Engliſh meaſure. The queſtion is, how many Ells Engliſh will ſerve my turn.*

Anſwer.

Anſwer. Foraſmuch as three Ells *Engliſh* are worth five Ells *Flemmiſh*, therefore put three Ells *Flemmiſh* into his ſquare, in multiplying three by himſelf, which maketh nine. Likewiſe multiply the *Engliſh* Ell, which is five quarters, every way into himſelf ſquarely, and you ſhall finde 25. Then multiply 342, which is the length of the piece, by 2, which is the breadth, and thereof cometh 684, then ſay by the *Rule of three*, as before, if 25 Ells ſquare of *Flemmiſh* meaſure, be worth nine Ells ſquare of *Engliſh* meaſure, what are 684 of *Flemmiſh* meaſure? Multiply and divide, and you ſhall finde $246\frac{6}{25}$ Ells *Engliſh*.

The ſame is alſo wrought by the Backer *Rule of three*, in ſeeking the ſquares contained in the *Flemmiſh* Ell of two Ells broad (which are 18) and alſo in ſeeking the ſquares contained in the *Engliſh* Ell (which are 25:) then ſay by the *Rule of three* backward, If 18 quarters require 342 Ells, what ſhall 25 quarters give? Multiply and divide by the *Rule of three* Reverſe, and you ſhall finde as before $246\frac{6}{25}$ Ells *Engliſh*.

Item, *more, at three ſhillings four pence the Flemmiſh Ell, what is the Engliſh Ell worth after the rate?*

Anſwer. Say, if three quarters give $3\frac{1}{3}$ s. what giveth five quarters? Multiply and divide, and you ſhall finde 5 s. $6\frac{2}{3}$ d.

Item, *more, at* 8 s. 4 d. *the Flemmiſh Ell ſquare, what is the Engliſh Ell worth after that rate?*

Anſwer. According to the reaſon of the laſt Queſtion, conſider that a *Flemmiſh* Ell ſquare is equall to nine quarters of a yard *Engliſh*, and an *Engliſh* Ell ſquare is equall to 25 quarters of a yard. There-

fore say by the *Rule of three*, if 9 quarters give $1\frac{2}{3}$ s. what 25 quarters? Work and finde 23 s. $1\frac{7}{9}$ pence. And so is the *English* Ell worth.

Item, *more, at* 6 s. 8 d. *the Ell square, what shall a piece of Cloth cost that is $7\frac{1}{2}$ Ells long, and $3\frac{1}{4}$ Ells broad?*

Answer. Multiply the breadth by the length, and you shall finde $24\frac{3}{8}$ Ells square cost $6\frac{2}{3}$ s. what $24\frac{3}{8}$ Ells? Multiply and divide, & you shall find 8 pounds, 2 s. 6 pence, and so much the same piece of cloth cost.

Item, *more, a Mercer sold 3 pieces of Silke, to wit $24\frac{1}{4}$ $13\frac{2}{3}$ and 25 yards, at $9\frac{3}{4}$ s. the yard, and was glad to receive in part of payment againe a cloth containing $34\frac{1}{3}$ yards at $7\frac{2}{3}$ shillings the yard. The question is now, what the Debtor is in the Creditors debt. Worke, and you shall finde he oweth the Mercer* 17 li. 8 s. 11 d $\frac{1}{4}$.

The tenth Chapter treateth of reducing of Pawns of Geans into English yards.

NOte that 100 *Pawns doe make* 26 *yards, whereupon three Pawns $\frac{11}{13}$ doe make one yard, and one Pawn after the rate and proportion is $\frac{13}{50}$ of a yard.*

In 4563 *Pawns of Geans, how many yards English?*

Answer. Say by the *Rule of Three*, if a hundred Pawns doe make 26 yards, what will 4563 Pawns make? Multiply and divide, and you shall find 1186 yards $\frac{19}{50}$. So many yards doe 4563 Pawns make.

Otherwise take some other number at your pleasure,

ſure,as ten pawns,which is the $\frac{1}{10}$ part of 100,then to finde his proportion, take the $\frac{1}{10}$ part of 26, which is $2\frac{3}{5}$ and then alſo by the *Rule of three*, if ten pawns give $2\frac{3}{5}$ yards,what will 4563 pawns give? Work, and you ſhall finde $1186\frac{39}{50}$ yards, as before.

More, at 2 s. 6 d. *the Pawns of Geans, what will the Engliſh yard be worth after the rate?*

Anſwer. Say by the *Rule of three*, if $\frac{12}{50}$ of a yard coſt $2\frac{1}{2}$ s. what one yard? Multiply and divide, and you ſhall finde 9 s. $7\frac{5}{13}$ d.

More, if $346\frac{1}{2}$ *Pawnes coſt* 30 li. 13 s. 4 d. *ſterling, what is that the Engliſh yard after the rate?*

Anſwer. Say by the *Rule of three*, if $346\frac{1}{2}$ Pawns coſt $30\frac{2}{3}$ pounds,what are $3\frac{11}{13}$ pawns worth (for ſo many Pawns make a yard?) Multiply and divide, and you ſhall finde $\frac{9200}{27027}$ parts of a pound, which in known numbers is worth 6 s. 9 d. $\frac{6271}{9009}$.

The eleventh Chapter treateth of Rules of Loan and Intereſt, with certain neceſſary queſtions and proofs incident thereunto, &c.

ITem, *I lent my friend 326 pounds for* $5\frac{1}{2}$ *months ſimply without any Intereſt, upon condition to have the like courteſie again when I need. But when I came to borrow, he could ſpare me but* 149 li. 8 s. 4 d. *The queſtion is now, how long time I ought to have the uſe thereof, to countervaile my friendſhip before-time ſhewed him.*

Anſwer. Say by the backer *Rule of three*, if 326 pounds give $5\frac{1}{2}$ months,what time will $149\frac{5}{12}$ pounds give?

give? Multiply and divide, and you shall find twelve months, and so long time ought I to use his money.

The Proofe.

Item, *lent my friend* 149 li. 8 s. 4 d. *for twelve months. The question is now, how much money he ought to lend me again for* $5\frac{1}{2}$ *months to recompence my friendship shewed him.*

Answer. Say by the Backer or Reverse *Rule of Three*, If twelve months give $149\frac{5}{12}$, what shall $5\frac{1}{2}$ months give? Work, and you shall find 326 pounds, and so much ought he to lend me to requite my gentleness or good turn.

Two other branches yet more, for proof out of the same question.

Item, *lent my friend* 149li. 8s. 4d. *for* 12 *months, to have the like friendship again when I need. And coming to borrow of him, he very courteously took me* 326 *pounds (for that he could well then spare the same.) The question is now, how long I ought to occupy it, not usurping friendship, but in his due time to restore it again.*

Answer. Say by the *Rule of Three* reverse, If $149\frac{5}{12}$ pounds give 12 months, what shall 326 pounds give? Multiply and divide, and you shall finde that at $5\frac{1}{2}$ months term I ought to restore it again.

Proofe.

Item, *Lent my friend* 326 *pounds for* $5\frac{1}{2}$ *months. The question is now, how many pounds he ought to lend me for* 12 *months to recompence this pleasure again.*

Scholar. Work by the *Rule of Three* reverse, as you have

have done before, and you ſhall finde 149 li.—8 s. —4 d.

Again, four other ſelected queſtions of Loan and Intereſt, all out of one branch, and each one alſo a neceſſary queſtion, and a particular proof to other.

ITem, *Lent my friend* 430 *pounds at Intereſt for three months, to receive after the rate of* 8 *pounds in the* 100 *pounds for* 12 *months. The queſtion is, what the Intereſt cometh to.*

You may, if you pleaſe, work it at two workings by the *Rule of three* direct, in ſaying, if 12 months give 8 pounds, what giveth three months? Multiply and divide, and it giveth 2 pound.

Then for the ſecond work ſay, If a hundred pound yield 2 pounds, what yieldeth 430 li? Multiply and divide, and you ſhall finde 8 li. 12 s. and ſo much comes the loan of 430 li. to for 3 months after the rate of 8 pound in the 100 li. of 12 months.

Otherwiſe wrought thus by the *Rule of three* at twice alſo.

If 100 pounds give 8 pounds, what giveth 430 li? Multiply and divide, and you ſhall finde 34 li. $\frac{2}{5}$. Then again for the ſecond work ſay, If 12 months give 34 pounds $\frac{2}{5}$, what giveth three months? Work, and finde 8 li. 12 s. as before.

Otherwiſe yet at one working, by the firſt part of the rule of five numbers forward, in ſaying, If 100 pounds in twelve months gaine eight pounds, what ſhall 430 pounds gain in three months? Multiply

tiply the first by the second for your Divisor, and the other three, the one into the other, for the Dividend, and you shall finde 8 pounds 12 shillings, as aforesaid.

Proofe.

Item, *A friend of mine received of me* 8 *pounds* 12 *shillings for the Interest and Use of* 430 *pounds for three months term. The question is now, what he took in the* 100 *pound for* 12 *months after that rate.*

Answer. For most brief, say by the first part or rule of five numbers forward, If 430 pounds in three moneths did pay 8 pound 12 shillings, what doth 100 pound in 12 months take after the rate? Work, and you shall finde 8 pounds, and so much he took upon the 100 pounds for 12 moneths.

A third Question and proof also by the Backer Rule of five Numbers.

ITem, *I lent my friend* 430 *pounds, to receive for the Interest thereof after the rate of* 8 *pounds in the* 100 *for* 12 *months. The question is now, how long time my friend ought to give the Use thereof, that it may be returned with* 8 *pounds* 12 *shillings gains.*

You may work it, if you please, by the Rule of Three direct at twice, in saying, If 100 pounds yield 8 pounds, what yieldeth 430 pound? Multiply and divide, and finde 34 pound and $\frac{2}{5}$.

Then again for the second work say, if 34 $\frac{2}{5}$ pounds give twelve months, what giveth 8 $\frac{3}{5}$ pounds? Multiply and divide, and you shall finde three months, and so long time ought my friend to use it to return with 8 li. 12 s. gain.

Other-

Otherwiſe at one working by the Backer Rule of 5 numbers, in ſaying, if 100 li. in 12 months do gain 8 li. how much time ſhall 430 li. be a gaining of 8 li. 12 s? Multiply the firſt and the ſecond into the laſt for your Dividend, and the third and fourth multiply together for your Diviſor, and then divide, and you ſhall finde three months, the juſt time that my friend ought to uſe it to return it with 8 li. 12 s. gain.

A fourth derived Queſtion out of this Branch, which is a proof of this laſt, and alſo of the other two going before.

ITem, *How much money ought a Merchant to deliver after* 8 li. *in the* 100. *for twelve months, that in three months he may gain* 8 *pounds twelve ſhillings?*

Anſwer. You may alſo, if you pleaſe, work it by the Golden Rule of Three at twice: firſt ſaying, If three months give $8\frac{3}{5}$ pound, what 12 months gain? You ſhall finde $34\frac{2}{5}$. Then ſay again, If 8 pounds be come of 100 pounds, what ſhall come of 34 pounds 8 ſhillings? Work, and you ſhall finde the anſwer to the queſtion, which is 430 pounds, and ſo much ought the Merchant to deliver.

But moſt briefly it is anſwered by the Backer Rule of five numbers, where I argue thus, ſaying, If 100 pounds be 12 months a gaining of 8 pound, then but for three months terme onely to take 8 pounds, 12 s. muſt needs be a good round ſum: to work it, ſet your numbers thus, 100——12——8——3——$8\frac{3}{5}$ multiplying the firſt into the ſecond, and alſo by 43

 the

the product of the fifth, for your Dividend, and the third and fourth together with 5. the Denominator of your fraction for your Diviſor: then divide, and you ſhall finde as before 430 pounds, the true ſolution to your queſtion.

The twelfth Chapter treateth of the making of Factors, which is taken in two ſorts.

THE firſt is, when the eſtimation of the Factor is taken upon the ſending of the Merchant, as if the eſtimation of his perſon be $\frac{1}{4}$, it is underſtood that he ſhall have $\frac{1}{4}$ of the gain, the Merchant the other $\frac{3}{4}$.

The other ſort is, when the eſtimation of his making is out of the ſending of the Merchant, as if the order and agreement between them were ſuch, that the Merchant ſhall put in 800 li. and the Factor for his making ſhall have $\frac{1}{4}$: neverthleſſe he ſhall have but $\frac{1}{5}$ of the gain or profit, for the $\frac{1}{4}$ of 800 is 200 (for the eſtimation of his making) which with the 800 pounds in all make 1000 pounds, whereof the 200 li. is $\frac{1}{5}$.

A Merchant doth put in 800 pound into the hands of his Factor, under ſuch condition, that the ſaid Factor ſhall have the $\frac{1}{4}$. And after certain time they finde in profit 124 li. 6 s. 8 d. I demand how much the Merchant ſhall have hereof, and how much ought the Factor to have.

Anſwer. *When the eſtimation of the Factor is out of the ſending of the Merchant, it maketh,*

li.	s.	d.		
99	9	4	*for the*	*Merchant.*
24	17	4		*Factor.*

But if that his estimation be at the sending of the Merchant, then it maketh but

li.	s.	d.		
93	5	0	*for the*	*Merchant.*
31	1	3		*Factor.*

For the Merchant is then to have $\frac{3}{4}$ *, and the Factor* $\frac{1}{4}$.

A Merchant doth put into the hands of his Factor 800 pounds, and the Factor 400 li. to have the $\frac{1}{5}$ part of the profit: I demand now for how much his person is esteemed, when the same is counted upon the sending of the Merchant.

Answer. *According to the tenour and order before prescribed in the first Rule, that is, if his estimate be* $\frac{1}{4}$, *he shall have the* $\frac{1}{4}$ *of the gain. Therefore say by the* Rule of Three direct, *If* $\frac{1}{4}$ *taken put in 400 pound, what is the estimate or putting in of* $\frac{1}{5}$ *taking? Multiply and divide, and you shall finde 320 pounds, and so much is the person of the Factor estimated.*

Otherwise.

To finde the estimation of the person of the Factor, you shall consider, that seeing it was agreed between them that the Factor should take the $\frac{1}{5}$, then the Merchant shall have the residue, which are $\frac{4}{5}$: wherefore the gain of the Merchant unto that of the Factor is in such proportion as 5 unto 4. Then if you will know the estimation of the person of

of the Factor, say, If 5 give 4, what will 400 give? Multiply and divide, and you shall finde 320 pound. And so much is the person of the Factor esteemed to be worth.

Other conditions then these aforesaid, may also be between Merchants and Factors, without respect either of sending, or not sending of the Merchant, where most commonly the estimation of the body of the Factor is in such proportion of the stock which the Merchant layeth in, as the gain of the said Factor is unto the gain of the Merchant. As thus, If a Merchant doe deliver into the hands of his Factor 400 pound, and he to have half the profit, the person of the said Factor shall be esteemed to be worth 400 pound: and if the Factor do take but $\frac{1}{3}$ of the gain, he should have but $\frac{1}{2}$ so much of the gain as the Merchant taketh, which must have $\frac{2}{3}$, wherfore the person of the Factor is esteemed but the $\frac{1}{2}$ of that which the Merchant layeth in, that is to say, two hundred pound.

And if the Factor did take the $\frac{2}{5}$ of the gain, then the Merchant shall take the residue, which are $\frac{3}{5}$; wherefore the gain of the Merchants unto the Factor is then in such proportion as 3 unto 2: whereupon if you will then know the estimation of the person of the Factor, say, if 3 give 2, what shall 400 give? Work, and you shall finde 266 $\frac{2}{3}$ pounds. And so much is the person of the Factor esteemed to be worth.

And if the Merchant should deliver unto his Factor 400 pound, and the Factor would lay in 80, and his person, to the end he might have the $\frac{1}{2}$ of the gain, I demand, how much shall his person be esteemed?

Answer. *Abate* 80 *from* 400, *and there will remain* 320. *And at so much shall his person be esteemed.*

A

A Merchant hath delivered unto his Factor 900 pounds to govern in the Trade of Merchandize, upon condition that he ſhall have the $\frac{1}{3}$ of the gain, if any thing be gained, and alſo to bear the $\frac{1}{3}$ of the loſſe, if any thing be loſt. Now I demand how much his perſon was eſteemed at.

Anſwer. *Seeing that the Factor taketh the $\frac{1}{3}$ of the gain, his Perſon ought to be eſteemed as much as $\frac{1}{2}$ of the ſtock which the Merchant layeth in: that is to ſay, the $\frac{1}{2}$ of 900 pound, which is 450. The reaſon is, becauſe $\frac{1}{3}$ of the gain that the Factor taketh is the $\frac{1}{2}$ of the $\frac{2}{3}$ of the gain that the Merchant taketh, and ſo the Factor his perſon is eſteemed to be worth 450 pounds.*

A Merchant hath delivered unto his Factor 600 pounds, and the Factor layeth in 250 pounds, and his perſon. Now becauſe he layeth in 250 pounds, and his perſon, it is agreed between them that he ſhall take the $\frac{2}{5}$ of the gain. I demand for how much his perſon was eſteemed.

Anſwer. *For as much as the Factor taketh $\frac{2}{5}$ of the gain, he taketh $\frac{2}{3}$ of that which the Merchant taketh, for $\frac{2}{5}$ are the $\frac{2}{3}$ of $\frac{3}{5}$. And therefore the Factors laying in ought to be 400 pound, which is $\frac{2}{3}$ of 600 pound that the Merchant laid in. Then ſubtract 250. which the Factor did lay in, from 400 pound which ſhould have been his whole ſtock, and there remaineth 150 pound for the eſtimation of his perſon.*

More, a Merchant hath delivered unto his Factor 800 li. upon condition that the Factor ſhall have the gain of 160 li. as though he laid in ſo much ready mony: I demand what portion of the gain the Factor ſhall take.

Anſwer. *See what part the 160 (which the Factor laid in) is of 960, which is the whole ſtock of their company,*

company, and you ſhall finde $\frac{1}{6}$*: And ſuch part of the gain ſhall the Factor take.*

But in caſe that in making their Covenants it were ſo agreed between them, that the Factor ſhould have the gain of 160 *pound of the whole ſtock which the Merchant layeth in, that is to ſay, of the* 800 *pound, then ſhould the Factor take* $\frac{1}{5}$ *of the gains, for* 160 *is* $\frac{1}{5}$ *of* 800 *pound.*

The thirteenth Chapter treateth of Rules of Barter, and exchanging Merchandize, which is diſtinct into ſeven Rules, with divers other neceſſary queſtions incident thereunto.

The firſt Rule.

TWO Merchants, willing to change their Merchandize the one with the other, the one hath 24 broad clothes at 10 li. 10 s. the piece, the other hath Mace at 12 ſhillings the pound. The queſtion is, how many pounds of Mace he ought to give for his Cloth, to ſave himſelf harmleſs, and be no loſer.

Anſwer. *Seek firſt by the* Rule of three *what the* 24 *Clothes coſt at* 10 *pound* 10 *ſhillings the piece, and you ſhall finde* 252 *pound: Then to finde the quantity of Mace, ſay again by the* Rule of three, *If* 12 *ſhillings buy one pound, what ſhall* 252 *pound buy me? Work, and you ſhall finde* 420 *pound of Mace: And ſo many pound ought he to give for his Clothes.*

The Proof.

Two barter. The one hath 420 pounds of Mace, at

at 12 s. the pound, to barter or change broad Clothes at 10 pounds 10 shillings the piece. The question is, how many broad Clothes he ought to give for all his Mace.

Answer. *First say, If one cost 12 shillings, what 420? you shall finde 5040 shillings. Then say again, If $10\frac{1}{2}$ pounds give one cloth, what shall 5040 shillings give? Work, and you shall find 24 Clothes, your desire.*

The second Rule.

Two change merchandize for merchandize. The one hath Pepper at two shillings four pence the pound, to sell for ready money; but in barter he will have no lesse then three shillings the pound. And the other hath Holland at five shillings six pence the Ell ready money. The question is now, at what price he ought to deliver the Ell in the barter to save himself harmless.

Answer. *Say by the* Rule of Three direct, *If $2\frac{1}{3}$ ready money give 3 shillings in barter, what shall $5\frac{1}{2}$ give in barter? You shall finde $7\frac{1}{14}$ shillings, and at that price ought the second Merchant to sell his Holland in barter.*

The Proof.

Two barter. The one hath Holland at 5 s. 6 pence the Ell, to sell for ready money; and in barter he will have $7\frac{1}{14}$ shillings. The other hath Pepper at 2 s. 4 pence the pound, to sell for ready money. The question is now, how he ought to sell in barter.

Answer. *Say by the* Rule of Three direct, *If $5\frac{1}{2}$ ready money give $7\frac{1}{14}$ shillings in barter, what ought $2\frac{1}{3}$ to take in barter? Multiply and divide, and you shall find 3 shillings, your desire.*

The

The third Rule.

Two barter. The one hath cloth of Arras at 30 s. the Ell ready money, but in barter he will have 35 $\frac{1}{2}$ s. And the other hath white Wines which he delivered in barter for 16 pounds the Tun. The question is now, what his Wines cost the Tun in ready mony.

Answer. *Say by the* Rule of Three direct, *If* 35 $\frac{1}{2}$ *shillings in barter, give but* 30 *shillings ready money, what did* 16 *pound in barter cost? Worke, and you shall finde* 13 *pound* 10 *shillings* $\frac{30}{71}$. *And so much cost his Wines for a Tun ready money.*

The proof.

Two barter Merchandize for Merchandize: The one hath white Wines at 13 pounds 10 s. $\frac{30}{71}$ s. the Tun to sell for ready mony; but in barter he delivered it for 16 li. The other, to make his match good and save himselfe harmlesse, delivereth Arras at 35 $\frac{1}{2}$ s. the Ell. The question is now, what an Ell of his Arras cost in ready money.

Answer. *Say by the* Rule of Three direct, *If* 16 *pounds in barter give but* 13 *pounds* 10 $\frac{30}{71}$ *shillings in ready money, what shall* 35 $\frac{1}{2}$ *shillings yeeld in barter? Worke, and you shall finde* 30 *shillings, your desire.*

The fourth Rule.

Two barter: The one hath Kerseyes at 14 *pounds the piece ready money; but in barter he will have* 18 *pounds: and yet he will have the* $\frac{1}{3}$ *part of his overprice in ready money. And the other hath Ginger at eight groats the pound to sell for ready money. The question is, how he ought to deliver the Ginger by the pound in barter to save himselfe harmlesse, and make the barter equall.*

Answer.

Anſwer. *Item*, for the working of this queſtion, and ſuch other like, you muſt underſtand, if the party over-ſelling his wares, require to have alſo ſome portion in ready money, as $\frac{1}{2}\frac{2}{3}\frac{3}{4}$, &c. then ſhall you firſt rebate the ſame demanded part, whatſoever it be, from the over-price, and alſo from the juſt price. And thoſe two numbers that ſhall remain after the ſubtraction is made, ſhall be the two firſt numbers in the *Rule of Three*. And the juſt price of the ſame Merchandize ſhall be the third number, which by the operation of the *Rule of Three direct*, ſhall yeeld you a true ſolution how, and at what price you ſhall over-ſell that your merchandize, to ſave your ſelf harmleſſe, and make the barter equall.

Example.

Take the $\frac{1}{3}$ (of eighteen) which is the over-price of his Cloth, which $\frac{1}{3}$ of eighteen is ſix, which you muſt ſubtract from 18, there reſt 12. And alſo abate it from 14, which is the juſt price of the Cloth, and there remaineth 8, which 8 and 12 are the two firſt numbers in the *Rule of Three*. Then take eight groats, or $2\frac{2}{3}$ ſhillings for the third number. Then ſay by the *Rule of Three direct*, If eight pounds give 12 pounds, what ſhall $2\frac{2}{3}$ s. give? Multiply and divide, and you ſhall finde 4 ſhillings. And for ſo much ſhall the ſecond Merchant ſell his Ginger, or his commodity in barter, to ballance the ſame equall.

14 ——	18
6	*6*
8	12

The Proof.

Two barter: The one hath fine Kerſeys, at 14 *pounds the piece ready money; but in barter he will have* 18 *pounds:*

pounds: and yet he will have the $\frac{1}{3}$ part of his overprice in ready money. And the other hath Ginger, which he having cunning enough to make the barter equall, delivered in barter for 4 shillings the pound. The question is now, what his Ginger cost him in ready money.

Answer. After you have made the subtraction, abating 6 the $\frac{1}{3}$ part of 18, both from 18 and 14 (as before was taught you) then will there remain 8 and 12 for your two first numbers in the *Rule of three.* Then say, if 12 give 8, what shall come of 4 the overprice of the pound of Ginger? Multiply and divide, and you shall finde 2 s. 8 pence, your desire.

Two Merchants barter Merchandize for Merchandize. The one hath Devonshire *whites at 7 pound 13 shillings 4 pence the piece ready mony: but in barter he doth them away for 8 pound 3 shillings 4 pence, and yet he will have the $\frac{1}{3}$ part of his price in ready money. And the other hath Cottens at three pounds the piece ready money. The question is now, at what price he ought to sell or exchange his Cottens in barter to save himself harmlesse, and make the barter equall.*

li.	s.	d.	li.	s.	d.
7	13	4	8	3	4
2	14	$5\frac{1}{3}$	2	14	$5\frac{1}{3}$
4	18	$10\frac{2}{3}$	5	8	$10\frac{2}{3}$

Answer. First seek the $\frac{1}{3}$ part of 8 li. 3 s. 4 d. which is 2 li. 14 s. 5 $\frac{1}{3}$ d. which rebated from 8 li. 3 s. 4 d. there resteth, as appeareth by the Example abovesaid, 5 li. 8 s. 10 $\frac{2}{3}$ d. which is $\frac{2}{3}$ parts of 8 li. 3 s. 4 d. also rebated from 7 li. 13 s. 4 d. there resteth 4 li. 18 s. 10 $\frac{2}{3}$ d. the two first numbers in the *Rule of Three*, and the three pounds, which is the neat price

of

of the piece of Cotten, is the third number. Then say by the *Rule of three direct*, as was taught before, If 4 li. 18 s. 10 $\frac{2}{3}$ d. give 5 li. 8 s. 10 $\frac{2}{3}$ d. what shall three pounds give? Multiply and divide, and you shall finde three pounds 6 $\frac{6}{89}$ s. the just price that he ought to deliver his Cottens in barter at.

The fifth Rule.

Two Merchants will change Merchandize for Merchandize. The one hath Kerseys at 40 s. *the piece, to sell them for ready money; and in barter he will sell them for* 56 s. 8 d. *and he will gain after ten pound upon the* 100 *pound. And yet he will have the* $\frac{1}{2}$ *of his over-price in ready money. The other hath Flax at* 3 d. *the pound ready money, The question is now, how he shall sell the pound of his Flax in barter.*

Answer. See first at 10 pound upon the 100 pounds, what the 56 $\frac{2}{3}$ s. cometh to, in saying (by the *Rule of Three direct*) If 100 pounds give 110 pounds, what 56 $\frac{2}{3}$ s? Multiply and divide, and you shall finde 3 pound 2 shillings 4 pence, of which the $\frac{1}{2}$ that he demandeth in ready money is 1 pound 11 shillings 2 pence; the same 31 s. 2 d. abated from 40 shillings, and also from 56 s. 8 d. there will remain 8 s. 10 d. and 25 s. 6 d. for the two first numbers in the *Rule of Three*, and 3 pence the price of the pound of Flax for the third number. Then multiply and divide, and you shall finde 8 $\frac{35}{53}$ d. And for so much shall he sell the pound of Flax in barter.

The sixth Rule.

Two are willing to exchange Merchandize. The one hath Norwich *Grograns at* 25 s. *the piece ready*

money; and in barter he will have 30 s. and he will have the $\frac{1}{4}$ part of his over-price in ready money. The other hath Norwich *Stockins at* 40 s. *the dozen to sell for ready money. But in as much as the first Merchants Grograns are no better, he would deliver them so to ballance the barter, that he may gain* 10 *pounds in the* 100 *pounds. The question is now, how he shall sel his hose the dozen in barter, according to his request.*

Answer. Say, if 100 give 110 li. what shall 40 s. give, which is the just price of the dozen of stockins? Multiply and divide, and you shall finde 44 s. Then take the $\frac{1}{4}$ of the 30 s. which is 7 s. 6 d. and subtract it from 25 s. and also from 30 s. and there will remaine 17 s. 6 d. and 22 s. 6 d. for the two first numbers in the *Rule of three,* and 44 shillings, which is the just price (with his gaine in the dozen of Stockins) for the third number. Then multiply and divide, and you shall finde 56 s. $6\frac{6}{7}$ d. and for so much he is to sell his dozen of stockins in barter.

The seventh Rule.

Two Merchants will change their Merchandize one with the other. The one hath 720 *Ells of Cambrick at* 5 s. *the Ell to sell for ready money, but in barter he requireth* 6 s. 8 d. *And yet notwithstanding, he loseth by it after* 10 *pounds in the hundred pounds, whereupon he requireth one halfe of his over-price in ready money: and the other Merchant having skill enough to make the barter equall, delivered* English *Saffrons at* 30 s. *the pound. The question is now, what his Saffrons cost the pound in ready money.*

Ans. You must first seek what is lost upon the 100 li. which to do, you may say (if you please) if 100 pound

pound lose 10, what shall 6 $\frac{2}{3}$ lose? Work, and you shall find $\frac{2}{3}$ s. (or 8 d.) which must be rebated from 6 s. 8 d. so resteth 6 s. still. Or you may say, If 100 pound give me but 90 pounds, what shall 6 s. 8 d. give? Work this way either, and you shall finde also as before directly in your quotient 6 s. your desire. Then are you next to cast up what the 720 Ells of Cambrick cometh to at 6 s. 8 d. the Ell, and you shall finde 240 pounds: the $\frac{1}{2}$ whereof the Cambrick Merchant will have in ready money (which is 120 pounds.) Next, you must cast what the Cambrick cometh to after his losse in the 100 pound, which as you found, is but 6 s. an Ell, and you shall find 216 pounds. Now must you subtract his ready money (which is 120 pounds in all) out of 240 pound, and also out of 216 pound, and there will remaine 120 pounds, and 96 pounds for your two first numbers in the *Rule of three*, and 30 shillings is the over-price of your Saffron for the third number: Then multiply and divide, and you shall finde 24 shillings. And so much did his Saffron cost in ready money.

Two Merchants barter; the one hath fifty Clothes to put away for ready money at 11 *pounds the Cloth, and in barter putteth them away for* 12 *pounds, taking Holland Cloth at* 20 d. *the Flemmish Ell, which was worth no more but* 18 d. *The question is now, what Holland payeth for the Cloth, and what he winneth or loseth by the bargain.*

Answer. Fifty Clothes at 11 pounds the Cloth come to 550 pounds, and put away at 12 pounds the piece, make 600 pound. Then to finde what Holland payeth for the Cloth, say by the *Rule of three direct*, If 20 d. buy one Ell, what 600 pounds?

 Work,

Worke, and you shall finde 7200 Ells. Now to finde the Estate of his gain or losse, you must seek what his 7200 Ells cometh to at 18 d. the Ell. Work by the Rule of proportion direct, and you shall finde 540 pounds, which is not so much as his Clothes were worth in ready money by ten pounds: and so much lost the first Merchant by his Exchange.

A Venetian *hath in* London 100 *pieces of Silke, to put away for ready money at* 3 li. *the piece. But in barter he delivered them for* 4 li. *the piece, taking Wools of a Fellmonger at* 7 li. 10 s. *the C. weight, which was worth no more but six pounds the C. ready money. The question is now, what Wools payeth for the Silks, and which of them winneth or loseth by the barter.*

Ans. A hundred pieces of Silk at 3 li. is in all 300 li. and at 4 li. is 400 li. Then to finde what Wools pay for the Silke, say by the *Rule of three direct*, If $7\frac{1}{2}$ li. buy me C. weight, what 400? Work, and finde $53\frac{1}{3}$ C. weight of Wool. Now to finde the Estate of their gain and losse, cast up his Wool at 6 li. the C. (for so much they were worth ready money) and you shall finde 320 pound, which is 20 pound more then the Silks were to be sold for ready money, whereby the *Venetian* gained 20 pounds by the Barter.

A Merchant hath $53\frac{1}{2}$ *weight of Wool at 6 pounds the C. to sell for ready money; but in barter he will have* 7 *pounds* 10 s. *and another doth barter with him for Silks, which are worth three pounds a piece ready money. The question is now, how he ought to deliver his Silks the piece in barter, and how many payeth for the Wools.*

Answer. Say by the Rule of proportion, (or by the *Rule of three direct*) If 6 pounds for C. weight ready money

money yield me 7 li. 10 s. what will 3 li. yield, which is the juſt price of a piece of Silk in barter, to make the Truck equall? Work, and find 3 li. 15 s. the price of a piece of Silk in barter: Then ſay, If 3 li. 15 s. require one piece of Silk, how many pieces of Silk are bought with 400 pound, which is the value of $53\frac{1}{3}$ C. weight of wool, at 7 li. 10 s? Work by the *Rule of Three direct*, and you ſhall find 106 pieces of Silk, and $\frac{2}{3}$ of a piece, and ſo many of Silk pay for the Wool, and neither party hath advantage of other.

Two men will change Merchandize the one with the other. The one of them hath beer at 6 s. 8 d. the barrell, to ſell for ready money; but in barter he will ſell the barrell for 8 s. and yet he will gain moreover after 10 pound upon the 100 pounds. And the other hath white Spaniſh *Wooll at 20 s. the Rove, to ſell for ready money. The queſtion is now, how he ſhall deliver the Rove of Wooll in barter to ſave himſelf harmleſs.*

Anſwer. Say, if $6\frac{2}{3}$ s. which is the juſt price of the barrell of Beer, be ſold in barter for 8 ſhillings, for how much ſhall 20 ſhillings (which is the juſt price of the Rove of Wooll) be ſold in barter? Work by the Rule of Three direct, and you ſhall finde 24 s. Then becauſe the firſt Merchant will gain after 10 pounds upon the 100 pounds, he maketh his 100 pounds 110 pounds. And therefore ſay by the Rule of Three. If the ſecond Merchant of 110 pounds doe make but 100 pounds, how much ſhall he make of 24 s? Multiply and divide, and you ſhall finde 21 s. 9 d. $\frac{9}{11}$ of a peny. And for ſo much ſhall he ſell the Rove of Wooll to be delivered in barter, to the end the firſt Merchant may give 10 in the 100.

Two Merchants will change their Commodities the

one with the other. The one of them hath white Paper at 4 s. the Ream, to sell for ready money; and in barter he will doe it away for 5 s. and yet he will gaine moreover after the rate of 10 pounds upon the 100 pounds. And the other hath Mace at 14 s. 6 d. the pound weight to sell in barter. Now I demand what the pound did cost in ready money.

Answer. Say, if 5 s. (which is the over-price of the Paper in barter) be come of 4 s. the just price, of how much shall come $14\frac{1}{2}$ shillings, which is the surprize of the pound of Mace in barter? Multiply and divide, and you shall finde $11\frac{3}{5}$ s. Then because the first Merchant of Paper will gain after 10 upon the 100. say, If 100 doe give 110, what shall $11\frac{3}{5}$ shillings give? Work, and you shall finde 12 s. $9\frac{8}{25}$ d. and so much did the pound of Mace cost in ready money.

The fourteenth Chapter treateth of exchanging of money from one place to the other.

EXchange is no other thing, then to take or receive money in one City, to render or pay the value thereof in another City, or else to give money in one place, and receive the value thereof in another, as term of certain dayes, months, or fairs, according to the diversity of the place.

But this practice chiefly consisteth in the knowledge of the Money or Coyns in divers places, of which for thy benefit (after a few examples given to the Introduction of this work) I will set down certain notes of the diversity

diversity of the common and usuall Coynes in most places in Christendome for traffick.

And first I will begin at *Antwerp*, where they use to make their accounts by *Deniers de grosse*, that is to say, pence *Flemmish*, whereof 12 doe make 1 s. *Flemmish*, and 20 s. doe make one pound *de grosse*.

Item, *A Merchant delivered at* Antwerp 400 *pound* Flemmish *to receive in* London 20 s. *sterling, for every* 23 s.—4 d. Flemmish: *The question is now, how much sterling money is to be received at* London *for* 400 *pounds* Flemmish.

Answer. Say by the Rule of Three, If $23\frac{1}{3}$ *Flemmish* give 20 s. sterling, what 400 pounds *Flemmish*? Worke, and you shall finde 342 li.—17 s.—$1\frac{5}{7}$ pence: and so much sterling shall I receive in *London* for the said 400 pounds *Flemmish*.

Otherwise also wrought by Rules of Practice, in taking the $\frac{1}{7}$ of the *Flemmish* money delivered, and abating the same from the principall, the rest is *English* money, as before.

400 li.	0 s.	0 d.
57	2	$10\frac{2}{7}$
342	17	$1\frac{5}{7}$ sterling.

A Merchant at London *delivered* 200 l. *sterling for* Antwerp, at 23 s.—5 d. Flemmish *the pounds sterling: the question is, how much he must receive at* Antwerp.

Answer. Say by the Rule of Three, If 1 pound sterling give 23s. 5d. *Flemmish*, what 200 li. sterling? Work, and thou shalt finde 234 li.—3 s.—4 d. So many pounds *Flemmish* shall he receive at *Antwerp* for the said 200 pounds sterling.

Otherwise by Practice.

1—3—5—200			
3 s. 4 d.	33	6	8
1 d.		16	8
maketh sterling	234 li	3s.	4d.

In London 20 *pound sterling is delivered by Exchange for* Antwerp, *at* 23 s. 9. d. Flemmish *the pound sterling: the question is, at what rate the* Flemmish *money ought to be returned to gain four pounds upon the* 100 *pound sterling at* London.

Answer. First say by the Rule of Three direct, If 1 pound sterling give $23\frac{3}{4}$ Flemmish, what 200 pounds sterling? Multiply and divide, and you shall finde 237 pounds 10 shillings. The which to return to gain 8 pounds sterling in *London*, say by the backer Rule, If 200 pounds sterling require the exchange 23 s. 9 d. Flemmish, what the exchange to make 208 li. sterling? Work by the Rule, and finde 22 s. $10\frac{1}{26}$ d. Flemmish, the effect in the question required.

If I take up money at Antwerp *after* 19 s. 4 d. Flemmish, *to pay for the same at* London 20 s. *sterling, and when the day of payment is come, I am forced to return the same money again in* London, *to pay my Bill of Exchange, so that for* 20 *shillings which I take up here at* London, *I must pay* 19 s. 6 d. *at* Antwerp, *I demand whether I doe win or lose, and how much in or upon the* 100 *pounds of money.*

Answer. Say by the Rule of Three, If $19\frac{1}{2}$ give $19\frac{1}{3}$, what will 100 pounds give? Multiply and divide, and you shall finde 99 li. $2\frac{106}{117}$ s. which being abated from 100 pounds, there will remain 17 s. $\frac{11}{117}$ and so much I do lose upon the 100 pounds of money.

If

If I take up at London *20 shillings sterling to pay at* Antwerp 22 s. 4 d. *and when the day of payment is come, my Factor is constrained to take up money again at* Antwerp, *wherewith to pay the aforesaid summe, and there he doth receive* 23 s. 4 d. Flemmish, *for the which I must pay* 20 s. *at* London: *The question is now, whether I doe win or lose, and how much upon the* 100 li. *of money after that rate.*

Answer. Say by the Rule of Proportion, If 22 $\frac{1}{3}$ s. give 23 $\frac{1}{3}$ s. what will 100 pounds give? Multiply and divide, and you shall finde 104 pounds 9 shillings $\frac{37}{67}$, from the which abate 100 pounds, and there will remain 4 pounds 9 shillings $\frac{37}{67}$, and so much is there gained upon the 100 pounds of money.

In Antwerp *is delivered* 200 *pounds* Flemmish *by exchange for* London, *at* 20 *shillings sterling for every* 23 *shillings* 4 d. Flemmish. *The question is, at what rate the same is to be returned to gain* 10 *pounds upon the* 100 *pounds* Flemmish *in* Antwerp.

Answer. First say by the Rule of Three, If 23$\frac{1}{3}$ Flemmish give 20 s. what shall 200 pounds gain? Work, and you shall finde 171 pounds 8 s. 6 $\frac{6}{7}$ d. Then say again by the Rule of Three direct, If 171 pounds 8 s. 6$\frac{6}{7}$d. sterling, give me 210 pounds Flemmish, what shall 20 s. sterling give? Work, and you shall finde 24 s. 6 d. Flemmish. And at the same rate ought the same to be returned at *Antwerp*, to gain 10 pounds upon the 100 Flemmish.

A Merchant of Antwerp *delivereth* 234 l. 3 s. 4 d. Flemmish, *to receive at* London 200 li. *sterling. The question is now, how the exchange goeth after this rate.*

Answer. Say by the Rule of Three direct, If 200 give 20. what 234$\frac{1}{6}$? Multiply and divide, and you shall

ſhall finde 23 s.—— 5 d. And for ſo much goeth the exchange.

Item, *the exchange from* London *into* France *is not like as it is to* Flanders, *but it is delivered by the French Crown, which is worth* 50 Soulx Turnois *the piece.*

Whereupon alſo you muſt note that in *France* they make their accounts by Franks, Soulx, and Deniers Turnois, whereof 12 Deniers make one Soulx Turnois, and 20 Soulx make one pound Turnois, which they call a Liure or Frank. But the Merchants to make their Accounts, doe uſe French Crowns, which is current among them for 51 Soulx Turnois. But by exchange it is otherwiſe, for it is delivered but for 50 Soulx Turnois the Crown, or as the taker up of the money can agree with the deliverer. And note that this △ Character repreſenteth the Crown by exchange, and is ever 50 Soulx Turnois or French money.

A Merchant delivereth at London 240 *pounds ſterling, after five ſhillings ſix pence the Crown, to receive at* Paris 50 *Soulx Turnois for every Crown. I demand how much* Turnois *or* French *money payeth the Bills for the ſaid* 240 *pounds ſterling.*

Anſwer. Say by the Rule of Three, If $5\frac{1}{2}$ s. ſterling give me 50 s. Turnois, what ſhall 240 pounds ſterling give? Reduce the pounds into ſhillings, then multiply and divide, and you ſhall finde 2181 Liures, 16 Soulx, 4 Deniers, and $\frac{4}{11}$ Turnois, and ſo much payeth the Bills at *Paris* for the 240 pound ſterling.

A Merchant delivereth at Roan, *or elſewhere in* France, 1430 *pounds or franks, the which frank or pound is* 20 *Soulx, or a pound* Turnois, *to receive in* London

London 6 s. 4 d. *sterling for every* △ *of* 50 *Soulx* Turnois. *The question is, how much sterling money I ought to receive at* London *for my* 1430 *pound* Turnois.

Answer. Say, if $2\frac{1}{2}$ pounds give me $6\frac{1}{3}$ s. what will 1430 give me? Work, and you shall find $3622\frac{2}{3}$ shillings sterling, which maketh 181 li. 2 s. 8 d. and so much money is to be received at *London* for the said 1430 Liures Turnois, after 6 s. 4 d. for every △ of 50 Soulx.

In London *is delivered* 200 *pound sterling by exchange for* Paris, *at* 5 s. 9 d. *the* △ *of* 50 *Soulx* Turnois. *The question is, at what price the said* △ *is to be returned to gain* 6 *pounds upon the* 100 *pounds sterling at* London.

Answer. First, say (by the Rule of Three direct) if $5\frac{3}{4}$ s. sterling give 50 Soulx Turnois, what shall 200 pound sterling give? Work, and you shall finde 1739 Franks or Liures, $2\frac{14}{23}$ Soulx. Then the which to returne, and gaine 6 pounds upon the hundred pounds in *London*, say by the Rule of Three direct, if 1739 Franks $2\frac{14}{23}$ Soulx yield 212 pound, what the △ of 50 Soulx? Work and finde 6 s. $1\frac{7}{50}$ d. the effect required in the question.

A Merchant delivered in London 160 li. sterling, *to receive in* Biscay *for every* 5 s. 6 d. *one Ducat of* 374 Marvides. *The question is, how many* Marvides *ought I to receive at* Biscay?

Answer. Say, if $5\frac{1}{2}$ s. sterling give 374 Marvides, what shall 160 pounds sterling give? Multiply and divide, and you shall finde 217600 Marvides, and so many I ought to receive at *Biscay* for my 160 pounds sterling.

A

A Merchant delivered in Baion 40000 Marvides, *to receive in* London 5 s. 8 d. *ſterling for every Ducat of* 374 Marvides. *The queſtion is now, how much ſterling money payeth the Bills of Exchange for the ſaid* 40000 Marvides ?

Anſwer. Say, if 374 Marvides make one Ducat, what 40000 Marvides ? Multiply and divide, and finde $106\frac{178}{187}$

Then ſay again, if 1 Ducat give $5\frac{2}{3}$s. what giveth $106\frac{178}{187}$ Ducats ? Work, and finde 30 l. 6 s. $\frac{34}{561}$ Otherwiſe it is wrought more brief at one working, as in the laſt queſtion before, in conſidering that 5 s. 8d. containeth one Ducat, or 374 Marvides. Therefore ſay by the Rule of Three, If 374 Marvides give $5\frac{2}{3}$ s. what 40000 Marvides? Work, and you ſhall alſo find in your quotient 30l. 6 s. $\frac{34}{561}$ s. And ſo many pounds ſterling is to be received for the 40000 Marvides.

In London 200 *pounds delivered by exchange for* Vigo, 374 *Marvides the Ducat of* 5 s. 10 d. *ſterling, maketh* $256457\frac{1}{7}$ *Marvides : the which to return and gain* 10 li. *upon the* 100 *pounds in* London, *ſay by the* Rule of Three direct, *If* 220 li. *require* $256457\frac{1}{7}$ *Marvides, what* 5 s. 10 d ? *Work, and finde* 340 *Marvides, the price of every Ducat in return, which is the effect in the queſtion required.*

These may ſeem ſufficient for inſtructions.

N*Otwithſtanding for thy further aid and benefit, hereafter follow ſix ſpeciall and moſt brief* Rules of Practice, *for* Engliſh, French, *and* Flemmiſh *money.*

How

1	teacheth	How to turn Flemmish to English sterling.
2		How to turn English sterling to Flemmish.
3		How to turn Flemmish to French.
4		How to turn French into Flemmish.
5		How to turn Sterling into French.
6		How to turn French into Sterling,

The fifteenth Chapter treateth of the said six Rules of brevity, and of valuation of *English*, *Flemmish* and *French* money, and how each of them may easily be brought to others value.

How briefly to reduce pounds, shillings and pence Flemmish, *into pounds, shillings and pence* English *sterling*.

IT is to be noted, that 7 pounds *Flemmish* make but 6 pounds sterling: 7 s. *Flemmish* make 6 s. sterling, and 7 d. *Flemmish* 6 d. sterling; so that 7 yieldeth but 6. Wherein is evident that then is lost $\frac{1}{7}$, (if it may be so called) when it is reduced into *English* money: wherefore to know how much 233 li. 13 s. 4 d. *Flemmish* maketh *English*, you must subtract from it $\frac{1}{7}$, beginning with the pounds, &c. and that which resteth after this subtraction, is the summe required: so that 233 li. 13 s. 4 d. *Flemmish*, maketh 200 li. 5 s. 8 $\frac{4}{7}$ d. sterling. 1 Rule.

Example.

Example. li.	s.	d.	Another Example. li.	s.	d.
233	13	4	311	0	0
$\frac{1}{7}$ 33	7	7 $\frac{3}{7}$	$\frac{1}{7}$ 44	8	6 $\frac{6}{7}$
200	5	8 $\frac{4}{7}$ Ster.	266	11	5 $\frac{1}{7}$

Rule 2. *To reduce pounds, ſhillings and pence ſterling, into pounds, ſhillings and pence* Flemmiſh.

Note that a pound ſterling maketh 1 li. 3 s. 4 d. *Flemmiſh*: that is, 1 $\frac{1}{6}$ li. 1 s. ſterling maketh 1 $\frac{1}{6}$ s. *Flemmiſh*, and 1 d. ſterling maketh 1 $\frac{1}{6}$ d. *Flemmiſh*. So that there is gained (if it may be ſo called) $\frac{1}{6}$ of the ſumme being thus reduced to *Flemmiſh*, for of 1 $\frac{1}{6}$ is made $\frac{7}{6}$ which is one whole and $\frac{1}{6}$. Then to know how much 237 li. 7 s. 6 d. ſterling maketh *Flemmiſh*, ſubtract from your ſterling the $\frac{1}{6}$ of the whole ſum, and adde it to the ſame ſum, and it maketh 276 li. 18 s. 4 d. which is the ſum required.

Example. li.	s.	d.	Another Example. li.	s.	d.
237	7	6 Ster.	337		
$\frac{1}{6}$ 39	11	3	$\frac{1}{6}$ 56	3	4
276	18	9 Flem.	393	3	4

Rule 3. *To reduce pounds, ſhillings and pence* Flemmiſh, *into pounds, ſhillings and pence* French.

Ye ſhall note, that the equality of Flemmiſh and French money is this, that is to ſay, the pound Flemmiſh maketh 7 pound $\frac{1}{5}$ French, or Turnois; 1 s. Flemmiſh maketh 7 $\frac{1}{5}$ s. French; and a groat Flemmiſh maketh 7 $\frac{1}{5}$ d. French.

Where-

Wherefore to know how much 143 li. 4 s. 9 d. Flemmish maketh French, ye must multiply the whole number twice by 6, beginning at pence, and so forward, and the Product of your second multiplication divide by 5, so the worke is finished. Or multiply the said said summe by 7, and take out of it $\frac{1}{5}$ adding it to the Product of your multiplication by 7, and that is your number required. So that as well by the one as by the other, 143 li. 4 s. 9 d. Flemmish, maketh 1031 li. 6 s. 2 $\frac{1}{3}$ d. French or Turnois.

Example.

li.	s.	d.	
143	4	9	Flem.
		6	
859	8	6	
		6	
5156	11	0	Fren.
$\frac{1}{5}$ 1031	6	2 $\frac{2}{5}$	Fren.

The same otherwise.

li.	s.	d.
143	4	9
		7
1002	13	3
$\frac{1}{5}$ 28	12	11 $\frac{2}{5}$
1031	6	2 $\frac{2}{5}$

Another Example.

143 l. Flem.
6

858
6

$\frac{1}{5}$ 5148 French.

1029 li. $\frac{3}{5}$ or 12 s. French.

Or thus:

143
7

1001
$\frac{1}{5}$ 28 — 12

1029 li. — 12

Rule 4. *To reduce pounds, ſhillings and pence* French, *into pounds, ſhillings and pence* Flemmiſh.

Multiply 233 li.—8 s.—4 d, French by 5, and divide the Product twice by 6, that is, the ſaid number by 6, and the Product or Quotient again by 6, and the Quotient of this ſecond Diviſion is the thing required. So that 233 li.—8 s.—4 d. French, maketh 32 li.—8 s.—4 $\frac{5}{9}$ d. Flemmiſh.

Example.				Another Example.			
li.	s.	d.		li.	s.	d.	
233	8	4	Fren.	753			French.
		5		5			
1167	1	8		3765			
$\frac{1}{6}$194	10	3 $\frac{1}{3}$		$\frac{1}{6}$627	10		
$\frac{1}{6}$32	8	4$\frac{5}{9}$	Flem.	$\frac{1}{6}$104	11	8	Flem.

Rule 5. *To reduce pounds, ſhillings and pence Sterling, into pounds, ſhillings and pence* French, *or Turnois.*

The pound Sterling maketh 8 li. 8 s. *French*, that is to ſay, 8 $\frac{2}{5}$ pounds: the ſhillings make 8 $\frac{2}{5}$ ſhillings, and the peny 8 $\frac{2}{5}$ d. *French.* Wherefore to know what 231 li. 13 s. 4 d. Sterling maketh *French*, ye muſt multiply the whole ſumme by 42, that is, by 7. and the Product of it by 6. and divide this ſecond Product by 5. and that is the ſumme required.

Otherwiſe, multiply the ſumme Sterling by 8, and adde twice to the Product $\frac{1}{5}$, and it ſhall produce the ſumme required. So that both wayes 231 li. 13 s. 4 d. Sterling, maketh 1946 pound *French*, as hereunder followeth.

Example.

Example.			Ster.
li.	s.	d.	
231	13	4	
		6	
1390	0	0	
		7	
$\frac{1}{5}$ 9730	0	0	
$\frac{1}{5}$ 1946	0	0	Fren.

The same otherwise.			Sterling.
li.	s.	d.	
231	13	4	
		8	
1853	6	8	
46	6	8	
46	6	8	
1946	0	0	French.

Another example.

753	
6	
4518	
7	
31626	
$\frac{1}{5}$ 6325 — 4	Fren.

The same.

Ster.	753		Sterling.
	8		
	6024		
$\frac{1}{2}$	150	12	
$\frac{1}{2}$	150	12	
	6325	4	French.

To reduce pounds, shillings and pence French, *into pounds shillings and pence sterling.* Rule 6.

To know how much 1256 li. 12 s. 6 d. *French* maketh in sterling money: multiply the summe by 5, and divide the Product by 7 and 6 at twice, and the last Quotient shall be the thing required, that is to say, 1256 li. 12 s. 6 d. maketh 149 pounds, 11 s. 11 $\frac{4}{7}$ d. sterling.

Example.				Another Example.			
	li.	s.	d. French.		li.	s.	d.
	1256	12	6		2531	0	0 French.
			5		5		
	6283	2	6		12655		
$\frac{1}{6}$	1047	3	9	$\frac{1}{6}$	2109	3	4
$\frac{1}{7}$	149	11	$11\frac{4}{7}$ Ster.	$\frac{1}{7}$	301	6	$2\frac{2}{7}$ Ster.

Note, that when any mony is given by exchange at *London* for *Roan* at $71\frac{1}{2}$ d. or rather $71\frac{1}{7}$ for the Crown of 50 s. French, there is neither gain nor losse: for it is one money for another, accounting 8 li. 8 s. French for one pound sterling. So the giver loseth the time of payment, which is about 15 dayes, and he that taketh it hath the gain of the same.

They of *Roan*, that put forth or take money by exchange for *London*, ought to have like consideration.

Item, When any man giveth at *London* 64 pence $\frac{1}{3}$, or rather $65\frac{2}{7}$ d. to have at one of the fairs of *Lions* a Crown *de Marc*, he that so giveth the money, loseth the time, and he that taketh it gaineth the same: for 62 pence $\frac{3}{7}$ is equall in value to 45 s. French. He that putteth or taketh mony at *Lions* for *London*, ought to consider the same.

Item, when any deliver in *Antwerpe* 75 pence, to receive at *Lions* a Crown *de Marc*, he that putteth it forth loseth the time, and he that taketh it gaineth the same. For 75 groats Flemmish is equall in value to 45 s. French.

Thus for this time I make an end of the Practice of Exchange,

Exchange, and the instructions thereunto belonging, and according to my promise: yet farther to gratifie such as are desirous to know the common Coynes used for traffick among Merchants in these Cities following, a brief declaration of their moneys, and the reckonings and account of them, I will here set down.

The sixteenth Chapter containeth a Declaration of the valuation and diversity of Coyns of most places of Christendome for traffick; and the manner of exchange in those places from one City or Town to another: which known, is right necessary for Merchants, by means whereof they do finde the gain or losse upon the exchange.

I*Tem*, forasmuch as the greatest diversity of money of exchange is at *Lions*; therefore I will begin duely of the money of that place.

At *Lions* they use Franks, Soulx, and Deniers Turnois. A Frank maketh 20 Soulx, and 1 Soulx 12 Deniers; but the Merchants to keep their Books of accounts, doe use French crowns of the marke at 45 Soulx the piece, and doe divide it into 20 Soulx, one Soulx is 12 Deniers.

Item, a mark of Gold maketh 65 △ of the mark, which serveth for exchange, and divide it into 8 ounces, the ounce into 24 pence or Deniers, the Denier into 24 grains. and so the summe or whole by imagination or guesse. △ This mark standeth for a Crown.

Also at *Lions* there are four Fairs in a year, at the which they doe commonly exchange, which are from

rom three moneths to three moneths.

At *Geans* they use the Soulx: one Ducat maketh 3 pound.

At *Naples* they use Ducats, Taries, and Grains; the Ducat maketh five Taries, and one Tary twenty Grains: but they take six Ducats which maketh 30 Taries for the ounce.

A Ducat maketh ten Carlins, and a Carlin ten Grains, so that two Carlins make a Tary, and 100 Grains make a Ducat.

At *Rome* they use the Ducats of the Chamber; one Ducat is worth 12 Guillis, and one Guillis 10 Soulx.

At *Venice* they use Ducats currant at 124 Soulx a piece, or 24 Deniers, & one Denier maketh 32 Picolis.

At *Palerm* and *Messina* they write after ounce, Tary and Grains, and one ounce is worth 6 Ducats of 30 Taries, and one Tary is 20 Grains, and one Grain 6 Picolis, one Ducat is also worth 24 Carlins.

At *Millain* they use li. s. d. of Ducat Imperials, and △ of exchange is worth 4 li.

At *Lucques*, *Florence*, and *Ancone*, they use the △ of Gold: in Gold the *French* Crown is worth 7 li. but at *Boloigne* 3 li. 10 s.

At *Barcelone* they use the Soulx: the Ducat of exchange is worth 22 Soulx.

At *Valence* and *Saragosse* they use the Liver, Soulx, and Denier: the *French* Crown of exchange is worth 20 Soulx, and one Soulx is 12 Deniers.

At the Fairs of *Castile* they use Marvides, the Ducat is worth 375 Marvides.

At *Lisbone* they use the Rayes: one Ducat of exchange is worth 400 Rayes.

At *Norenburg*, *Frankford*, and *Augurst* in *Germany*

many, they uſe the *Krentzers*, whereof 60 make a Floren.

At *Antwerp* they uſe li. s. d. *de Gros*, and they exchange into the Denier *de Gros*, to wit, our Engliſh peny.

At *London* they uſe the li. the s. and d. ſterling, and they exchange in pence ſterling.

The exchange of Lions *at ſundry places.*

Item, at *Lions* there is exchange in three ſorts, at the Cities and Towns following.

Firſt, they deliver at *Lions* one mark, to have or receive at *Naples* almoſt 41 $\frac{1}{2}$ Ducats, at *Venice* 70 Ducats current, at *Rome* 63 Ducats of the Chamber, at *Lucques* and *Florence* 65 △ of Gold, at *Millan* 82 △.

And contrariwiſe, at the ſaid Cities aforeſaid they doe give ſo much of money, to have a mark of *Lions*.

Secondly, they give at *Lisbon* one of △ of mark of 45 Soulx *Turnois* a piece, to have at *Geans* almoſt 68 Soulx, at *Palerme* and *Meſſine* almoſt 24 Carlins, at *Barcelone* 22 Soulx, at *Valence* or *Saragoſſe* 20 Soulx, at the Faire at *Caſtile* 350 Marvides, at *Lisbon* 360 Rayes, in *Antwerp* 57 Deniers *de Gros*, and at *London* 70 d. ſterling.

And contrariwiſe, they give in the ſaid Cities almoſt as much of their money to have a French Crown of the mark at *Lions*.

Thirdly, they doe give at *Lions* a △ of the Sun, to have almoſt 93 Krentzers at *Frankeford*, *Ausburg*, *Noremburg*, or other Cities in *Almain*.

Alſo at *Lions* onely they do pay, they change the $\frac{2}{3}$ in Gold, and $\frac{1}{3}$ in money, or elſe all in money, in giving 1 $\frac{1}{2}$ for the hundred.

Changes at Naples, *and other Towns.*

Item, at *Naples* they give or deliver almost 112 Ducats, to receive at *Rome* 100 Ducats of the Chamber at the old value.

Through *Lucques* and *Florence* they deliver 100 Ducats Carlins, to receive there almost 86 ▲ of Gold.

Through *Palerm* and *Messine*, one Ducat of 5 Tary, to receive there almost 164 grains.

Through *Millain*, one Ducat, to receive there almost 90 Soulx.

Through *Geans*, one Ducat to receive there almost 65 Soulx. The whole summe to be paid within ten dayes after the sight of the Bill of exchange.

Also at *Naples* they deliver one Ducat, to receive in *Antwerp* almost 67 d. or Deniers *de Gros*, within two months. At *London* almost 60 d. sterling in three months. At *Barcelone* almost 20 Soulx within two months.

At *Valence* almost 18 Soulx within two months. At *Lisbon* 333 Rayes within three months: and at the Fair at *Castile* almost 340 Marvides.

Change of Venice *to other places.*

At *Venice* they deliver 100 Ducats current, to receive in *Almain* almost 140 Florens at 60 Krentzers the piece.

At *Lucques* and *Florence* almost 108 △ of Gold in 10 dayes.

Likewise at *Venice* they deliver a Ducat current, to receive at *Palerm* and *Messine* almost 21 Carlins: at *Millain* almost 93 Soulx: at *Geans* almost 69 Soulx, the whole at 10 dayes end.

Of

Of the Pair *or* Pari.

As touching the exchange, it is necessary to understand or know the *Pair*, which the *Italians* call *Pari*, which is no other thing then to make the money of the change of one City or Town to or with the money of another, by means whereof they doe find the gains or losse upon the exchange.

Example.

Item, having received Letters of credit of one of *Antwerp*, that the △ of the Sunne is there worth 7 Soulx: the question is, what the same is worth at *London*, when the *Pair* of exchange goeth for 23 shillings.

Answer. *Say, if* 23 *give but* 20, *what giveth* 7? *Worke, and finde* 81. 1. $\frac{1}{5}$ d. *and so much is the* △ *of the Sun worth at* London.

The seventeenth Chapter containeth also a Declaration of the diversity of the Weights and Measures of most places of Christendome for traffique. At the end of which discourse are two Tables, the one for Weight, and the other for Measure, proportionate and reduced to an equality of our *English* Measure and Weight; by the aid whereof, the ingenious may easily by the *Rule of Three*, convert the one into the other at pleasure, *&c.*

AT *London*, and so all *England* through, are used 2 kinds of Weights and Measures, as the *Troy* weight, and the *Haberdepoise*. From the *Troy* weight is derived the proportion and quantity of all kinde of dry and liquid Measures, as Pecks, Bushels, Quarters, &c.

wherewith is bought and ſold all kinde of grain and other Commodities mete by the Buſhell; and in liquid, Ale, Beere, Wine, Oyle, Butter, Honey, &c. Upon theſe grounds and ſtatutes is bread made, and ſold by the *Troy* weight: and ſo is Gold, Silver, Pearle, precious Stones, and Jewels. The leaſt quantity of this *Troy* weight is a grain: twenty four of theſe graines make a penny weight, twenty penny weights an ounce, and twelve ounces a pound; two pounds or pints of this weight make a quart. And ſo aſcending into bigger quantities, is produced the Meaſures whereby are ſold our other naturall ſuſtenance, *viz.* Ale or Beere, with all other neceſſary commodities, as Butter, Honey, Herrings, Eeles, Sope, &c. All which laſt before rehearſed, though their Meaſures (wherein they are contained) be framed and derived from the *Troy* weight, yet are they in traffique with divers Commodities, as Lead, Tinne, Flax, Wax, with all other commodities, both of this Realme, and of other forraine Countryes whatſoever, bought and ſold by the *Haberdepoiſe* weight after ſixteen ounces to the pound, and 112 pound to the hundred weight. And to every hundred is allowed but 12 pound weight at the common beam. From hence is alſo derived the weight of *Suffolke* Cheeſe, which containeth thirty two Cloves, 8 pound to a Clove, and weigheth in all 256 pounds. And alſo the Barrell of *Suffolke* Butter is, or ſhould be of like weight with the weight of Cheeſe, *viz.* 256 pounds. More 14 of theſe pounds make a Stone, and 26 Stone containeth a Sack of *Engliſh* Wool: Forrain Wool, to wit, *French*, *Spaniſh*, and *Eſtrich*, is alſo ſold by the pound, or C. weight, but moſt commonly by the Rove,

See further of theſe Weights and Meaſures in Reduction, beginning pag. 111.

Rove, 25 pound to a Rove. Other commodities of Tale are bought and sold by the C. fivescore to the C. except headed ware, to wit, Cattel, Nails, and Fish, which are sold after sixscore to the C. There are also two other sorts of measures, to wit, the Ell and the Yard. By the Ell is usually mete Linnen cloth, as Canvas, *&c.* And by the Yard Silks, woolen clothes, *&c.*

Antwerp.

At *Antwerp* are also two sorts of weights; their Gold and Silver weight, and their common weight. Gold and Silver is weighed by the mark, the mark is 8 ounces, the ounce 20 esterlings, and the esterling 32, as our grains. The Goldsmiths divide that into smaller, but not the Merchants. The proof of Gold is made by Karects, whereof 24 make a marke of fine Gold, the Karect is 24 grains: the proof of the money is made by Deniers: 12 Deniers is 1 s. fine, that is, a mark of fine silver: the Denier also is divided into 24 grains, and the grain into foure quarters.

Item, 100 marks in *Antwerp* Troy weight maketh at *Lions* 103 marks, $2\frac{1}{2}$ ounces, and 20 grains 23d. at *Noremburg* 103 marks, $2\frac{1}{2}$ ounces, 2 Quints, 3 Deniers; at *Frankeford* 105 marks; at *Amburg* 104 marks, 3 ounces, 1 Quint; at *Venice* 103 marks, 1 ounce, 7 Deniers, 18 grains; at *London 66* pounds.

The Mark of gold or silver at Antwerp, Troy *weight, which is 8 ounces, maketh $7\frac{1}{2}$ ounces common weight, with which all other Merchandize is weighed. So that the* Troy *weight is greater then the common weight by $6\frac{1}{4}$ in the* C. *By this weight of* Troy *they also weigh* Musk, Amber, Pearl, *&c.*

All silks are bought at Antwerp *by the* Burges Ell, *which*

which is greater then the common measure, by which they retaile by 2 in the hundred. Their common Ell is $\frac{3}{4}$ *of our yard, and* $\frac{3}{5}$ *of our Ell.*

Lions.

At *Lions* is used 3 sorts of weight, whereof the first is the common Towne weight, with which they weigh all kind of Spicery, and divers other Merchandize. The second is called *Geneva* weight, which is 8 in the C. greater then the common weight, with which they weigh Silks, *&c.* The third is *French* weight, called commonly the mark weight, and 100 pounds thereof maketh 106$\frac{1}{4}$ li. *Geneva*, and 114$\frac{3}{4}$ of their common weight: with which *French* weight are weighed all things that pay Custome or Toll.

At *Lions* is also used two sorts of Ells or Aulnes. The one wherewith they measure grosse clothes, as Canvasse, and such like. The other is called the *French* Ell or Aulne, with which they measure all other kinde of Merchandize, whereof 7 common Town Ells make 11 ordinary *French Ells.*

Roan.

At *Roan* 6$\frac{1}{2}$ Muides of Salt, being the measure of the place, make a C. at *Armuiden* in *Zeland*, and the C. of *Brouage* measure of *Armuiden* maketh at *Roan* 11 Muides. 30 Muides make a Last of Corn, and 16 a Last of Oats. 100 pound weight there, maketh at *London* 114$\frac{1}{2}$, and 190$\frac{1}{4}$ at *Antwerp.* And 200 Ells make at *London* 115$\frac{1}{4}$.

Noremburg.

A 100 pound weight at *Noremburg* maketh at *London*

London 111 $\frac{3}{4}$; at *Antwerp* 107 $\frac{1}{7}$. and 100 Ells at *Noremburg* make at *London* 75$\frac{1}{5}$, at *Antwerp* 95$\frac{3}{50}$ &c.

Lisbon.

The C. weight at *Lisbon* maketh 4 Roves, every Rove 32 pounds, ſo that their C. weight is 128 pounds, and their pound containeth 14 Ounces, and 100 pounds of their weight maketh at *London* 113 $\frac{1}{8}$.

Their Silk, Cloth of Gold, and Woollen, is meaſured with a meaſure which they call a cubit, containing about $\frac{3}{4}$ of a Varre of *Caſtile*. Howbeit their common meaſure is called a Varre, which maketh five Palms, and containeth 1$\frac{1}{4}$ of a Varre of *Caſtile:* our Ell of *London* is equall with the Varre of *Lisbon.*

All kind of Merchandize brought from *Flanders*, *Roan*, or *Britain*, payeth at *Lisbon*, as a duty or cuſtome to the King, 20 in the C. which they call the tenth in Merchandize, and the other tenth in money.

Note alſo, that all kinde of Merchandize coming to *Lisbon* by land, payeth leſſe in cuſtome then that that cometh by water.

Sivill.

The Rove of *Sivill* is 30 pound, 4 Roves make their C. weight, which is 120 li. The 100 pounds of *Sivill* maketh at *London* 102 pounds. Their other common meaſure is a Varre, whereof 100 maketh at *London* 74 Ells, and at *Rome* 40 Canes, &c.

Venice.

At *Venice* be two ſorts of weight, the one called *la Groſſe*, the other *la Suttle*; with the groſſe is

weighed

weighed all kinde of great wares, and with the small all kind of spicery, and such like: 96 pounds of gross weight there make at *London* 100 pound, and 100 pounds of spicery there, without any tare or allowance, make at *London* 94. and with tare 65.

Their owne common Measure are Braces, whereof 100 make at *London* $55\frac{1}{2}$ Ells, at *Antwerp* $92\frac{1}{2}$, &c.

Florence.

At *Florence* the 100 l. weight maketh at *Aquila*, for Saffron, 110. and 145 pounds of *Florence* make at *Roan* but 100 pounds; the weight of *Florence* and that of *Lucque* is all one.

Their other measures are Braces, whereof 100 maketh at *Antwerp*, *Burges* measure, $81\frac{2}{5}$ Ells; 100 Braces there make at *London* 49 Ells, &c.

Lucque.

The *Lucque* Sattens commonly sold at *Lions* by weight, and $133\frac{1}{3}$ pound, make at *Lions* 100 pound, so that 1 pound $\frac{1}{3}$ maketh at *Lions* but one pound.

Their other measures are Braces, whereof 100 of them make at *London* 50 ells, at *Antwerp* $83\frac{1}{3}$ ells, &c.

Aquila.

At *Aquila* their 100 pounds make at *London* $71\frac{1}{4}$, their $136\frac{2}{3}$ pounds of saffron make at *Geneva* but 100. and 11 li. of *Geneva* maketh 15 li. at *Aquila*.

Valentia.

At *Valentia* be two sorts of weights, a great and

a ſmall. The C. weight or great weight containeth four Roves, the Rove 36 li. ſo the C. great weight is 144 li. and the C. weight ſmall containeth but 120 pounds, and is alſo parted into four Roves, which is 30 pounds to a Rove. By the ſmall is ſold the ſcarlet grain, with all other kind of ſpicery, and by the great is ſold Wool, with all ſuch like groſſe wares. The $1\frac{1}{3}$ pounds of Silk at *Valentia* maketh at *Lions* one pound *Geneva* weight. The charge of great Merchandize at *Valentia* containeth 432 pounds, and in ſmall wares 360 pounds,

The weight here and at *Barſellone* is all one.

Their 100 pound weight maketh at *London* 78 pound, at *Antwerp* 75.

Dantzick.

At *Dantzick* or *Spruceland* the rule is, that whoſoever buyeth any merchandize there, buyeth it by the ſhip-pound, which is 320li. 20 Liſpounds make a ſhip-pound, and the Liſpound containeth 16 pound, which ſhip-pound of *Dantzick* maketh at *Antwerp* $266\frac{2}{3}$ li. Their 100 li. weight maketh at *London* 86, &c.

Their other common meaſures are Ells, whereof 100 maketh at *London* $72\frac{1}{4}$, and at *Antwerp* $120\frac{1}{2}$ Ells.

Tolouſe.

At *Tolouſe* 6 cabes of Woad make a charge, two Ciſterns of corn meaſure and all kinde of grain make a Charge, the Ciſtern weigheth 160 li. weight of that place. Their 100 in weight maketh at *London* but $91\frac{1}{4}$ pound.

Geans.

Geans.

At *Genua* or *Geans*, 100 li. of their weight maketh at *London* 71 $\frac{1}{4}$, and at *Antwerp* 68 $\frac{2}{5}$. 100 li. weight at *Genua*, maketh at *Venice*, to wit, Suttle, 106 li.

Their other common Measures are *Palms*, whereof 100 make at *London* 20 $\frac{2}{5}$ Ells, and at *Antwerp* 34 $\frac{3}{5}$.

The rest are supplied in two Tables, which hereafter follow: whereby the ingenious may gather his desire.

The Table of the agreement of the weights of divers Countries, the one with the other, being reduced to an equality, as followeth.

112 pounds weight at London, make at		112 pounds weight at London, make at	
Antwerp	107 $\frac{1}{5}$	*Venice grosse weight*	105 $\frac{3}{8}$
Frankeford	099	*Venice suttle weight*	166 $\frac{1}{7}$
Colen and *Ausburgh*	102 $\frac{1}{4}$	*Aquina*	157 $\frac{1}{7}$
Noremburg	100 $\frac{1}{3}$	*Vienna*	089 $\frac{3}{8}$
Roan	098	*Preslaw*	134 $\frac{2}{6}$
Paris	102 $\frac{1}{4}$	*Leipsig*	101 $\frac{1}{4}$
Lions	118 $\frac{1}{2}$	*Dantzick*	129 $\frac{1}{4}$
Diep	100 $\frac{1}{4}$	*Lubeck*	097 $\frac{3}{8}$
Geneva	090 $\frac{1}{3}$	*Barcellon*	144 $\frac{1}{2}$
Tolouse	122 $\frac{1}{4}$	*Lisbone*	099
Rochell	124 $\frac{1}{4}$	*Geans*	157 $\frac{1}{5}$
Marseilles	124 $\frac{3}{4}$		
Sivill, &c.	109 $\frac{1}{4}$		

The other Table of agreement of Measures of divers Countryes reduced unto an equality, by the aid whereof you may with the use of the Rule of three, convert either more or lesse of any one Measure unto the other.

Antwerp

60 Ells, or 75 Yards at *London* make at

Place	Number	Measure
Antwerp	100	Ells.
Noremburg	$104\frac{1}{2}$	Ells.
Frankford	125	Ells.
Leipsig	125	Ells.
Preslaw	125	Ells.
Dantzick	183	Ells.
Vienna in *Austria*	87	Aulnes.
Lions in *France*	$60\frac{3}{4}$	Aulnes.
Paris in *France*	57	Aulnes.
Roan in *Normandy*	52	Aulnes.
Lisbone	60	Varres.
Sivil in *Spain*	81	Varres.
Castile in *Spain*	81	Varres.
Methera Isles	62	Varres.
Venice	108	Braces.
Lucques	120	Braces.
Florence	$122\frac{1}{2}$	Braces.
Millain	138	Braces.
Rome	90	Canes.
Geans.	$288\frac{6}{13}$	Palms.

The eighteenth Chapter treateth of Sports and Pastimes, done by number.

IF you would know the number that any man doth think or imagine in his mind, as though you could divine, bid them triple it, or put twice so much more to it as it is, which done, ask him whether it be even or odd; if he say odd, bid him take one to it, to make it even, and for that one, keep one in your minde. Now after he hath taken one to it, to make it even,

even, bid him give away half, and keep the other half for himself, which when he hath done, bid him triple that half, and again, after he hath tripled it, ask him whether it be even or odde: if he say odde, then bid him take one to make it even again, and for that last one keep two in your minde: now after he hath made his number even bid him cast away the one half, and keep the other still, from which half that he keepeth, cause him subtilly to put away or give you nine out of his number, as oft as he can, and for each 9 that he giveth you, keep in minde, and thereunto joyn the 3 which I bad you keep, and you shall have your desire.

Example.

Imagine he thought 7, the triple whereof is 21, and because it is odde, he is to take 1 to make it even, which first 1 given, is for you to keep in minde. Then the half of his 22 being cast away, he reserveth still 11, which after you have bid him triple, it maketh 33: then in giving of him one again to make it even, upon that last 1 reserve 2 in your minde, then his half of 34 maketh 17; from whence he can give you 9 but once. Therefore that yeelding to you 4, and the 3 that you keep, maketh 7 your desire.

Another kinde of Divination, to tell your friend how many pence of single pieces, reckoning them one with another, he hath in his purse, or should think in his minde.

Which to doe, first bid him double the pieces he hath in his purse, or the number he thinketh (if he participate his number or secrecy unto some one friend that sitteth by him that can but multiply, and adde never so

ſo little, if their number be great, then ſhall they work as you bid them ſo much the ſurer.)

Now after he hath doubled his number, bid him adde thereunto 5 more; which done, bid him multiply that his number by 5 alſo; which done, bid him tell you the juſt ſumme of his laſt multiplication: which ſumme the giver thinketh is nothing available, becauſe it is ſo great above his pretended imagination; yet thereby ſhall you preſently, with the help of Subtraction, tell his propoſed number.

The Rule is this,

Imagine he thought 17, *double* 17, *and it maketh* 34, *whereunto if you adde* 5, *it maketh* 39, *which multiplyed by* 5, *as here is practiſed, it yieldeth* 195, *which* 195 *is the ſumme delivered you in the work: then for a generall Rule, you ſhall evermore cut off the laſt figure toward your right hand, with a daſh of your pen, as here is performed, as a figure nothing available unto your work, and then rebate* 2 *from your firſt figure, after* 5 *is cut off, and the reſt ſhall evermore be your deſire, as by this example doth appeare.*

17
2
34
5
39
5
195
2
17

Another of a Ring.

If in any company you are diſpoſed to make them merry by manner of divining, in delivering a Ring unto any one of them, which after you have delivered it unto them, that you will abſent your ſelfe from them, and they to deviſe after you are gone, which of them ſhall have the keeping thereof, and that you at your returne will tell them what

Ii perſon

perſon hath it, upon what hand, upon what finger, and what joynt: To doe this, cauſe the perſons to ſit down all in a row, and to keep likewiſe an order of their fingers. Now, after ye are gone out from them to ſome other place, ſay unto one of the lookers on, that he double the numbers of him that hath the Ring, and unto the double bid him adde 5. and then cauſe him to multiply the Addition by 5. and unto the product bid him adde the number of the finger of the perſon that hath the Ring. And laſtly, to end the work, beyond that number, towards his right hand, let him ſet down a figure ſignifying upon which of the joynts he hath the Ring; as if it be upon the ſecond joynt, let him put down 2. Then demand of him what number he keepeth, from the which you ſhall abate 250. and you ſhall have three figures remaining at the leaſt. The firſt towards your left hand ſhall ſignifie the number of the perſon which hath the Ring, the ſecond or middle number ſhall declare the number of the finger, and the laſt figure towards your right hand ſhall betoken the number of the joynt.

Example.

Imagine the ſeventh perſon is determined to keep the Ring upon the fifth finger, and the third joynt: firſt, double 7, *it maketh* 14. *thereto adde* 5, *it maketh* 19. *which multiplyed by* 5, *yieldeth* 95. *unto which* 95, *adde the number of the finger, and it maketh* 100. *and beyond* 100, *towards the right hand, I ſet down* 3, *the number of the joynt, all maketh* 1003. *which is the number that is to be delivered you, from which abating* 250. *there reſteth* 753. *which prefigureth unto you the ſeventh perſon, the fifth finger, and the third joynt.*

But

But note, that when you have made your ſubtraction, if there doe remain 0, *in the place of tens, that is to ſay, in the ſecond place, you muſt then abate* 1, *from the figure which is in the place of the hundreds, that is to wit, from the figure which is next your left hand, and that ſhall be worth* 10 *tenths, ſignifying the tenth finger; as if there ſhould remain* 803, *you muſt ſay, that the ſeventh perſon upon his tenth finger, and upon his third joynt, hath the Ring.*

Another of three Dice.

If a man doe caſt three Dice, you may know the points of one of every of them: For if you cauſe him to double the points of one Die, and to the double to adde 5, and the ſame ſumme to multiply by 5, and unto the product adde the points of one of the other Dice, and behind the number towards the right hand, to put the figure which ſignifieth the points of the laſt Die, and then ask what number he keepeth, from which abate 250, and there will remain three figures, which doe note unto you the points of every Die.

Another of things hidden.

If three divers things are to be hidden of three divers perſons, and you to divine which of the three perſons hath the three divers things, doe thus: imagine the three things to be repreſented, *A*, *B*, *C*. Then ſecondly, keep well in your minde which of the perſons you mean to be the firſt, ſecond, and third. Then take 24 Counters or Stones, and your three things, and give *A* to the party whom you imagine to be your firſt man, and therewithall give him one of your 24 Counters in his hand, & *B* unto your ſecond man,

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 and

and therewithall 2 Counters, and *C* unto your third man, and therewithall 3 Counters, and leave the reſt, which are 18, ſtill among them: which done, ſeparate your ſelfe from them, and afterwards bid them change the things among them as they ſhall thinke good: which done, after they are agreed, bid him that hath ſuch a thing, as before you have repreſented by *A*, for every Counter that he hath in his hand, to take up as many more, and for him that hath *B*, for every one in his hand to take up 2. and for him that hath *C*, for every one in his hand to take up 4, and the reſt of them to leave ſtill upon the board. Theſe 3 things, and the three perſons being fully printed in your minde, come to the Table, and you ſhall evermore finde out of theſe 6 numbers, 1, 2, 3, 5, 6, or 7. If therefore one remain ſtill upon the board, then have they made no exchange, but keep them ſtill as they were delivered unto them. So that the firſt man hath *A*, the ſecond *B*, and the third man *C*. But if 2 remain, then the firſt man hath *B*, your ſecond man *A*, and your third man *C*. The reſt of the work and the order thereof are here apparent by the Table following.

	1	*A*		1	*B*
1	2	*B*	5	2	*C*
	3	*C*		3	*A*
	1	*B*		1	*C*
2	2	*A*	6	2	*A*
	3	*C*		3	*B*
	1	*A*		1	*C*
3	2	*C*	7	2	*B*
	3	*B*		3	*A*

Another

Another divination of a number upon the casting of two Dice.

First, let the Caster cast both the Dice, and mark well the number: then let him take up one of them, it maketh no matter which, and look what number it hath in the bottom, and adde all together: then cast the Die again, and keep in his minde what all together maketh: then let the Dice stand, and bring seven with you, and thereunto adde the rest of the pits that you see upon the upper side of the Dice, and so many did the Caster cast in all.

F I N I S.

An Appendix concerning the Resolution of the *Square* and *Cube* in Numbers, to the finding of their sides: By *Ro. Hartwell.*

A Figurate Number is a number made by the multiplication of one number or more by another. A figurate number what.

The sides of a figurative number are the numbers by whose multiplication it is made. The sides of a figurate number what.

A Figurate number is two-fold, a { *Plain.* / *Solid.* }

And it is { *Of one Multiplication.* / *Or consequently of many.* } *as a* { *Plain.* / *Solid.* }

And in each { *Both Æquilater.* / *And Inæquilater.* }

A figurate number made of one multiplication by two sides or numbers multiplyed together, is called a plain figurate number.

A plain figurate number.

For every number made by the mutuall multiplication of two numbers, may be called a *Plain*, because it bringeth forth a right-angled parallelogramme, according to his unites dispoſed in *length* and *breadth*, the *sides* whereof are the two multiplying numbers. As the number 20, made by the mutuall multiplication of 4 and 5, is called a *Plain*, and the *sides* thereof are 4 and 5, as here

* * * * *
* * * * *
* * * * *
* * * * *

Becauſe the unites thereof diſpoſed in *length* and *breadth*, as the *sides* doe expreſſe, doe bring forth an *inequaliter Parallelogramme*, for that the numbers or *sides* are inequall.

By like reaſon 36 made by multiplication of 6 by 6, is called an *Æqualiter plain*, for the *sides* thereof 6 and 6 are equall.

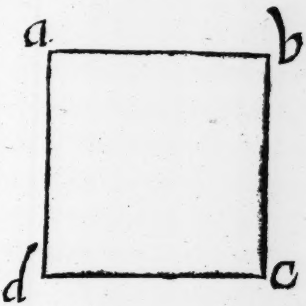
Moreover, one and the ſame *plain number* may have many *sides*, as the *plain number* 24, hath *sides* 4 and 6, 3 and 8, 2 and 12. For it is produced from the mutuall multiplication of theſe numbers. Whereupon for the invention of the ſides to wit, in *inæqualiter Plains*, it is needfull to give one of the *sides*, by which the *plain* it ſelfe divided, the other *side* is made known. As the *plain* 48 being divided by the *side* 8, the *quotient* 6 is the remaining *side*. Notwithſtanding another reſolution and inquiſition doth happen in the *sides* of the *Æquilater plains*.

An Equilater Plain or quadrat what.

An Æquilater plain is a number made by two equall sides, or by any number multiplyed by it ſelfe. It is vulgarly called a ſquare or quadrat; by the Arabians

Arabians Zenſus : *it is commonly expreſſed by this note* z, *by us* q.

A quadrat or *ſquare* in *Geometry* is called a right-lined *plain figure* , made by four equall right lines , and ſo many right Angles ; and every one of the lines is called the *ſide* of the *quadrat*, as this *figure a b c d*, whoſe *ſide* is *a b* , or *b c* , as alſo *c d*, and *a d*.



To the ſimilitude hereof, that number is called a *Quadrate* which is made by the multiplication of two equall numbers, or of one in it ſelf: of which manner 36 is made, by 6 multiplyed in it ſelfe,or by the mutual multiplication of 6 and 6. For if 36 unites be placed in plain form , it bringeth forth a perfect *Geometricall Quadrat* , having in every ſide ſix unites, as here.

* * * * * *
* * * * * *
* * * * * *
* * * * * *
* * * * * *
* * * * * *

The number whereof the Quadrat is produced by multiplication in it ſelfe , is called the ſide or root of the Quadrat.

The ſide or root of a number what.

Concerning the extraction of the Quadrat or ſquare Root.

THerefore to finde the *Quadrat Root* , or the *ſide* of any *Quadrat number*, is to ſearch a number, which brought or multiplyed in it ſelf, maketh the number propounded : concerning the finding whereof, as it is requiſite that the *ſides* (being leſſer then 10) of

the *squares* under an hundred should be gathered by the Table of Multiplication; so the *sides* of the *greater squares* are to be sought out by *Art*. First, the *squares* whose *sides* are simple numbers, are here set down as you see.

The roots.	1	2	3	4	5	6	7	8	9
The squares.	1	4	9	16	25	36	49	64	81

The knowledge of a square is by finding out his side expressed by a whole number.

Although the finding out of the *side* of a *square* be applyed to each number given, as to a *square*, yet *square numbers* onely have a *side* to be expressed by a certain number of unites, or by rationall numbers; the other are to be expressed but onely in power. The *sides* are commonly called *Roots* by a *Metaphoricall phrase*.

The Root or side of a square is to be found by the Theorem following.

If the odde degrees of a *square number* being marked from the right toward the left hand with points, you subduct from the number given the particular *square* of the last *period*, setting the *side* thereof alone by it selfe,

Then going on, if you divide the remainder (if there be any) with the figure going before it, by the double of the side set alone by it selfe,

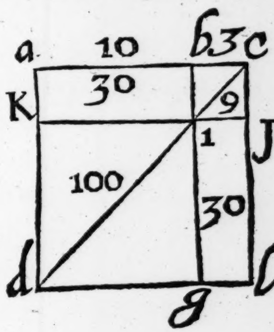
And multiply the *quotient* found out (being placed by the side, which was first set alone by it selfe, and also before the doubled number on the right hand) by both the numbers (namely, by the double number, and the Figure set by it selfe) being counted as one *Divisor*, subducting the products from the given number, and then renew this last work of division so many

times

times as there are pricks remaining, the *ſide* of the *ſquare* ſhall be found out.

This artificiall device is taken out of the 4 *Pro.* 2.l.*of* Euclid. *Where by demonſtration it is proved, that if a right line be cut into two* ſegments, *howſoever the* ſquare *of the whole line is equall to the* ſquares *of the* ſegments, *and the two* right-angled figures *made of the* ſegments *as in* the figure *annexed, the two* Diagonals, *k g, and b f, are the* ſquares *of the* ſegments, *a b, and b c. Alſo the* complements *b k, and f g, are the* right-angled figures *made by multiplying the line a b, by b c.*

To extract the ſquare root.



The ſelfe-ſame parts are to be found in any *ſquare number.* As for *example*, let the *number* be 169, whoſe *ſide* is 13. This *ſide* being divided into two pieces, 10 and 3, multiplying each pieces by it ſelfe once, namely, 10 by 10, and 3 by 3. then multiply one by another, as 10 by 3, and 3 by 10, ſo ſhall you have 4 *plain numbers*, whereof 2 are *ſquare*, as here you ſee.

The firſt example.

Therefore as the *ſquare* 169 is made by adding together of theſe 4 *plaine numbers*, ſo by ſubducting them ſeverally it is reſolved.

Firſt therefore, I marke each odde place with points, becauſe the *particular ſquares* are to be found in the odde places. Then for ſo much as the unity ſtanding under the firſt point next the left hand, and repreſenting the laſt *period*, is both a *ſquare* and the *ſide* of a *ſquare*:

10	3
10	3
100	
30	
30	
9	
169	

ſquare : that *figure* therefore being ſet alone in the *quotient*, and being ſubducted from the unity ſtanding over the point, there remaineth nothing.

This unity ſet alone by it ſelf in the *quotient*, ſhall ſignifie 10 when another *figure* is ſet by it, repreſenting the ſide of ſome other particular *ſquare*. Whereupon I ſay, that the greater *Diagonall k g*, is now ſubducted from the whole *ſquare*, and the ſide of it *k i*, or *a b*, (for they are equall one to another) and alſo the *ſide* of the *complement* is found out.

This is the firſt ſtep to this Reſolution.

Moreover, I double the *figure* found out, becauſe being doubled, it is the ſide of both the *complements* taken joyntly together, namely, *k i*, and *g i*. Then ſetting 2 the doubled number under 6, I divide 6 (which in this place is as much as 60, and repreſenteth both the *complements*) by 2, the *quotient* is 3, repreſenting the other *ſide* remaining of the *complement*, namely, *i f* for *b c*, which number I ſet in the *quotient*, and count it for the *ſegment remaining* of the right line given. Wherefore becauſe this number 3 is the ſide of the remaining *Diagonall*, that is to ſay, of the leſſer *ſquare b f*, therefore being ſet by the *diviſor* on the right hand, and multiplyed by it ſelf and alſo by the *diviſor*, it bringeth forth three *plain numbers*, namely, the *ſquare b f*, and the two *complements*, *a i*, and *i l*, which being ſubducted from the numbers ſtanding over them, there remaineth nothing.

The

The example is thus.

169(12 *Which is all one,* 169

~~122~~ *as if you had put down the numbers found out in this manner.*

~~2~~

69

~~100~~ *The greater Diagonall.*

~~60~~ *The complements twofold.*

~~9~~ *The lesser Diagonall.*

169

The subtilty of this invention is illustrated by many examples.

Let the *square* given be 1764. This number being marked with two points, telleth us, that the side thereof is to be written with two Figures. The second example.

First therefore, beginning at the point on the left hand, I seek the side of the last period, namely, 17. But for so much as it is no square number, I take 4 the side of the next lesser square, which I set alone by it self in the quotient, and then multiply it by it self; the *Product* is 16. which being subducted from 17. there resteth 1. Moreover, I double the side found out, the product is 8. I place this doubled number under 6, and by it divide 16 standing above it, the *quotient* is 2, which must be set by 4. This *quotient* 2, must be set before the *Divisor* 8, on the right hand under the point, and then it must be multiplyed both by it self, and into 8, the product is 164. which being subducted from the figures standing over them, there remaineth nothing: whereby I gather, that the number given is a just *square*.

The

The Example ſtandeth thus.

$\cancel{1}$

$\cancel{1}\cancel{7}\cancel{6}\cancel{4}$ (42

$168\cancel{2}$

$\cancel{2}$

164

1764 *The Collection.*

The ſame manner of working is to be followed in greater ſquare numbers given, ſaving that the former part of the worke is to be uſed but once, but the latter part is to be followed ſo many times as there are points remaining excepting the laſt.

The third example. As in 5 47 56, I ſay, that the *ſide* of the *ſquare* next unto 5 is 2. therefore 2 being ſet in the *quotient*, and multiplyed by it ſelf, makes 4, and taken from 5, the remainder is 1. Moreover, I double the *quotient*, the product is 4, which I ſet under the next *figure* toward the right hand, and thereby divide 14, the *quotient* is 3, which three being ſet both in the *quotient*, and alſo before the *Diviſor* towards the right hand, I multiply both the numbers by it, the *product* is 129: this being ſubducted from 147 ſtanding above it, the remainder is 18. But becauſe there is yet one point remaining, with which I have not medled, I therefore againe double all the whole *quotient*, for in this caſe I muſt take 23 for the ſide of one former *ſquare*, Note. and generally in great numbers, when I light upon more particular *ſquares* then two, I muſt eſteem them but as two, and take the ſides which are firſt found out, but as the *ſides* of one onely *ſquare*. Therefore twice 23 is 46: by this I divide 185, the number to be ſet in the *quotient* is foure, which

number

number alſo muſt be ſet before the *Diviſor* on the right hand: then muſt 464 be multiplyed by 4: the *product* is 1856, this *product* being ſubtracted from the numbers ſtanding over it, there remaineth nothing. The example ſtandeth thus.

~~1~~ ~~1~~ 8
~~5~~ ~~4~~ ~~7~~ ~~5~~ ~~6~~ (234
~~4~~ ~~4~~ ~~3~~ ~~6~~ ~~4~~
129

4
1856
54756 *The Collection.*

See alſo the Example following.

10942864 (3308.

Therefore out of this invention is this *conſectarie.*

The number whoſe ſide cannot be expreſſed by whole numbers, is not a ſquare number. 4 Example of a ſurd number.

Such are all *prime numbers*, and (the *ſquares* themſelves excepted) all other *compound numbers.* For if in them you deſire to finde out the *ſquare ſide*, you ſhall labour in vain, becauſe they are not *ſquares*, for to the whole numbers ariſing in the *quotient*, there will be ſome *fraction* adjoyned, whereby it cometh to paſſe, that the number of the *ſide* is not to be expreſſed by a true number, and it is commonly called a *ſurd number.*

Notwithſtanding, if you adjoyn to the *ſide* found out, the number remaining, taking his *denomination* from the double of the *ſide augmented* by an unity, you

you ſhall finde the next ſide that may be like to the *ſide of a ſquare.*

As if from 40 you take the neareſt *ſquare*, to wit, 36, the remainder is 4. here therefore the ſide ſought for of the *ſquare* exceedeth not the *ſide* found out by an unity, but either by one, or more parts of ſome whole number: wherefore I double 6, the ſide found out, and adde an unity to it being doubled, the totall is 13. this number I ſet under 4 the remainder, and ſay that the ſide of 40 demanded as near as may be is $6\frac{4}{13}$: the *Denominator* of the *Fraction* being added to the greateſt *ſquare* in the number given, namely unto 36, maketh the next greateſt *ſquare* above it, namely 49, whoſe ſide is 7. But this ſurd ſide, to wit, $6\frac{4}{13}$, multiplyed by it ſelf, maketh $39\frac{133}{169}$, which are not juſt equall to 40, the given number.

Judge the like concerning the reſt which are not *ſquares.*

Thus much concerning plain figurate numbers, but eſpecially ſuch as are ſquare numbers.

Concerning ſolid figurate Numbers.

A *Solid figurate Number is made of two multiplications by three numbers or ſides multiplyed together, admitting length, breadth, and thickneſſe.*

A ſolid figurate number.

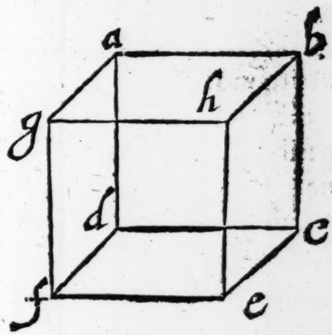
Therefore every *number* made by the mutuall multiplication of three numbers, may be called a *ſolid*, becauſe it bringeth for *a right angled Parallelipipedon*, diſpoſed according to his unities in *length, breadth and thickneſſe*, the *ſides* whereof are the three multiplying *numbers.* As the *number* 30 made by the mutuall

tuall multiplication of 2, 3, and 5, is called an *Inequilater solid number*, and the sides thereof are 2, 3, and 5; because the unities thereof disposed by a certain distance one from another, in *length*, *breadth*, and *depth*, as the sides doe expresse, doe bring forth in resemblance an *Inequilater Parallelipipedon*, for that the *numbers* or sides are inequall.

By like reason 216 made by multiplication of 6 by 6, and the product thereof by 6, is called an *Equaliter solid*, for the sides thereof 6, 6, and 6, are equall. An Equilater Solid or Cube.

An Equilater is a number made by three equall sides, or by any number multiplyed by it self, and that product againe by the foresaid number. And it is called an Equilater and Equiangled Parallelipipedon or Cube, and is commonly represented by us thus, C.

A Cube in Geometry is a right angled Parallelipipedon, having six equall *surfaces*, and 8 *solid angles*, and 12 sides, as this figure *a. b. c. d. e. f. g. h.* whose side is *a b*, or *a d*, also *b c*, or *c d*, either *c e*, or *e f*, likewise *c h*, or *h g*, also *g f*, or *d f*, or *d a*, and *g a*.



The number whereof the Cube is produced by Multiplication in it selfe twice, is called the side or root of the Cube; which being found out in whole numbers, the Cube is known. The side or root of Cube.

Con-

Concerning the extraction of the Cubick Root.

THerefore every *Cube* in numbers hath such a side as may be expressed in whole numbers, but in magnitudes it is not alwayes so, as indeed in magnitudes there are many things not to be expressed in *whole number*. Now for as much as the side of any *Cube* under 1000, is a simple figure, it is necessary, before we undertake to finde out the side of any great number, to know what *Cube* is made of each simple figure, and what is the side of any *Cube* lesser then 1000, as I have here set them down.

Roots.	1	2	3	4	5	6	7	8	9
Squares.	1	4	9	16	25	36	49	64	81
Cubes.	1	8	27	64	125	216	342	512	729

But in searching out the *sides* of greater *Cubes*, we are to proceed as the *Theorem* following teacheth us.

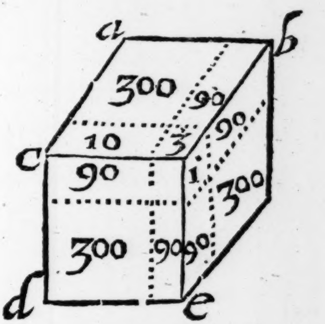
If you distinguish with *points* as it were into periods, the given *Cube*, beginning at the first figure on the right hand, and omitting each two figures continually, and first of all subduct the *particular Cube* of the last *period* from the given number, setting the *side* thereof in the *quotient*, and then set triple of the *quotient* under the *figure* next following the former point, on the right hand, and the *square* of the *quotient* being tripled beneath it one *degree* more towards the left hand, and afterward divide the number above written by the triple of the *square*, setting the *quotient* by it selfe, and then multiply the *Divisor* by the *quotient* found out, and the triple *square* by the *square*

ſquare of the *quotient*, and the *quotient cubically*, ſubducting the *products* (ſo orderly added together, that each *figure* may anſwer the *numbers* whereof it was multiplyed) from the *number* given, and renew this laſt manner of diviſion ſo many times as there are *points* remaining, the *ſide* of the *Cube* ſhall be found out.

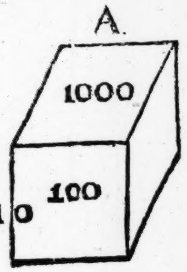
This artificiall device is drawn out of that *theorem* which *Ramus* made, imitating that of *Euclide* concerning *ſquare numbers*, in this manner. ;

If a right line be cut into two ſegments, the Cube of the whole line ſhall be equall to the Cubes of the ſegments, and the two ſolid figures comprehended three times under the ſquare of his ſegment, and the ſegment remaining. The extraction of the Cubick ſide or root.

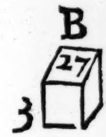
As the line *c i*, which is 13, is cut into two *ſegments*, 10 and 3, therefore the *Cube* of the whole line, namely 2197, is equall unto the *Cubes* of the *Segments*, namely unto 100, and 27 alſo to *the two-fold Solids* or *Parallelipipedons thrice* taken, whereof three have like *ſolidity*, the *ſolidity* of each of the three



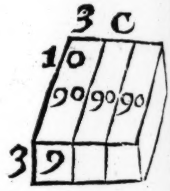
leſſer is 90, being made of the *Square of the Segment* 3, that is to ſay of 9, multiplied by the other *Segment* 10. Theſe three *Parallelipipedons* joyntly taken together, make 270. But of the three greater *Parallelipipedons* each containing 300, being made of 100, the *Square* of the greater *Segment* 10, multiplyed by the *leſſer Segment* 3, & they being taken joyntly together, make 900.



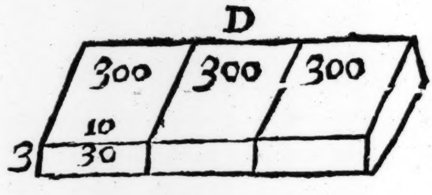
The Cube of the greater segment 10.



The Cube of the lesser segment 3.



The three lesser Parallelipipedons.



The three greater Parallelipipedons.

The *Cube* therefore hath eight *particular* 10 3
solids in number, which are made of the 10 3
parts of the number given, namely of 10
and 3 in this manner: First, let there be 9
foure *plaine numbers* made, each part be- 30
ing multiplyed by it selfe, and one by 30
another. 100

If

If againe I multiply the *Plaines* by the same parts, there will arise 8 *solids*, as you see here,

9	9	
30	30	
30	30	
100	100	
3	10	
27	90	*All these being added*
90	300	*together, are equall to*
90	300	*the* Cube *of the whole, to*
300	1000	*wit,* 2197.

Therefore the same way that is kept in making the Cube, *is also to be followed in resolving the* Cube.

As for example, I marke the *Cube* given with points in this manner, 2197. The first example to extract the Cubick root.

Then I subduct the particular *Cube* of the number set under the last point but for so much as that number is no *Cube*, I take the nearest to it, namely, an unity, which also I set in the *quotient*. This unity in the number given is 100, but in the *quotient* it is but 10, the unity subducted from 2, the remainder is 1, which must be written over the number given. So that the greater *Cube A*, is to be supposed to be subducted from the number given.

This is the first step of this worke,

1
2197 (1
~~2~~

After I triple the *quotient* found out, (that is to say, I multiply it by 3) this triple representeth the three *sides* (joyntly taken together) of the three lesser *solids* marked with *C*, I place the tripled number under 9. Againe, I multiply the *quotient* square-wise, and triple the *product*, which

maketh likewise 3. This *product* resembleth the three *square sides* (taken joyntly together) of the three *greater solids*, marked with *D*. I place the *product* on a degree lower towards the left hand, underneath 1. With it I divide 11, which written above it, the quotient is 3. This *segment* or *quotient* 3, being multiplyed by 3, the *Divisor* maketh 9, which in respect of the place wherein it standeth, is 900, and representeth the three greater *solids* marked with *D*, taken joyntly together. Furthermore, the same *quotient* being multiplyed square-wise, maketh 9, and multiplyed afterward by the *triple number* standing under 9, it maketh 27, which in respect of the place wherein it standeth, is 270, and representeth the 3 *lesser solids* marked with *C*. Last of all, the same *quotient* multiplyed *cubically*, breedeth the lesser *Cube B*. These 3 *products* therefore being added together, and the totall subducted from the *numbers* standing over it, there remaineth nothing, which importeth the given number is a *Cube*.

The example is as you see.

1	2197 (13
2197 (13	1000 *The greater Cube.*
1 3	3
3	3
9 Or thus:	900 *The 3 greater Parallelipipedons.*
27	270 *The lesser Parallelipipedons.*
27	27 *The lesser Cube.*
2197	2197

The second example of the Cubick.

The matter may be explained by many examples.

Let the side of the given *Cube* 16387064, be sought out, contrive it therefore (as it were) into certain periods

periods with points. Then firſt of all, ſearch out the *ſide* of the *Cube* next to the left hand. But foraſmuch as 16 is no *Cube*, take 2 the *ſide* of the next *Cube* under it, that is to ſay, of 8, and ſet in the *quotient*, and ſubduct 8 the *Cube* thereof from 16, there remaineth 8. The firſt work is not to be renewed throughout the whole *number*, but the *rules* following muſt be repeated as often as there are *points* remaining.

The firſt ſtep to finde out the root, is in this manner,

8
16387064 (2
8

Moreover, triple the *quotient* now found out, and the *product* is 6, which is to be placed under 8, namely, under the figure following the next prick towards the right hand. Then multiply the *quotient* by this tripled *number* (or which is all to one purpoſe, ſquare the *quotient*, and then triple the *product*) it maketh 12, ſet that number in a lower place one degree nearer the left hand, and make it the *Diviſor*: divide 83 by 12, obſerving this rule in chuſing your *quotient*, that it be no greater then that the numbers afterward produced by multiplication may not exceed the numbers ſtanding over it. So that here you ſhall take 1 in 8, but 5 times. Afterward by this number 5, multiply the *Diviſor* 12, and by the ſquare of 5, multiply the tripled number 6, and laſt of all multiply 5 cubically: ſo ſhall you produce three numbers, namely, 60, 150, 125, to be deſcribed in ſuch ſort as you ſee. Theſe numbers added together, and ſubducted from 8387, the remainder is 762.

The second step to find out the root, in this manner.

```
   8762
 16387064 (25
    6
   12
 --------
   60
  150
   125
 --------
  7625
```

And because there is yet one point remaining, this last manner of Division must be wrought againe.

First, therefore I triple the *quotient*, the product is 75. which must be so placed, that the first figure thereof, namely 5, may stand under 6, the second under the 0. Again, multiply the *quotient* by this *tripled number* (or which is all one, square the quotient, and triple the product) it maketh 1875, which must be the *Divisor*, whose first figure, namely 5, must be placed under 7, the *last figure* of the *tripled number*. Then see that 1 may be contained in 7, many times, but I can take it but 4 times, I set 4 in the quotient, and multiply the *Divisor* by 4, the product is 7500. afterward I square 4, it maketh 16, which I multiply by the tripled number 75, the product is 1200. Last of all, I multiply 4 *cubically*, it maketh 64, these products added all together, make 762064, which number being subducted from the *Cube* given, there remaineth nothing, whereby I gather that the number given is exactly *cubicall*.

The

The third ſtep to finde out the ſide is in this manner.

762
~~163~~87064 (254

75
1875

7500
1200
64

762064

Behold alſo the Example following. The third example of the Cubick root.

614125000 (850

Another manner of working.

Hitherto the Princely high-way to finde out the ſide of the Cube *hath been declared.*

But there are moreover certaine other wayes alſo bending thereto, and leaning to the ſame principles, whereof this is one.

Having found out in the Table of ſimple *Cubes*, the firſt figure repreſenting the ſide of the *Cube* contained in the number ſtanding under the firſt *point* on the left hand, ſet it in the *quotient*, and ſubduct the particular *Cube* of that figure as you did before: then *ſquare* that *figure*, and triple that *ſquare*, the *product* ſhall be the *Diviſor*, the firſt *figure* whereof ſhall be ſet under that *figure* which is on the right hand next of all to the point (now examined) before going. The ſecond form.

See how many times the *Diviſor* is contained in the number written over it, and multiply the *Diviſor* in the *quotient*, and ſubduct the *product* from the *dividend*: yet here you muſt take heed that you take not a greater *quotient* then that the *products* made after-

afterward thereby may be ſubducted from the *number* given.

The ſubduction being done, triple the firſt *figure* which was ſet in the *quotient*, and adde to the triple the laſt *number* which was ſet in the *quotient* on the right hand of the *product*.

This totall multiplyed by the ſquare of the *figure* laſt found out, ſet downe the *product* ſo, that the firſt *figure* thereof toward the right hand may ſtand under the *point* next before-going on the ſame hand, and finally, ſubduct the ſame from the *number* given.

The fourth example of the Cubick root.

As in 804357. the particular *Cube*, namely 729, being taken from the number ſtanding under the laſt *period* upon the left hand, there remaineth 75357, the ſide of that particular *Cube* being 9, I ſet in the *quotient*. Then I *ſquare* that *ſide*, it maketh 81, and triple the *ſquare*, the *product* 243 is my *Diviſor*, which I ſet under the given number, ſo that 3 may ſtand under 3 with this *Diviſor*; divide the number ſtanding over it, you ſhall find 2 to be contained in 7 three times. Therefore I ſet 3 in the quotient, and multiply the *Diviſor* by it, the *product* is 729, which being ſubducted from 753, the remainder is 24.

The Induction is thus:

2
~~7~~ ~~5~~ ~~4~~
1 ~~0~~ ~~4~~ ~~3~~ 5 7 (93
2 4 3
~~7~~ ~~2~~ ~~9~~

Moreover I triple 9, the product is 27, by which on the right hand I ſet 3 the quotient laſt found out, the totall is 273.

This

This *Number* I multiply by 9 the ſquare of the quotient laſt found out, the product ſhall be 2457, which being ſubducted from the ſuperiour number, there remaineth nothing.

The Induction is thus:

```
  24
804357 (93
  732
    9
-------
 2457
```

Another manner.

THE ſelfe-ſame worke may be diſpatched another way, a little differing from the former, in this manner;

The third forme.

The figure in the *quotient* being found out by ſubducting the particular *Cube*, and alſo the ſecond figure in the quotient being found by diviſion, let the totall quotient be tripled, and let the tripled number be multiplyed by the former figure in the quotient. Then let the product be multiplyed againe by the latter figure found out, and let a *cipher* be ſet on the right hand of that *product*. Laſt of all, let the *Cube* of the latter figure found out be added to this *product*, and let the totall ſumme be ſubducted from the number given. As in 373248.

The

The firſt induction is in this manner.

30
~~373~~248 (7
343

The fifth example.

Moreover, I *ſquare* the ſide found out, it maketh 49, and triple the ſquare, the *product* is 147, which ſhall be the *diviſor*, by this I divide 302, the number written over it, the *quotient* is 2. Now I triple the *totall quotient* 72, it maketh 216, and multiply this triple by 7, the former *figure* in the *quotient*, the product is 1512. I multiply this *product* alſo by 2, the latter *figure* of the *quotient*, and ſet a *cipher* on the right hand of it, ſo as it maketh 30240, unto this number laſt of all I adde 8, the *Cube* of the latter *figure* found out, the totall is 30248, which being ſubducted from this *figure* above it, there remaineth nothing.

The ſecond induction is thus.

30
~~373~~248 (72
147
30248

All the *points* of the *number* given being examined, if any thing remaine, it ſignifieth the *number* given is no *Cube*: wherefore the true *ſide* of it cannot be exactly given in *numbers*. Yet if it pleaſe you to ſiſt out the neareſt *ſide* that may be, by the firſt kinde of reduction of *mixt numbers*, you ſhall reduce the

number

number given unto a *cubicall* fraction of a greater *denomination*, and afterward ſeeke out the *cubicall ſide* of that *fraction*.

For example ſake, becauſe 120 is no *Cube*, therefore let it be reduced into ſixty *cubicall* parts, after this manner. Multiply 60 *cubically* in it ſelfe, it maketh 216000, by this being taken for the *Denominator* of the *fraction*, multiply 120 the *number* given, the *product* is 25920000, whoſe *cubicall ſide* is $\frac{295}{60}$ that is $4\frac{11}{12}$ the neareſt to the true *ſide* that can be.

To finde the neareſt Cubick root in a ſurd number.

For the extraction of all ſorts of roots, the Tables of Logarithmes ſet forth by Mr. Briggs *are moſt excellent and ready.*

FINIS.

A Table of Board and Timber-measure more perfect then ever hath been made, shewing also the Squares between 4 and 37 from quarter to quarter, calculated by Robert Hartwell.

Board measure.	Inches & quarters.	Squares.	Timber-measure.	Board-measure.	Inches & quarters,	Squares.	Timber-measure.
36.0.0	4	16	108.0.0	16.0.0	9	81	21.3.3
33.8.8	1	18	96.0.0	15.5.7	1	85	20.3.3
32.0.0	2	20	86.4.0	15.1.6	2	90	19.2.0
30.3.1	3	22	78.5.4	14.7.7	3	95	18.1.9
28.8.0	5	25	69.1.2	14 4.0	10	100	17.2.8
27.4.3	1	27	64.0.0	14.0.2	1	105	16.4.6
26.1.8	2	30	57.6.0	13.7.1	2	110	15.7.1
25.0.4	3	33	52.3.6	13.3.9	3	115	15.0.3
24.0.0	6	36	48.0.0	13.0.9	11	121	14.2.8
23.0.4	1	39	44.3.0	12.8.0	1	126	13.7.1
22.1.5	2	42	41.1.4	12.5.2	2	132	13.0.9
21.3.3	3	45	38.4.0	12.2.5	3	138	12.5.2
20.5.7	7	49	35.2.6	12.0.0	12	144	12.0.0
19 8.6	1	52	33.2.3	11.7.5	1	150	11.5.2
19.2.0	2	56	30.8.6	11.5.2	2	156	11.0.8
18 5.8	3	60	28.8.0	11.2.9	3	162	10.6.7
18.0.0	8	64	27.0.0	11.0.7	13	169	10.2.2
17.4.5	1	68	25.4.1	10.8.7	1	175	9.8.7
16.9.4	2	72	24.0.0	10.6.7	2	182	9.4.9
16.4.6	3	76	22.7.4	10.4.7	3	189	9.1.4

Board-

Board-measure	Inches & quarters.	Squares.	Timber measure	Board-measure.	Inches & quarters.	Squares.	Timber measure
10.2.8	14	169	8.8.1	6.8.6	21	441	3.9.2
10.1.0	1	203	8.5.1	6.7.7	1	451	3.8.3
9.9.3	2	210	8.2.3	6.6.9	2	462	3.7.4
9.7.6	3	217	7.9.6	6.6.2	3	473	3.6.5
9.6.0	15	225	7.6.8	6.5.4	22	484	3.5.7
9.4.4	1	232	7.4.4	6.4.7	1	495	3.4.9
9.2.9	2	240	7.2.0	6.4.0	2	506	3.4.1
9.1.4	3	248	6.9.7	6.3.3	3	517	3.3.4
9.0.0	16	256	6.7.5	6.2.6	23	529	3.2.7
8.8.6	1	264	6.5.4	6.1.9	1	540	3.2.0
8.7.3	2	272	6.3.5	6.1.2	2	552	3.1.3
8.6.0	3	280	6.1.6	6.0.6	3	564	3.0.6
8.4.7	17	289	5.9.8	6.0.0	24	576	3.0.0
8.3.5	1	297	5.8.1	5.9.4	1	588	2.9.4
8.2.3	2	306	5.6.4	5.8.8	2	600	2.8.8
8.1.1	3	315	5.4.8	5.8.2	3	612	2.8.2
8.0.0	18	324	5.3.3	5.7.6	25	625	2.7.6
7.8.9	1	333	5.1.9	5.7.0	1	637	2.7.1
7.7.8	2	342	5.0.5	5.6.5	2	650	2.6.5
7.6.8	3	351	4.9.2	5.5.9	3	662	2.6.1
7.5.8	19	361	4.7.9	5.5.4	26	676	2.5.5
7.4.8	1	370	4.6.7	5.4.8	1	689	2.5.1
7.3.9	2	380	4.5.5	5.4.3	2	702	2.4.6
7.2.9	3	390	4.4.3	5.3.8	3	715	2.4.2
7.2.0	20	400	4.3.2	5.3.3	27	729	2.3.7
7.1.1	1	410	4.2.1	5.2.8	1	742	2.3.2
7.0.2	2	420	4.1.1	5.2.3	2	756	2.2.8
6.9.4	3	431	4.0.1	5.1.9	3	767	2.2.5

Board measure	Inches & quarters.	Squares.	Timber measure	Board measure	Inches & quarters.	Squares.	Timber measure
5.1.4	28	784	2.2.0	4.3.6	33	1089	1.5.9
5.0.9	1	798	2.1.6	4.3.3	1	1104	1.5.6
5.0.5	2	812	2.1.2	4.3.0	2	1122	1.5.4
5.0.0	3	826	2.0.9	4.2.7	3	1139	1.5.2
4.9.6	29	841	2.0.5	4.2.3	34	1156	1.4.9
4.9.2	1	855	2.0.2	4.2.0	1	1173	1.4.7
4.8.8	2	870	1.9.8	4.1.8	2	1190	1.4.5
4.8.4	3	885	1.9.5	4.1.4	3	1208	1.4.3
4.8.0	30	900	1.9.2	4.1.1	35	1225	1.4.1
4.7.6	1	915	1.8.9	4.0.8	1	1242	1.3.9
4.7.2	2	930	1.8.6	4.0.5	2	1260	1.3.7
4.6.8	3	945	1.8.3	4.0.3	3	1278	1.3.5
4.6.4	31	961	1.7.9	4.0.0	36	1296	1.3.3
4.6.1	1	976	1.7.7	3.9.8	1	1313	1.3.1
4.5.7	2	992	1.7.4	3.9.4	2	1331	1.2.9
4.5.3	3	1008	1.7.1	3.9.1	3	1350	1.2.8
4.5.0	32	1024	1.6.9	3.8.9	37	1369	1.2.6
4.4.6	1	1040	1.6.6	3.8.7	1	1388	1.2.4
4.4.3	2	1056	1.6.4	3.8.4	2	1406	1.2.2
4.4.0	3	1072	1.6.1	3.8.1	3	1425	1.2.1

The use of this former Table.

IF upon a *Scale or Ruler* you divide one *inch* into ten *equall parts or primes*, and again by *diagonals*, and *parallel-lines* you ſubdivide each of them into ten *equall parts or ſeconds* with your *compaſſes*, you may take a more exact running meaſure for *board and timber*, then by any other meanes whatſoever, and ſo

ſo place the ſame, or this *Table* if you will, upon any *Ruler.*

Alſo by means of the *columns of ſquares*, you may readily finde a *ſquare* equall to any *Parallelipipedon*, or piece of *timber*, which is thicker then it is broad. As for example, ſuppoſe a piece of *timber* to be ten *inches thicke*, and 9 *inches broad:* if I multiply thoſe *ſides* one by another, they will produce 290. then ſeeking the *column of ſquares* for 290, which I finde not, but I finde 289, the neareſt number to 290, to ſtand againſt 17: therefore I ſay 17 *inches ferè* will make a *ſquare* equall to ſuch an unlike *ſquared piece:* then looking in the *column of timber meaſure* againſt 17, you ſhall finde that 5 *inches*, 9 *primes*, or $\frac{9}{10}$ and 8 *ſeconds*, or $\frac{98}{100}$ of an *inch in length*, of that piece will make a *foot of timber.*

Likewiſe for *board meaſure*, you may finde how much in *length* or *breadth* of *board* muſt be in one *foot.*

By the like meanes, ſuppoſe for example that a *board*, appointed to be meaſured, is 15 *inches* $\frac{3}{4}$ *broad*, if I deſire to know how much in *length* thereof will make a foot; I ſeek in the *columns* that ſtand under *unites* and *quarters*, for 15$\frac{3}{4}$, and alſo againſt the ſame in the *columne* under the title of *board meaſure*, where I finde 9 *inches*, 1 *prime*, or *tenth* of an *inch*, and 4 *ſeconds*, or *hundreds* of an *inch* will make a *foot* at that *breadth:* The like may be practiſed for any other *breadth of board* whatſoever.

Cer-

Certain *Tables* ſhewing the Intereſt of any *ſumme of money* whatſoever unto 40 years; how much *Annuities* reſpited or forborne come unto. And for *buying* or *ſelling* of *Annuities* for the ſaid time; and alſo the ſame in *reverſion* after any *number of yeares* unto 30. What they may be worth in preſent ready money. By *R.C.* and now diligently corrected and amended by *Robert Hartwell.*

Definition of Intereſt.

PRincipall *is the ſumme from which the* Intereſt *is reckoned.*

2 Intereſt *is the ſumme reckoned for the lending or forbearance of the* Principall *for any terms or time.*

3 Intereſt ſimple *is that which is counted from the* Principall onely.

4 Intereſt compound *is that which is counted for the* Principall, *together with the* Arrerage.

5 Intereſt profitable *is that which is added to the* Principall.

6 Intereſt damageable *is that which is to be ſubtracted from the* Principall.

The uſe *per annum* of	1 li.	is	2 s.
	10 s.		12 d.
	5 s.		6 d.
	2 s. 6 d.		3 d.
	1 s.		1, d. $\frac{1}{5}$ of a peny.

A Table shewing what 1 li. *with interest, and interest upon interest after* 10 *in the* 100, *comes to every year under* 41 *years. As followeth.*

years.	li.	s.	d.	li.	s.	d.	years.
1	1	2	0	7	8	0	21
2	1	4	2	8	0	9	22
3	1	6	7	8	19	1	23
4	1	9	3	9	16	11	24
5	1	12	2	10	16	8	25
6	1	15	5	11	18	4	26
7	1	18	11	13	2	2	27
8	2	2	10	14	8	5	28
9	2	7	1	15	17	3	29
10	2	11	10	17	8	11	30
11	2	17	1	19	3	10	31
12	3	2	9	21	2	3	32
13	3	9	0	23	4	6	33
14	3	15	11	25	10	11	34
15	4	3	6	28	2	0	35
16	4	11	10	30	18	0	36
17	5	1	1	34	0	0	37
18	5	11	2	37	8	1	38
19	6	2	3	41	2	10	39
20	6	14	6	45	5	2	40

By

By the former Table, if you deſire to know what 1 li. cometh to with *intereſt*, and *intereſt* upon *intereſt* after 10 in the 100, for any *number* of years unto 40. look in the *row* or *margent* (over which is written *years*) and againſt it on the right hand cloſe unto it in the *row or margent of pounds, ſhillings, and pence*, (which is titled thus, li. s. d.) you ſhall finde your deſire.

Example.

I would know what 1 li. *with intereſt, and intereſt upon intereſt, cometh to in* 7 *years.*

I look in the *row* of yeares for the *number* 7. And againſt it on the right hand I finde 1 li. 18 s. 11 d. Alſo what it cometh unto in 13 years. I ſeeke among the years for 13, and againſt it I finde 3 li. 9 s.

Againe, for 21 yeares. I looke for 21 among the years, and I finde 7 li. 8 s. 0 d. But if you would know for a greater ſumme then 1 li. then multiply your ſumme by that ſumme of 1 li. in the Table for any of thoſe years, and you ſhall eaſily finde it. As thus, I would know what 10 li. cometh to for 7 years with intereſt, &c. I ſee that 1 li. cometh to 1 li. 18 s. 11 d. in that time. Then ſay I, that 10 li. muſt be ten times as much in that ſpace, which is 19 li. 9 s. 2 d. Alſo of 10 li. in 13 years, I ſee that 1 li. in that time cometh unto 3 l. 9 s. Then muſt 10 li. be ten times as much in that ſpace, which is 34 li. 10 s. Alſo what 10 li. cometh to in 21 yeares. I finde firſt, that 1 li. in that ſpace cometh to 7 li. 8 s. Then I ſay, 10 muſt be 10 times as much, which is 74 li. Laſtly, I would know what 100 li. cometh to in 7 years, I ſee it muſt be 100 times as much as 1 li. cometh to in that

that ſpace, which is 194 li. 11 s. 8 d. Hereby you ſee the common ſaying is not true, that 100 li. doth double it ſelfe in 7 years, for it wants thereof 5 li. 8 s. 4 d. But in 8 years 100 li. cometh to 210 li. 8 s. 4 d. which you ſee is more then double it ſelfe by 10 li. 8 s. 4 d. And in this ſort may any that can but caſt with Counters, or indeed by memory, finde the increaſe of any ſum whatſoever for any of the number of years in the foreſaid Table, after they have found what 1 li. cometh unto for that time, as before is ſpecified.

A Table ſhewing if 1 li. *annuity to endure for any number of years under* 41, *be all reſpited or forborne, untill the laſt payment grow due, and then all be received together, with intereſt, and intereſt upon intereſt after* 10 *in the* 100 per annum, *what they will amount unto by any of the ſaid number of years. As followeth.*

years.	l.	s.	d.	li.	s.	d.	years.
1	1	0	0	64	0	0	21
2	2	2	0	71	8	0	22
3	3	6	2	79	10	10	23
4	4	10	10	88	9	11	24
5	6	2	1	98	6	11	25
6	7	14	3	109	3	7	26
7	9	9	8	121	1	11	27
8	11	8	8	134	4	2	28
9	13	11	7	148	12	7	29
10	15	18	8	164	9	10	30
11	18	10	7	181	18	10	31
12	21	7	8	201	2	9	32
13	24	10	5	221	5	0	33
14	27	19	5	245	9	6	34
15	31	15	6	271	0	5	35
16	35	18	11	299	2	6	36
17	40	10	10	330	0	9	37
18	45	11	11	364	0	10	38
19	51	3	2	401	8	11	39
20	57	0	6	442	11	10	40

By this Table you may know what any Annuity being respited or forborn for any number of years unto 41. with interest upon interest after 10 in the 100, will come unto: first seeking in the *Table* what 1 li. will come unto in that time; and that being found, to multiply it by the summe you desire to know.

Example.

Example.

First, I would know what 1 li. *Annuity, being forborn or respited for* 14 *years, cometh unto.*

I look in this last Table (which is for the purpose) and I finde 27 li. 19 s. 5 d.

Againe, what 1 li. *Annuity* respited for 21 *years* cometh to, I looke in the said Table for 21 *years*, and I finde 64 li. Also the like for 1 li. for 30 *years* respited, I look, and finde it to be 164 li. 9 s. 10 d. as by the said Table may appeare. Now for greater *Annuities*, as 30 li. *per annum*, respited or forborne, what it amounteth to in 16 *years*, I seek first for 1 li. in this last *Table* before for 16 *years*, and against it I finde 35 li. 18 s. 11 d. Then say I, that 30 li. *per annum* being respited for that time, will come to 30 times as much, which is 1078 li. 7 s. 6 d. Also if there be an *Annuity* of 45 li. due and unpaid for 12 *years*, I look in the said Table what 1 li. cometh to, 12 *years* being respited, and I finde it is 21 li. 7 s. 8 d. Then I conclude that 5 li. must be 45 times as much, which is 962 li. 5 s.

Lastly, I have an Annuity of 50 li. per annum, *which hath been behind for* 16 years, *and must be answered unto me with interest, and interest upon interest, all at one payment, what shall or ought I to receive in all at the* 16 years *end.*

I seek what 1 li. comes unto in that time (as before taught) and I finde 35 li. 18 s. 11 d. Then must my 50 li. *per annum*, forborne for that time, come to 50 times as much, which is 1797 li. 5 s. 10 d. And thus may you finde any other *summe*, great or small, for any *number of years* contained in the aforesaid *Table*, without the helpe of *Arithmeticke*, if

you can but uſe your Counters, or by memory count well.

A Table ſhewing if 1 li. *Annuity (to endure for any number of years unto* 41.) *be to be ſold for preſent ready money, how much ought that ready money to be, reckoning* 10 per 100 per annum , *abating intereſt, and intereſt upon intereſt. As followeth.*

years.	l.	s.	d.	li.	s.	d.	years.
1	0	18	2	8	12	11	21
2	1	14	8	8	15	5	22
3	2	9	8	8	17	7	23
4	3	3	4	8	19	8	24
5	3	15	9	9	1	6	25
6	4	7	1	9	3	2	26
7	4	18	4	9	4	8	27
8	5	6	8	9	6	1	28
9	5	15	2	9	7	4	29
10	6	2	10	9	8	6	30
11	6	9	9	9	9	7	31
12	6	16	3	9	10	6	32
13	7	2	0	9	11	4	33
14	7	7	4	9	12	2	34
15	7	12	1	9	12	10	35
16	7	16	5	9	13	6	36
17	8	0	5	9	14	1	37
18	8	4	0	9	14	7	38
19	8	7	3	9	15	1	39
20	8	10	3	9	15	6	40

This

This Table before last specified is very necessary and commodious for all *Gentlemen*, or others, that shall have cause to buy or sell *Annuities*, or such like; for by this they shall know what they doe, whether they demand, or take too little or too much, after the rate of ten in the 100, by which proportion all these *Tables* are ruled.

As for example, I am to buy an Annuity of 16 li. per annum, *for* 12 *years, and am demanded for it ready money* 120 li. *I would know, if I give this rate, whether I give too much or too little, according to the proportion of ten in the* 100 per annum, &c.

I look in the *Table* last before what 1 li. is worth for 12 *years*, and I finde against 12 this *summe* 6 li. 16 s. 3 d. Now I say that 16 li. *Annuity* for that time, and after that proportion, cometh to 16 times as much, which is 109 li. So that I see the party demanded of me 11 li. too much after the rate of ten in the 100 *per annum*, and therefore I must draw him to a lower price, or leave it.

Again, I am offered an Annuity of 20 li. per annum *of* 14 *years for* 130 li. *I would know, if I give it, whether I give too much or too little, according to the proportion aforesaid.*

I seek first what 1 li. *Annuity* is worth for 14 *years*, and I finde in the said last *Table* 7 li. 7 s. 4d. Then say that the *Annuity* of 20 li. *per annum* will come to 20 times as much, and will be worth 147 li. 6 s. 8 d. according to the proportion before mentioned, and is more then his demand by 17 li. 6 s. 8 d. So that I see if I accept of it, I shall have a good bargaine. And thus may you know readily by looking in your Table, and finding what 1 li. is worth for

any time therein contained, how much any greater summe will come unto, if you multiply it by that summe of 1 li. as before is sufficiently shewed.

But suppose this, I have 300 li. *ready money, and would bestow the same for a valuable Annuity answerable thereunto according to the proportion aforesaid, I would know what Annuity to endure* 21 *years this* 300 li. *will buy.*

I look in the former Table what 1 li. Annuity will cost for that time, and I finde 8 li. 12 s. 11 d. Then I say by the *Rule of proportion*, If 8 li. 12 s. 11 d. will buy 1 li. Annuity for 21 years, what Annuity shall 300 li. buy or be worth for that time? I reduce the *sums* to the least denomination (which is pence) and I finde 34 li. 13 s. 11 d. And after this manner (by the helpe of this *Rule*) may you finde all other sums for any time contained in the foresaid last *Table.*

A

A Table ſhewing what 1 li. *in reverſion for any number of years under* 31 *is worth in ready money, the buyer ſtaying untill the thing be faln in hand.*

years.	li.	s.	d.	li.	s.	d.	years.
1	0	18	2	0	4	4	16
2	0	16	6	0	3	11	17
3	0	15	0	0	3	7	18
4	0	13	7	0	3	3	19
5	0	12	5	0	2	11	20
6	0	11	3	0	2	8	21
7	0	10	3	0	2	5	22
8	0	9	3	0	2	2	23
9	0	8	5	0	2	0	24
10	0	7	8	0	1	10	25
11	0	7	0	0	1	8	26
12	0	6	4	0	1	6	27
13	0	5	9	0	1	4	28
14	0	5	3	0	1	3	29
15	0	4	9	0	1	1	30

This laſt *Table* differeth, and is contrary to the other three before mentioned: For whereas the others increaſed more and more according to the *number of years* ſpecified, this doth grow and diminiſh leſſe and leſſe, as the *number of years* increaſeth As for example,

There

There is a Tenement, the fee-ſimple whereof after 7 years will be worth 40 li. *what am I to give for it in ready money now, ſtaying untill it fall in hand?*

To know this, I look in this laſt Table for 7 years, and againſt it I finde 10 s. 3 d. So that a thing that after 7 years will be worth 1 li. is worth now in ready money but 10 s. 3 d. Then ſay I, that the foreſaid Tenement (which after 7 years will be worth 40 li.) is now worth 40 times 10 s. 3 d. which is 20 li. 10 s.

Again, there is a Farm which after 9 years will be worth the Fee-ſimple 420 li. *what is it now worth in ready money, ſtaying untill it fall in hand?*

I look in the ſaid Table what 1 li. is worth in reverſion after 9 years, and I finde 8 s. 5 d. Then ſay I, that the Farm of 420 li. ſo long in Reverſion, will be now worth in ready money 420 times as much, which is 176 li. 15 s.

Laſtly, there is a Lordſhip to be ſold, the Fee-ſimple whereof after 14 *years will be worth* 7500 li. *I would know what the ſame is now worth in ready money for the Reverſion.*

I look in this laſt *Table* for 14 years, and againſt it I finde 5 s. 3 d. ſo much 1 li. is worth in reverſion after 14 years. Then ſay I, that 7500 li. is worth no more in reverſion for that time then 7500 times 5 s. 3 d. which is 1968 li. 15 s. And after this manner may you finde out any other ſumme whatſoever. And though ſome men of their own experience can aime (as they thinke) near enough the mark to ſerve their owne turnes: yet I dare undertake they ſhall never ſo exactly doe it, nor juſtifie what they doe, as if they did it by Art.

FINIS.

New *Tables of Interest* at 8 *per centum per annum*, exactly calculated for 30 years by *Robert Hartwell*, with necessary *questions* for the use of them.

The first Table expressing the increase of one pound principall, put out and forborn for any number of years under 31, *at* 8 per centum per annum.

years.	li.	s.	d.	q.	li.	s.	d.	q.	years.
1	1	1	7	0	3	8	6	0	16
2	1	3	3	3	3	14	0	0	17
3	1	5	2	1	3	19	6	0	18
4	1	7	2	2	4	6	3	3	19
5	1	9	4	2	4	13	2	2	20
6	1	11	8	3	5	0	8	0	21
7	1	14	3	1	5	8	8	3	22
8	1	17	0	0	5	17	0	0	23
9	1	19	11	3	6	6	9	3	24
10	2	3	2	0	6	16	1	2	25
11	2	6	7	2	7	7	11	0	26
12	2	10	4	1	7	19	9	0	27
13	2	14	4	2	8	12	6	2	28
14	2	18	8	3	9	6	4	0	29
15	3	3	5	1	10	1	3	0	30

The

The deſcription and uſe of the Tables *of Intereſt at* 8 per annum, *being profitable.*

The firſt of them.

THeſe Tables conſiſt of four *Columns*, in the firſt and fourth whereof is written over the head yeares, and under the firſt number of yeares deſcending from 1 to 15, likewiſe in the fourth the number of yeares deſcending from 16 to 30. and againſt every year in the ſecond Column, toward the right hand, the pounds, ſhillings, pence and farthings, which one pound, or 20 s. principall will amount unto, being put forth and forborne for the number of yeares ſet againſt it; (but the pounds, ſhillings, pence, &c. in the third Columne, belong to the yeares ſet in the laſt Columne.)

1 Example.

Let it be required what one pound, or 20 *ſhillings, being put forth and forborn for* 12 *years, ariſeth to at* 8 per centum per annum, *intereſt upon intereſt.*

Seek in the firſt Columne, under the title of yeares, for 12, the number of years propoſed in the queſtion, and right againſt it toward the right hand in the ſecond Column you ſhall find 2li—10s—4d—1q. which is the principall and increaſe thereof due for the time required.

2 Example.

If 100 li. *be put forth for* 17 *yeares according to the ſame intereſt, I demand what it will amount to in that time.*

Look in the Column under the title of years for 17, and right againſt it toward the left hand in the Table is found 3 li.—14 s.—0 d.—0 q. which is the

increaſe

increaſe of 1 li. by which you may thus gather the increaſe of 100 li. or any other *ſumme*; a hundred times 3 li. is 300 li. then 100 times 14 *ſhillings* is 70 li. both which added together doe make 370 li.——0 s.——0 d. which is the *increaſe* of 100 li. put forth and forborn 17 *years*, the ſolution to the queſtion.

li.	s.	d.	q.
300	0	0	0
70	0	0	0
370	0	0	0

3. Example.

Suppoſe 60 li. *be put forth for* 19 *years according to that rate, what will it increaſe to in that time?*

Seek 19 under the *title* of yeares, and againſt it toward the left hand is found 4 li.——6 s.——3 d.——3 q. now ſay 60 times 4 li. is 240 and 60 times 6 *ſhillings* is 360 *ſhillings*, or 18 li. and 60 times 3 d. is 180 d. or 15 *ſhillings*, and 60 times 3 *farthings* is 3 *ſhillings* 9 d. all added together make 258 li. 18 s. 9 d. the increaſe thereof demanded.

li.	s.	d.	q.
240	0	0	0
18	0	0	0
0	15	0	0
	3	9	0
258	18	9	0

The

The second Table *shewing what one pound annuity or yearly rent is worth at the end of any number of years under* 31, *being forborn*, *at* 8 per centum per annum.

years.	li.	s.	d.	q.	li.	s.	d.	q.	years.
1	1	0	0	0	30	6	5	3	16
2	2	1	7	0	33	15	0	0	17
3	3	4	6	0	37	9	0	0	18
4	4	10	1	1	41	8	11	0	19
5	5	12	2	3	45	15	2	3	20
6	7	6	8	0	50	8	5	2	21
7	8	18	5	1	55	9	1	2	22
8	10	12	8	3	60	17	10	1	23
9	12	9	9	0	66	15	3	2	24
10	14	9	8	3	73	2	1	1	25
11	16	12	10	3	79	19	1	0	26
12	18	19	0	2	87	7	0	0	27
13	21	9	10	3	95	6	1	2	28
14	24	4	3	2	103	19	3	3	29
15	27	3	0	2	113	5	7	3	30

The use of the second Table, *whose disposition is altogether like the former, according to the title thereof, being profitable.*

1 Example.

1 Example.

There is a Leaſe worth 28 li. per annum, *to endure* 14 *years. I demand what it will riſe unto at the end of thoſe years, being all forborne with the intereſt upon intereſt at the rate preſcribed in this* Table.

Looke in the third Table for 14 yeares, againſt which, toward the right hand, you ſhall finde 24 li.——4 ſ.——3 d.——2 q. Now multiply 28 li. by 24, there ariſeth 672li. then 28li. by 4s. yieldeth 112 s. or 5 li. 12 s.

Again, 28li. by 3d. produceth 84 d. or 7 s. Finally, 28. by 2 farthings yieldeth 56 farthings, or 1s.2d. Al which added together make 678li. os. 2d. to be received at the end of 14 years, the ſame rent or annuity being reſpited.

li.	s.	d.	q.
672	0	0	0
5	12	0	0
	7	0	0
	1	2	0
678	0	2	0

2 Example.

If 60 li. *yearly rent or annuity be forborn* 20 *years, I demand how much it will increaſe at the end of the ſaid term.*

In the Table I finde that 1 pound in 20 years will ariſe to 45li.—15s.——2d——3q. therefore 60li. in the like term will yield 60 times as much; which I will reckon thus, 60 times 45 li. is 2700 li. 60 times 15 s. is 900 s. or 45 l. 60 times

li.	ſ.	d.	q.
2700	0	0	0
45	0	0	0
	10	0	0
	3	9	0
2745	13	9	0

times 2 d. is 120 d. or 10 s. last of all, 60 times 3 q. is 180 farthings, or 3 s.—9 d. all which together amount unto 2745 li.—13 s.—9 d. the value thereof to be received at the end of the term.

3 Example.

The yearly rent of 6 li.—13 s.—4 d. *being behind and unpaid the space of* 7 *years, at the end of which term the Tenant is compelled to pay the same, with the interest thereof, according to the above-named rate; I demand what the payment ought to be.*

The increase of 1 li. *yearly rent* answering to 7 *years*, is 8 li. 18 s. 5 d. 1 q. which for 6 li. *rent* taken 6 times, ariseth to 53 li. 10 s. 7 d. 2 q. Now because 13 s. 4 d. is two third parts of 1 li. therefore I take $\frac{2}{3}$ of 8 li. 18 s. 5 d. 1 q. which is the increase of 1 li forborn for 7 *years*, that is 5 li. 18 s. 11 d. 1 q. which together make 59 li. 9 s. 7 d. 0 q. the *summe* to be received, as was required.

li.	s.	d.	q.
53	10	7	2
5	18	11	2
59	9	7	0

The

The third Table, declaring what one pound due at the end of any number of years under 31, *is worth ready money at* 8 per centum, per annum.

years.	li.	s.	d.	q.	li.	s.	d.	q.	years.
1	0	18	7	0	0	5	10	0	16
2	0	17	10	3	0	5	4	3	17
3	0	15	1	2	0	5	0	0	18
4	0	14	8	1	0	4	7	2	19
5	0	13	7	1	0	4	3	1	20
6	0	12	7	0	0	3	11	2	21
7	0	11	8	1	0	3	8	0	22
8	0	10	9	2	0	3	4	3	23
9	0	10	0	0	0	3	1	3	24
10	0	9	3	0	0	2	11	0	25
11	0	8	6	3	0	2	8	1	26
12	0	7	11	1	0	2	5	0	27
13	0	7	4	0	0	2	2	3	28
14	0	6	9	2	0	2	1	3	29
15	0	6	3	3	0	1	11	3	30

This third Table *is diſpoſed as the firſt , the uſe according to the Title thereof, being damageable.*

1. Example.

Suppoſe there is 750 li. *due to be paid at the end of 9 years , the Creditor would ſell this debt for preſent money, what ought that money to be at the rate deſcribed in the* Table.

Seek in this third Table for 9 years at the left ſide of the Table , and right againſt it toward the right hand you ſhall finde 10 ſhillings, which multiplyed or taken 750 times, yieldeth 7500 ſhillings , which is 375 li. the value of that debt in preſent money.

2. Example.

There is a Leaſe worth 500 li. *after the end of 7 years; what is it worth preſent money, according to the rate deſcribed in the Table, ſtaying till it fall ?*

I ſeek in the Table for the 7 years,and right againſt it I finde 11 s.—8 d. now I multiply 500 by 11 , it yieldeth 5500 ſhillings , or 275 li. then 500 times 8 d. maketh 4000 d. which is 16 li. 13 s. 4 d. which added together is 291 li. 13 s. 4 d. the value of the Leaſe to be paid before it fall in hand.

li.	ſ.	d.	q.
275	0	0	0
16	13	4	0
291	13	4	0

The

The fourth Table, *expressing what one pound yearly rent or annuity for any number of years not exceeding 30, is worth ready money at* 8 per centum per annum.

years.	li.	s.	d.	q.	li.	s.	d.	q.	years.
1	0	18	6	0	8	17	0	1	16
2	1	15	7	3	9	2	5	0	17
3	2	11	6	2	9	7	5	1	18
4	3	6	2	3	9	12	0	3	19
5	3	19	10	1	9	16	4	1	20
6	4	12	5	1	10	0	4	0	21
7	5	4	1	2	10	4	0	0	22
8	5	14	11	0	10	7	5	0	23
9	6	4	11	1	10	10	6	3	24
10	6	14	2	1	10	13	5	3	25
11	7	2	9	1	10	16	2	1	26
12	7	10	8	2	10	18	8	1	27
13	7	18	0	3	11	1	0	0	28
14	8	4	10	2	11	3	2	0	29
15	8	11	2	1	11	5	1	3	30

The fourth Table is disposed altogether as the former, and the use thereof in like sort being damageable.

1. Example.

There is an annuity or rent of 20 s. per annum *to endure* 25 *years, it is required what it is worth ready money.*

Look in the Table for 25 *years*, and right against it you shall finde 10 li. 13 s. 5 d. 3 q. which is the solution.

2. Example.

What is the Lease of certaine Land valued at 140 li. per annum, *to begin presently, and endure* 18 *years, worth ready money?*

Search in the *Table* for 18 *years*, the *term* named in the *question*, and right against it toward the left hand you shall find 9 li. 7 s. 5 d. 1 q. which expresseth that *one pound rent* to be bought for that *terme* is worth so much; therefore that summe 140 times is the value required. Now 140 times 9 li. is 1260, and 140 times 7 s. is 980 s. or 49 li. likewise 140 times 5 d. is 700 d. or 2 li. 18. 4 d. and 140 *farthings* is 2 s. 11 d. all which added together make 1312 li. 1 s. 3 d. for the value of the said *Lease* paying no *rent*.

li.	s.	d.
1260	0	0
49	0	0
2	18	4
	2	11
1312	1	3

3 Example.

3. Example.

A Lease taken for 21 *yeares at* 13 li. 6 s. 8 d. per annum, *which after* 5 *years expired, the Tenant is desirous to give a fine, and bring the rent down to* 8 li. per annum *for the rest of the term; the demand is, what fine is to be paid?*

Subtract 5 *years* from 21, the remain 16, is the time unexpired; likewise from the present *rent* abate 8 li. the rest will be 5 li. 6 s. 8 d. now the drift of the *question* is, what 5 li. 6 s. 8 d. *yearly rent* or *annuity* to endure 16 years is worth present money.

The value of 1 li. *rent* or *annuity* answering to 16 *years*, is 8 li. 17 s. 0 d. 1 q. Now 5 times 8 li. is 40 li. and 5 times 17 s. 4 li. 5 s. and 5 times one farthing, is 1 d. 1 q. and because 6 s. 8 d. is $\frac{1}{3}$ of 1 li. I take $\frac{1}{3}$ of 8 li. 17 s. 0 d. 1 q. which is 2 li. 19 s. 0 d. all which added together make 47 li. 4 s. 0 d. 1 q. which is the *fine* that ought to be paid to bring the *rent* to 8 li. *per annum*.

li.	s.	d.	q.
40	0	0	0
4	5	0	0
2	19	0	1
47	4	0	1

The fifth Table, declaring what yearly rent or annuity of one pound ready money will purchase for any number of years under 31, *at* 8 per centum per annum.

years.	li.	s.	d.	q.	li.	s.	d.	q.	years.
1	0	18	7	0	0	2	9	3	16
2	0	14	0	2	0	2	8	3	17
3	0	9	8	2	0	2	8	0	18
4	0	7	6	0	0	2	7	0	19
5	0	6	3	0	0	2	6	2	20
6	0	5	4	3	0	2	5	3	21
7	0	4	9	2	0	2	5	1	22
8	0	4	4	0	0	2	4	3	23
9	0	4	0	0	0	2	4	1	24
10	0	3	8	2	0	2	4	0	25
11	0	3	6	0	0	2	0	3	26
12	0	3	3	3	0	2	3	1	27
13	0	3	1	3	0	2	3	0	28
14	0	3	0	1	0	2	2	3	29
15	0	2	11	0	0	2	2	2	30

In

In the fifth Table *the* Numbers *and* Columns *are all diſpoſed as the former* Tables, *and needeth no further explanation but onely Examples.*

1. Example.

The Table *declareth at firſt ſight what yearly rent or annuity one Pound ready money will purchaſe for any terme in the* Table *expreſſed.*

But if the *ready money* be above one pound, then if any *value* or *rent* ſet down in this Table be multiplyed by the *number* belonging to the *years in queſtion*, the *product* will ſhew what *yearly rent* or *annuity* that *ready money* will purchaſe for the *time propoſed.*

2. Example.

A certain man hath 750 li. *to purchaſe an Annuity to endure* 27 *years, ſo as it may yield him the like profit as if it were put out according to the rate in the* Table *expreſſed, it is required what that annuity ought to be.*

Becauſe the *annuity* is to endure 27 yeares, ſeek out the value or rent ſet againſt 27 years, in this fifth Table, which is 2s.—3d.—1q. now this being the *Annuity* which 20 s. ready *money* will purchaſe for that *terme*, it muſt be multiplyed by 750 li. as followeth: becauſe 2 s. is the tenth part of 20 s. therefore take the tenth part of 750 li. which is 75 li.

li.	s.	d.	q.
75	0	0	0
9	7	6	0
	15	7	2
85	3	1	2

which set first down, then 750 times 3d. is 9li. 7s. 6d. which set under the former, last of all 750 *farthings* is 15 s. 7 d. 2 q. All which added together, produce 85 li. 3 s. 1 d. 2 q. the *yearly Annuity* required.

Deo soli laus, omnis honor & gloria tribuatur.
AMEN.

FINIS.

Compendious Tables *of Interest-money forborn any number of Dayes, Weeks, Months, or Years, under 22, exactly calculated at* 6 l. per Cent. per Annum.

Dayes.	Numbers.	Yeares.	Numbers.
I	100016.	I	106000.
II	100032.	II	112360.
III	100048.	III	119102.
IV	100064.	IV	126248.
V	100080.	V	133822.
VI	100096.	VI	141852.
Weeks.		VII	150363.
I	100112.	VIII	159385.
II	100224.	IX	168948.
III	100336.	X	179085.
		XI	189830.
Months.		XII	201220.
I	100487.	XIII	213293.
II	100976.	XIV	226090.
III	101467.	XV	239656.
IV	101961.	XVI	254035.
V	102458.	XVII	269277.
VI	102956.	XVIII	285434.
VII	103457.	XIX	302560.
VIII	103961.	XX	320714.
IX	104467.	XXI	339956.
X	104976.		
XI	105486.		

The

The uſe of this Table *of proportionall numbers; the* Radius 100000.

This Table is divided into 4 columns, and the firſt in 3 parts aſcending from 1 day unto 6. ſecondly, from 1 week to three, thirdly, from one month proceeding to 11. the third columne comprehends the years incluſive from 1 to 21. as by their numerall letters doe appeare, the ſecond and fourth columnes are proportionall numbers, in arithmetical characters, reſpectively anſwering the time or times compounded of Dayes, Weekes, Months and Years to 21, as by examples ſhall be evidenc'd.

An explanation.

What will 50 L. amount unto, if forborn 21 years, the Intereſt allow'd at 6 L. *per Cent. per An.*

Look in the columne of yeares for the terme of forbearance propounded, in this 21, whoſe *Decimall* is 339956, which multiplyed by the principall forborne, *viz.* 50 L. the product will be 169|97800, to be divided by the *Radius* 100000, therefore cut off 5 places (as in the margent) from the right hand, and

		339956
		50
L	169	97800
S	19	56000
		112000
D	6	72000
Q	2	88|000
		100

there will appeare 169 L. the remainder 97800 multiply by 20 s. or by 2, and annex a cipher at the right hand, the product is 19|56000, cut off 5 places as before; on the left hand you will finde 19 S. the remainder 56000, which increas'd by 12 D. produ-

ceth

ceth 6|72000, ſtrike off 5 places, you will diſcover the 6. remainder 72000 multiplyed by 4 Q. and worke as before, you ſhall finde $2\frac{22}{25}$ Q. ſo 50 L. forborne 21 yeares at 6 L. *per Cent. per An.* will amount unto the ſumme of 169 L. 19 S. 6 D. $2\frac{22}{25}$ Q. the demand ſolv'd.

If a queſtion conſiſts of ſeveral denominations, *viz.* Pounds, Shillings, Pence, ſee the rules of Practice in the propoſitions of Intereſt; if mixt, as in reſpect of time, *viz.* Moneths, Weeks, Dayes, &c. finde firſt the increaſe for the terme of yeares, and multiply all that by the proportionall number found in the former Table, for the parts of a yeare the reſult will anſwer gripple expectation: for Decimal Tables, (or what theſe Authors have not treated of) I referre the Reader unto my Books of *Natural* and *Artificial* Arithmeticke, or my Scales of Commerce and Trade, this place being not convenient to enlarge my ſelf, or build upon anothers ground, neither will I leſſen their works (by me now corrected) whereby to magnifie my owne, but have inſerted this Table, and here I ſubſcribe my name, hoping my labours may give you eaſe, I reſt,

Your friend, although unknown,

Thomas Willsford.

To every young Arithmetician, or Practicioner in Numbers, who ſhall peruſe theſe Bookes.

Candid Lector,

You will here receive an old Arithmeticke from the authority of Record, entail'd upon the People, ratified and ſign'd by the approbation of Time, with multitudes of ſurviving Witneſſes; ſo expect not from me to confer Encomiums, when I had written nothing here, but that through miſtakes and overſights of former Correctors, Errors have appear'd like infirmities incident to decrepit Age, involv'd within the ſheets, as if prepar'd for a Funeral, the Authors Senſes departed, or in a trance, confuſed and ambiguous, their names remaining like inſcriptions on a Tombe, where corruption of one have demonſtrated generations of others.

Some places I found obliterated, other parts diſlocated, or falſe numbers have uſurp'd

their

their roomes, and those established by sundry impressions, may animate some (ignorantly) to plead Prescription for them, although multiplyed into a numerous and adulterous offspring, which deterred many young beginners from their progresse in these Numbers, and invoked me to their assistance in the restitution of the Authors: notwithstanding I have published some Bookes of this subject already, differing in forme, the scope I aimed at being both Speculation and Practice, my intentions dedicating that and this to the Publick good, whereby purblinde Suspicion, or fond affection (the Parents of Partiality) are expulsed and vanished, and I elected as an impartiall Corrector of this Treatise, where many of the Tables *and* Rules *of direction were direct divided from the first Composers meaning, the grounds of Truth, or wayes of Art; some of which deviations I have rectified, subtracted others, and totally cancelled many, adding numbers in their places accommodated and reduced to the Authors sense, the Questions stated, and the pristin Copies; usher'd from the Presse againe by Mr.* Mellys *attended by* Hartwell, *and all these expose to publicke view, drest (without disguise) in their old attires; otherwise it would seem absurd, as to see grave Antiquity vested in* French *habit.*

Humanum

Humanum eſt errare; *ſo I will not promiſe that all the old errours are corrected (although above* 1000) *nor yet engage the Preſs ſhall commence no new: but ſo faithfully as I could, theſe Authors recovered are here preſented to your view; which is all that was required of me, and from you (Courteous* Reader) *a friendly acceptance (in recompence of my labours) deſiring to be number'd amongſt the Coadjutors of my Country-men; in teſtimony whereof I here ſubſcribe my name,*

Thomas Willsford.

FINIS.

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